SYDNEY GIRLS HIGH SCHOOL

3U ASSESSMENT TASK SEPTEMBER 1998

MATHEMATICS

YEAR ELEVEN

INSTRUCTIONS

- There are five questions.
- Questions are of equal value.
- Show all working.
- Marks will be deducted for careless or badly arranged work.
- Time Allowed: 75 minutes
- Start each question on a new page
- Write on one side of the paper only
- Diagrams are not to scale

Name [redacted]

Class 11M3

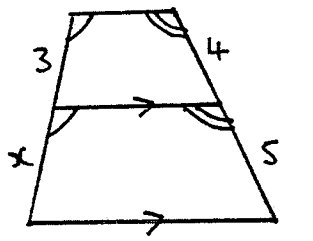
Question (1)

(a) i) Factorise $9a^2 - 25b^2$

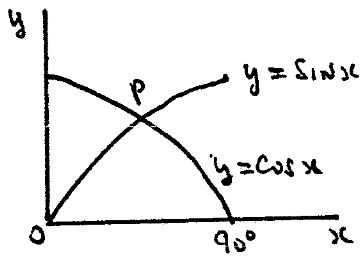
ii) Simplify $\dfrac{(3x)^{-1}}{2x}$

iii) Solve for x ; $\dfrac{a + bx}{b + ax} = c$

(b) i) Find the value of x (give a reason)



ii) Find the exact coordinates of P the point of intersection



iii) Simplify $\dfrac{3}{4} - \dfrac{2 - x}{5}$

(c) i) What is the domain and range of the following function $y = \sqrt{4 - x^2}$

ii) Find the value of x if $\sqrt{18} - \sqrt{2} = \sqrt{x}$

iii) Simplify $(\sqrt{3} - 2\sqrt{3})^2$

Question (2)

(a) A curve has equation $y = -x^3 + 4x^2 - 4x$

i) find where the curve cuts the x-axis

ii) find any stationary points

iii) sketch the curve for $-1 \leqslant x \leqslant 3$ clearly showing all relevant points

iv) by observation, is the function odd, even or neither

(b) Explain why the relation $x^2 + y^2 = 7$ is not a function

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(c) In a class of 25 students 4 study both Maths and Geography, 10 study Maths only and 9 study neither Maths nor Geography. If a student is selected at random, find the probability of the event

i) the student studies Maths only
ii) the student studies Geography only
iii) the student does not study Maths.

(d) Two students, Katrina and Natalia attempt a mathematical problem. Katrina has a 70% chance and Natalia a 60% chance of solving the problem Find the probability

i) both solve the problem
ii) only Natalia solves the problem
iii) Natalia solves the problem
iv) neither solves the problem

Question (4)

(a) Find $\frac{dy}{dx}$ if

i) $y = x^N + x^{3N}$

ii) $y = \sqrt{5 - 4x}$

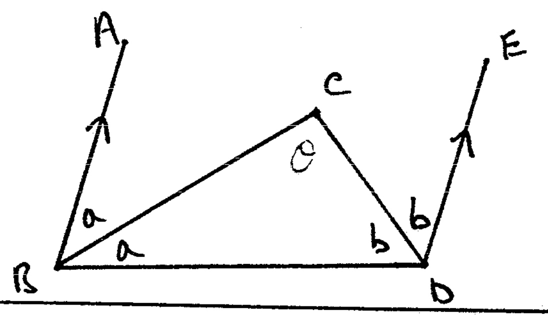
iii) $y = \frac{x^2}{x - 2}$

iv) $y = 4x - \frac{3}{x^2}$

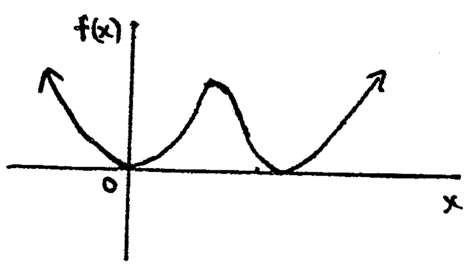
v) $y = 2x(x^2 + 4)^4$

(b) The normal to the curve $y = x^2 + 3x + 1$ at the point $(-1,-1)$ cuts the curve again at A. Find the coordinates of A.

