SYDNEY GIRLS HIGH SCHOOL

MATHEMATICS

Ext 1 (Ch 11,12,13,14,16)

Year 11 September 2001

Question 1

(a) The point P divides the interval AB in the ratio 3:2.
Given A(3,7) and B(-5,3) find the co-ordinates of P.

(b) Solve: $\frac{2}{x-1} < 1$

(c) A box contains 8 juggling balls: 3 red, 5 green. Kate takes two at random.

What is the probability:

(i) they are both red?

(ii) they are the same colour?

(iii) they are different colours?

Question 2

(a) Differentiate:

(i) $y = \frac{x+5}{3}$

(ii) $y = 3x^3 - 5x + 6$

(iii) $y = (24 - x^2)^{-\frac{1}{2}}$

(iv) $y = \frac{x+2}{5-x}$

(b) Find the equation of the tangent to $y = x - 2x^{-1}$ at the point (-1,1).

Question 3

(a) The curve of $y = x^2(x-3)$ is restricted to the domain $-3 \le x \le 4$.

(i) Find the stationary points and determine their nature.

(ii) Find the point of inflection.

(iii) Find where the curve cuts the x axis.

(iv) Sketch the curve showing all relevant features.

(v) What are the maximum and minimum values of the function over the specified domain?

(b) A large sweet box contains 'Caramello Koalas' and plain chocolate 'Koalas' in the ratio 3 : 2 . Three are selected at random.

What is the probability of at least one plain chocolate Koala?

QUESTION 5

a) From the top of a cliff 130 metres high the angle of depression to a boat at sea is 30°. A short time later, the angle of depression is 38°. **5**

i) Draw a clear diagram showing all the given information.

ii) Find how far the boat has moved.

b) Given $H(x) = x^2 - 3x + 2$, evaluate H(a+1) **3**

c) Determine whether the function $f(x) = x^3 - \frac{1}{x}$ is odd, even, or neither, giving reasons. **2**

QUESTION 6

a) In the Cartesian plane, A is the point with coordinates (1,5), B has coordinates (7, -1), and O is the origin. **8**

The interval OP is drawn so that it is perpendicular to AB, intersecting it at P.

Find

i) the equation of AB

ii) the gradient of OP, and hence the equation of OP

iii) the coordinates of P

iv) the length of interval AP

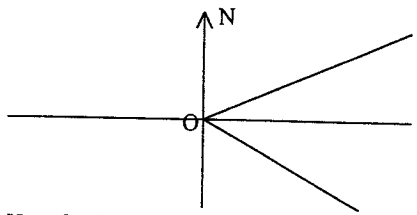
b) Find the value of k so that the lines $3x + y - 4 = 0$ and $2x - ky - 3 = 0$ intersect on the y-axis. **2**

QUESTION 7

a) Find the perpendicular distance from the point A(-2, 3) to the line $4x + 3y + 9 = 0$ **4**

Hence, or otherwise, find the equation of the circle with centre A which has the line $4x + 3y + 9 = 0$ as a tangent.

b) Joseph and Maria depart from the same location O. Joseph drives in a direction 050° at a speed of 80 km/h. Maria leaves 15 minutes after Joseph but travels in a direction SE of O at a speed of 100 km/h. **6**



i) How far does Joseph travel in 45 minutes?

ii) Copy and complete the diagram showing the position of Joseph and Maria, 30 minutes after Maria's departure.

iii) Hence, find the distance between them at this time. (to 2 significant figures)

iv) What is the bearing of Joseph, as seen from Maria?

QUESTION 8

a) Find the equation of the line through the point of intersection of $x + 3y - 4 = 0$ and $3x - 4y + 1 = 0$, which also passes through the point (3,-1). **3**

b) The base of a triangle is 5 m longer than the altitude. If the area of the triangle is 52 m², find the altitude. (Hint: let the altitude be h metres) **3**

c) Prove that **4**

$$\frac{(\cos\theta\cot\theta - \sin\theta\tan\theta)\sin\theta\cos\theta}{\cos\theta - \sin\theta} = 1 + \sin\theta\cos\theta$$

---------- END OF EXAM----------

Year 11 Mathematics September 2001

Q1.

a) $2.40701... \div 2.41$

b) i) $4y^2 - 10yt$
$= 2y(2y - 5t)$

ii) $9x^2 - 4p^2$
$= (3x+2p)(3x-2p)$

c) $\frac{x}{3} + \frac{3x-1}{2}$
$= \frac{2x + 9x - 3}{6}$
$= \frac{11x - 3}{6}$

d) i) $\sqrt{15} \times 2\sqrt{5}$
$= \sqrt{3} \times \sqrt{5} \times 2\sqrt{5}$
$= 2 \times 5\sqrt{3}$
$= 10\sqrt{3}$

ii) $3\sqrt{18} + 4\sqrt{12} - 2\sqrt{108}$
$= 3\sqrt{9 \times 2} + 4\sqrt{4 \times 3} - 2\sqrt{36 \times 3}$
$= 9\sqrt{2} + 8\sqrt{3} - 12\sqrt{3}$
$= 8\sqrt{3} - 3\sqrt{2}$

Q2.

a) $x = 180 - 71$
$= 109$

b) i) $\cos 135^\circ = -\cos 45^\circ$
$= -\frac{1}{\sqrt{2}}$

ii) $\tan(-60^\circ) = -\tan 60^\circ$
$= -\sqrt{3}$

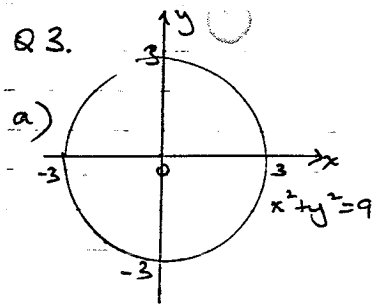
iii) $\sin 570^\circ = \sin 210^\circ$
$= -\sin 30^\circ$
$= -\frac{1}{2}$

c) i) $x^2 = 6x - 5$
$x^2 - 6x + 5 = 0$
$(x-5)(x-1) = 0$
$x = 5, 1$

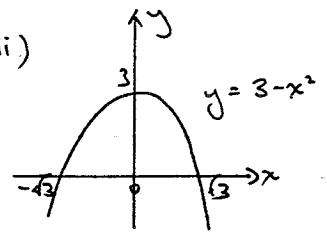
ii) $4 - 3x \geq 13$
$-3x \geq 9$
$x \leq -3$

Q3.

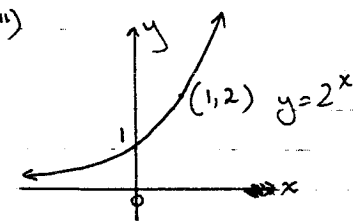
a)



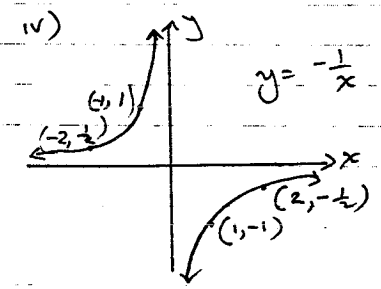
ii)



iii)



iv)

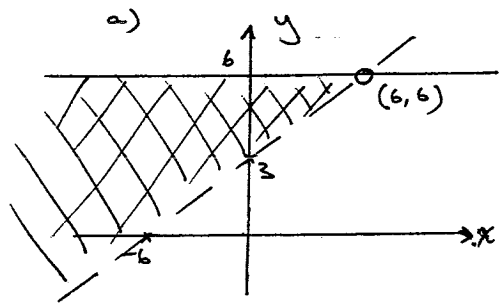


b) $\angle$ sum $= (8-2) \times 180^\circ$
$= 6 \times 180^\circ$

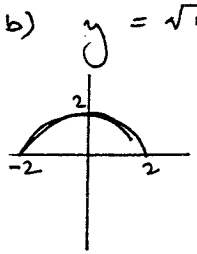
each $\angle = \frac{6 \times 180^\circ}{8}$
$= 135^\circ$

Q4.

a)



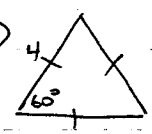
b) $y = \sqrt{4 - x^2}$



domain: $-2 \leq x \leq 2$

range: $0 \leq y \leq 2$

c)



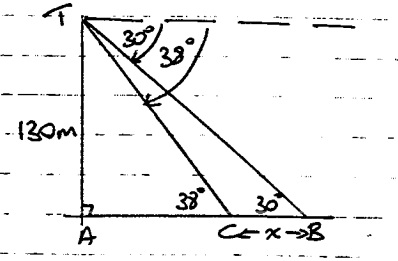
$A = \frac{1}{2}ab\sin C$
$= \frac{1}{2} 4 \times 4 \sin 60^\circ$
$= 8 \times \frac{\sqrt{3}}{2}$

area $= 4\sqrt{3}$ cm^2

d) $\frac{\sec\theta}{\cos\theta} - \frac{\tan\theta}{\cot\theta}$
$= \frac{1}{\cos\theta \cdot \cos\theta} - \tan\theta \cdot \tan\theta$
$= \sec^2\theta - \tan^2\theta$
$= 1$

(from $\sin^2\theta + \cos^2\theta = 1$
$\tan^2\theta + 1 = \sec^2\theta$
$\therefore 1 = \sec^2\theta - \tan^2\theta$)