(c) Find $< BCD$ giving reasons



(d) Copy the graph of the following function On a separate number plane sketch the graph of $f'(x)$



Queston (5)

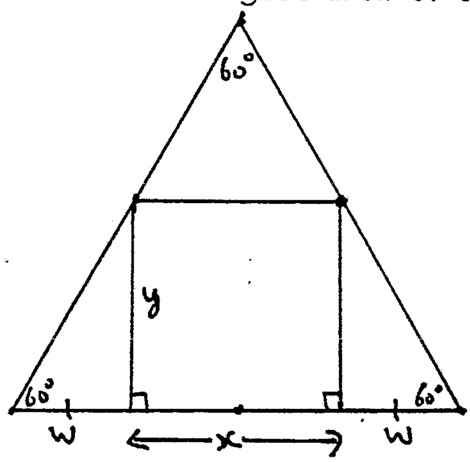
(a) The tangent to the curve $y = ax^2 + bx$ at the point (1,5), is parallel to the line $y = x - 2$. Find the values of a and b.

(b) A rectangle is inscribed in an equilateral triangle with sides 10cm as shown.

i) show that $y = w\sqrt{3}$

ii) show that the area of the rectangle is given by $A = 10\sqrt{3}\,w - 2\sqrt{3}\,w^2$

iii) hence find the largest area of the rectangle



(c) The angle of elevation of a hill OA, at the point P due south of the hill, is 38°. From Q, due west of P, the angle of elevation is 25°. If PQ = 4 km, find the height, h, of the hill to the nearest metre

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3 Unit Y 11
1998 Yearly. TOTAL (15)

Yr 11 3u Y '98

Question 1

(a) i

$9a^2 - 25b^2$

$= (3a-5b)(3a+5b)$ (2)

ii

$\frac{(3x)^{-1}}{2x}$

$= \frac{1}{(3x)(2x)}$

$= \frac{1}{6x^2}$ (1)

iii

$\frac{a+bx}{b+ax} = c$

$a+bx = c(b+ax)$

$a+bx = cb+cax$

$(b-ca)x = cb-a$

$\therefore x = \frac{(cb-a)}{(b-ca)}$ (2)

(b) i

$\frac{x}{3} = \frac{5}{4}$ (2)

$x = 3 \times \frac{5}{4}$

$= \frac{15}{4}$ (or $3\frac{3}{4}$)

Equal intercepts / ratios } between parallel lines

ii (y=) $\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$ (2)

$\tan x = 1$

$\therefore x = 45^\circ, \ y = 1/\sqrt{2}$ (.707)

(a) iii

$\frac{3}{4} - \frac{(2-x)}{5}$

$= \frac{15-4(2-x)}{20}$

$= \frac{15-8+4x}{20}$ (1)

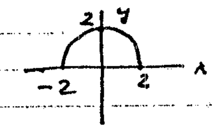
$= \frac{7+4x}{20}$

(c) i $y = \sqrt{4-x^2}$

$4-x^2 \geq 0$

$(2-x)(2+x) \geq 0$

$\therefore -2 \leq x \leq 2$ Dom (1)



$\therefore 0 \leq y \leq 2$ (1) Range

ii

$\sqrt{18} - \sqrt{2} = \sqrt{x}$

$3\sqrt{2} - \sqrt{2} = \sqrt{x}$

$2\sqrt{2} = \sqrt{x}$

$\sqrt{8} = \sqrt{x}$

$\therefore x = 8$

iii $(\sqrt{3} - 2\sqrt{3})^2$

$= 3 - 2(\sqrt{3})(2\sqrt{3}) + 4\times 3$

$= 3 - 12 + 12$

$= 3$ (1)

Q2

a. $y = -x^3+4x^2-4x$

at x axis $y=0$

$-x(x^2-4x+4) = 0$

$-x(x-2)(x-2) = 0$ (1)

$\therefore x = 0$ or 2. (1)

ii) $\frac{dy}{dx} = -3x^2+8x-4$

for stat pts $\frac{dy}{dx} = 0$ (1)

$-(3x^2-8x+4) = 0$

$-(3x-2)(x-2) = 0$

$x = 2$ or $\frac{2}{3}$

$\therefore$ st. pts $(2,0)$ + $(\frac{2}{3}, \frac{-32}{27})$ (1)

test (2,0)

x	2^-	2	2^+
$\frac{dy}{dx}$	+ve	0	-ve

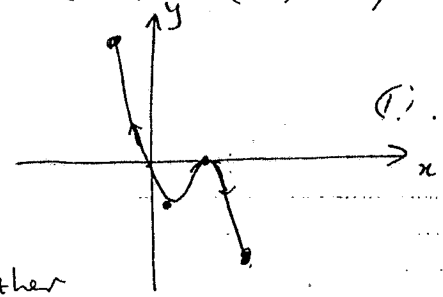
$\therefore$ max

½ test $(\frac{2}{3}, -\frac{32}{27})$

$\frac{2}{3}^-$	$\frac{2}{3}$	$\frac{2}{3}^+$
-ve	0	+ve

½ $\therefore$ min

iii)



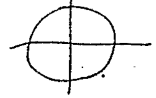
$x = -1 \quad y = 9$

$x = 3 \quad y = -3$

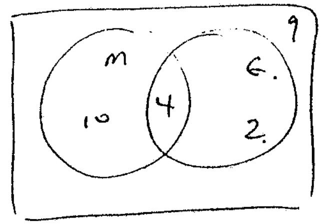
(1)

(1) iv) neither

b) $x^2+y^2=7$ is not a function as for each x there are 2 y values (1)



c)

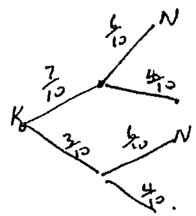


i) $\frac{10}{25} = \frac{2}{5}$ (1)

ii) $\frac{2}{25}$ (1)

iii) $\frac{11}{25}$ (1)

d)



(1) i) $\frac{7}{10} \times \frac{6}{10} = \frac{42}{100} = \frac{21}{50}$ 0.42

(1) ii) $\frac{3}{10} \times \frac{6}{10} = \frac{18}{100} = \frac{9}{50}$ 0.18

(1) iii) $\frac{7}{10} \times \frac{6}{10} + \frac{3}{10} \times \frac{6}{10} = \frac{60}{100} = \frac{3}{5}$

(1) iv) $\frac{3}{10} \times \frac{4}{10} = \frac{12}{100} = \frac{3}{25}$

Yr 11 3U Yrly SGHS 1998

Q 3.

a) i) $P(A) = P(B) = \frac{1}{2}$

$P(A, Lab, Lab) + P(B, Lab, Lab)$

$= 0.5 \times 0.25 \times 0.25 + 0.5 \times 0.65 \times 0.65$

$= 0.2425 \quad \left(\frac{97}{400}\right)$ ①

ii) $P(A, Lib, Lab) + P(B, Lib, Lab)$

$= 0.5 \times 0.6 \times 0.25 + 0.5 \times 0.25 \times 0.65$

$= 0.15625 \quad \left(\frac{5}{32}\right)$ ①

iii) $P(A, Any, I.N.) + P(B, Any, I.N)$

$= 0.5 \times 1 \times 0.15 + 0.5 \times 1 \times 0.1$

$= 0.125 \quad \left(\frac{1}{8}\right)$ ①

b) i) $P(WWW) = \frac{5}{20} \times \frac{4}{19} \times \frac{3}{18}$

$= \frac{60}{6840}$

$= \frac{1}{114}$ ①

ii) $P(\tilde{W}\tilde{W}\tilde{W}) = \frac{15}{20} \times \frac{14}{19} \times \frac{13}{18}$

$= \frac{2730}{6840}$

$= \frac{91}{228}$ ①

iii) P(at least one prize)