Q5



In $\triangle TAB$

$$\tan 30° = \frac{130}{AB}$$

$$\therefore AB = \frac{130}{\tan 30°}$$

In $\triangle TAC$

$$\tan 38° = \frac{130}{AC}$$

$$\therefore AC = \frac{130}{\tan 38°}$$

$$\therefore BC = AB - AC$$

$$= \frac{130}{\tan 30°} - \frac{130}{\tan 38°}$$

$$= 58.77$$

$\therefore$ distance $= 58.7$ m

b) $H(x) = x^2 - 3x + 2$

$$H(a+1) = (a+1)^2 - 3(a+1) + 2$$

$$= a^2 + 2a + 1 - 3a - 3 + 2$$

$$= a^2 - a$$

c) $f(x) = x^3 - \frac{1}{x}$

$$f(-x) = (-x)^3 - \frac{1}{-x}$$

$$= -x^3 + \frac{1}{x}$$

$$= -\left(x^3 - \frac{1}{x}\right)$$

$-f(x) = f(-x)$ $\therefore$ odd

Q6

A (1,5) B (7,−1)

OP ⊥ AB

i) $m(AB) = \frac{5 - -1}{1 - 7} = \frac{6}{-6} = -1$

$y - 5 = -1(x - 1)$

$y = -x + 1 + 5$

$y = -x + 6$ ①

ii) OP ⊥ AB

$\therefore m(OP) = 1$

$y - 0 = 1(x - 0)$

$y = x$ ②

iii) AB intersects OP

solve ① + ② simult.

$y = -x + 6$

$y = x$

$\therefore x = -x + 6$

$2x = 6$

$x = 3$

$\therefore P(3, 3)$

iv) $AP = \sqrt{(3-1)^2 + (3-5)^2}$

$= \sqrt{4 + 4}$

$= \sqrt{8}$

$AP = 2\sqrt{2}$ units

b) $3x + ky - 4 = 0$

y-int $= 4$

$2x - ky - 3 = 0$

y-int $= \frac{-3}{k}$

$\therefore 4 = \frac{-3}{k}$

$k = -\frac{3}{4}$

Q7.

$4x + 3y + 9 = 0$

$a = 4, b = 3, c = 9$

$(-2, 3) = (x_1, y_1)$

$$d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

$$= \left| \frac{-8 + 9 + 9}{\sqrt{4^2 + 3^2}} \right|$$

$$d = \left| \frac{10}{5} \right| = 2$$

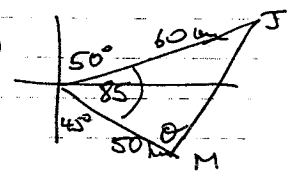
d = r for tangent: d = radius

circle:

$(x + 2)^2 + (y - 3)^2 = 2^2$

i.e. $(x + 2)^2 + (y - 3)^2 = 4$

b)



i) 45 min: $\frac{3}{4} \times 80 = 60$

distance = 60 km

iii) $JM^2 = 50^2 + 60^2 - 2(50)(60)\cos 85°$

$JM^2 = 5577.065...$

$JM = 74.679...$

distance $\doteq$ 75 km.

iv) $\frac{75}{\sin 85°} = \frac{60}{\sin\theta}$

$\therefore \sin\theta = \frac{60 \sin 85°}{75} = 0.796...$

$\theta \doteq 53°$ $\therefore$ bearing is 008°

Q8.

a) $(x + 3y - 4) + k(3x - 4y + 1) = 0$

subst (3,7)

$(3 - 3 - 4) + k(9 + 4 + 1) = 0$

$-4 + 14k = 0$

$\therefore k = \frac{2}{7}$

$(x + 3y - 4) + \frac{2}{7}(3x - 4y + 1) = 0$

$7x + 21y - 28 + 6x - 8y + 2 = 0$

$13x + 13y - 26 = 0$

$x + y - 2 = 0$

(OR pt. of intes. (1,1) with $m = -1$)

b)

$\frac{1}{2} h(h+5) = 52$

$h(h+5) = 104$

$\therefore h^2 + 5h - 104 = 0$

$(h + 13)(h - 8) = 0$

$\therefore h = -13, 8$

but $h > 0$, $\therefore$ height = 8m

c) LHS =

$$\frac{\left(\frac{\cos\theta \cdot \cos\theta}{\sin\theta} - \frac{\sin\theta \cdot \sin\theta}{\cos\theta}\right)\sin\theta\cos\theta}{\cos\theta - \sin\theta}$$

$$= \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$$

$$= \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin\theta\cos\theta + \sin^2\theta)}{\cos\theta - \sin\theta}$$

$= 1 + \sin\theta\cos\theta$

$=$ RHS

NB $(\sin^2\theta + \cos^2\theta = 1)$