$= 1 - P(\text{no prizes})$

$= 1 - \frac{91}{228}$

$= \frac{137}{228}$ ①

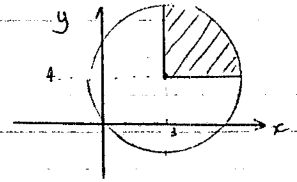
iv) $P(W\tilde{W}\tilde{W}) + P(\tilde{W}W\tilde{W}) + P(\tilde{W}\tilde{W}W)$

$= 3\left(\frac{5}{20} \times \frac{15}{19} \times \frac{14}{18}\right)$

$= \frac{3150}{6840}$

$= \frac{35}{76}$ ①

c)



②

$(x-3)^2 + (y-4)^2 \leq 25$

$x \geq 3$

$y \geq 4$ ①

d) $5AP : 3PB$ [also see alternate locus solution]

$\frac{5AP}{PB} = 3$

$\frac{AP}{PB} = \frac{3}{5}$

$AP : PB = 3 : 5$

$k_1 : k_2$

$x_1 = 1, y_1 = 2, x_2 = 7, y_2 = 4$

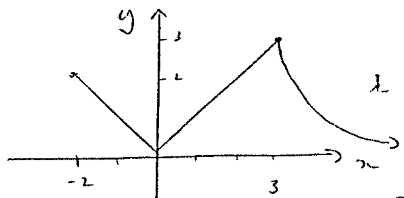
$x = \frac{k_1 x_2 + k_2 x_1}{k_1 + k_2}, \quad y = \_\_\_\_$

$x = \frac{3(7) + 5(1)}{3+5} \qquad y = \frac{3(4) + 5(2)}{3+5}$

$= \frac{26}{8}, \qquad y = \frac{22}{8}$

$= 3\frac{1}{4}, \qquad = 2\frac{3}{4}$ ②

e)



②

$R: 0 \leq y \leq 3$ 1.

YR 11

3UNIT

Q 4

a) i) $\frac{d}{dx}\left(x^N + x^{3N}\right) = Nx^{N-1} + 3Nx^{3N-1}$ ①

ii) $\frac{d}{dx}(5-4x)^{\frac{1}{2}} = \frac{-2}{\sqrt{5-4x}}$ ②

iii) $\frac{d}{dx}\left(\frac{x^2}{x-2}\right) = \frac{x^2 - 4x}{(x-2)^2}$ ②

iv) $\frac{d}{dx}\left(4x - 3x^{-2}\right) = 4 + \frac{6}{x^3}$ ①

v) $\frac{d}{dx} 2x(x^2+4)^4 = 2(x^2+4)^3(9x^2+4)$ ②

(accepted: $16x^2(x^2+4)^3 + 2(x^2+4)^4$)

b) $y = x^2 + 3x + 1$

$\frac{dy}{dx} = 2x + 3$

at $x = -1$, $\frac{dy}{dx} = 1$

$\therefore$ grad. of normal $= -1$

eqn. of normal

$y + 1 = -1(x+1)$

$y = -x - 2$ ②

find A.

$-x - 2 = x^2 + 3x + 1$

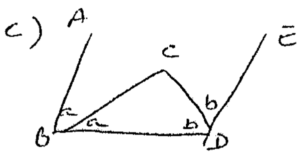
$x^2 + 4x + 3 = 0$

$(x+3)(x+1) = 0$

$\therefore x = -3, -1$ ②

$A(-3, 1)$

c)



$2a + 2b = 180°$ (coint ∠s, AB ∥ …)

$\therefore a + b = 90°$

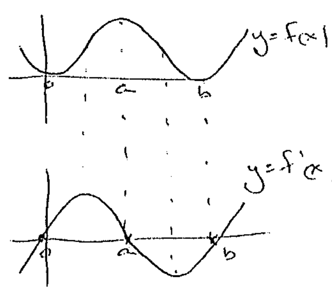
In $\triangle BCD$, $a + b + \angle BCD = 180°$

(∠ sum of △)

$\angle BCD = 180 - 90$

$= 90°$ ②

d)



①

Question Five

a) $y = ax^2 + bx$

$\frac{dy}{dx} = 2ax + b$

at $(1,5)$ $\frac{dy}{dx} = 1$

$\therefore 1 = 2a + b \quad -①$

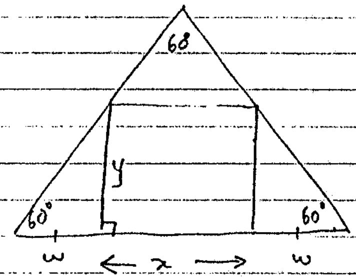
$5 = a + b \quad -②$

$1 = 2a + b$

$\left.\begin{array}{l} a = -4 \\ b = 9 \end{array}\right\}$

$\boxed{\therefore y = -4x^2 + 9x}$

b)



(i) $\tan 60° = \frac{y}{w}$

$\therefore \sqrt{3} = \frac{y}{w} \Rightarrow y = w\sqrt{3}$

(ii) Area of rectangle is xy.

But $x = 10 - 2w$ and $y = w\sqrt{3}$.

$\therefore A = w\sqrt{3}(10 - 2w) = 10\sqrt{3}w - 2\sqrt{3}w^2$.

(iii) For Max Area $\frac{dA}{dw} = 0$

$\frac{dA}{dw} = 10\sqrt{3} - 4\sqrt{3}w \Rightarrow 4\sqrt{3}w = 10\sqrt{3}$.

$w = \frac{10}{4} = \frac{5}{2} = 2.5$.

$\frac{d^2A}{dw^2} = -4\sqrt{3} < 0 \therefore$ MAX at $w = 2.5$

OR. $w < 2.5$. $\frac{dA}{dw} = +$

$w > 2.5$ $\frac{dA}{dw} = -$

$\therefore$ Max when $w = 2.5$

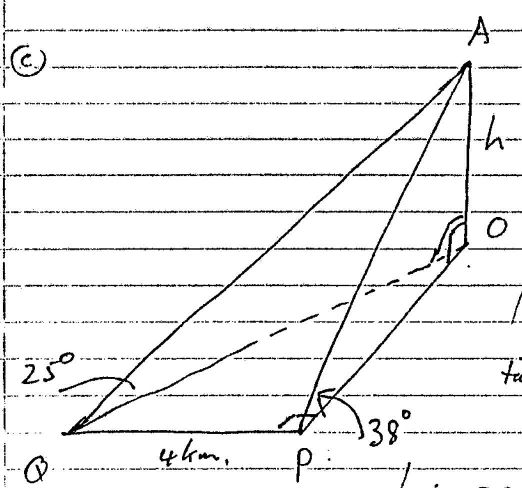
(iii) Area $= 10\sqrt{3}w - 2\sqrt{3}w^2$ when $w = 2.5 = \frac{5}{2}$,

$= 10\cdot\sqrt{3} \times \frac{5}{2} - 2\cdot\sqrt{3} \times \frac{5}{2} \times \frac{5}{2}$

$= 25\sqrt{3} - \frac{25}{2}\sqrt{3}$

$= \frac{25}{2}\sqrt{3}$ cm^2

(c)



$\tan 38° = \frac{h}{OP}$

$\therefore h = OP \tan 38° \quad ①$

$\tan 25° = \frac{h}{OQ}$

$\therefore h = OQ \tan 25° \quad ②$

$\frac{h}{\tan 25°}$

Now $OQ^2 = OP^2 + QP^2$.

$\therefore OQ^2 - OP^2 = 16$.

$\therefore \left(\frac{h}{\tan 25}\right)^2 - \left(\frac{h}{\tan 38}\right)^2 = 16$

$h^2\left(\frac{(\tan 38)^2 - (\tan 25)^2}{(\tan 25)^2 . (\tan 38)^2}\right) = 16$.

$\therefore h = \frac{4 . \tan 38 . \tan 25}{\sqrt{(\tan 38)^2 - (\tan 25)^2}}$

$h = 2.3247$

$h = 2325m$