Sydney Girls' High School

LABOR OMNIA VINCIT

2005
MATHEMATICS EXTENSION 1

YEAR 11
Assessment Task one
Time Allowed: 75 minutes

TOPICS: Exponential and Logarithmic functions, Integration and Locus

Directions to Candidates

- There are four (4) questions.
- Attempt ALL questions.
- Questions are **not** of equal value.
- Start each question on a new page.
- Write on one side of the paper only.
- Show all necessary working. Marks will be deducted for careless or badly arranged work.
- Diagrams are NOT drawn to scale.
- Board-approved calculators may be used.

Total: 70 marks

QUESTION ONE

a) Differentiate $y = e^{3x^2+5}$ (1)

b) Evaluate $\int_1^2 \frac{x^2-2x+1}{x}\,dx$ (2)

c) On the same diagram sketch the graph of $y = e^{3x}$ and $y = e^{-x}$ (2)

d) Differentiate $3x\log_e x$ (2)

e) If $\log_a 3 \doteq 1.4$ and $\log_a 2 \doteq 0.8$ evaluate $\log_a 12$. (2)

f) If $\int_0^a (4-2x)\,dx = 4$, find the value of a. (2)

g) Find the volume enclosed by the surface generated when the curve $x^2+4y^2=16$ is rotated about the x-axis (2)

h) A point $P(x, y)$ moves so that it is equidistant from $A(6,0)$ and the y-axis. (3)

i) Find the equation of its Locus
ii) Describe this Locus

i) $\log_{64} a = \frac{1}{3}$, find a (1)

QUESTION TWO

a) Find the following integrals (10)

+C

i) $\int(6-\sqrt{x})^2 dx$

ii) $\int\frac{4x}{1-5x^2}dx$

iii) $\int 3xe^{6x^2+1}dx$

iv) $\int\frac{e^x+e^{3x}}{e^{2x}}dx$

b) Find $\frac{dy}{dx}$; given (4)

i) $y=\frac{2x}{e^x+1}$

ii) $y=(e^{4x}+2)^6$

c) Find the coordinates of the centre and the radius of the following circle (3)

$$4x^2+4y^2+20x=24y-25$$

QUESTION THREE

a)

gradient

i) Find the equation of the tangent to the parabola $x^2=16y$ at point P where $x=4$. (2)

ii) The straight line joining P to the focus of the parabola intersects the parabola again at R. Find the coordinates of R. (3)

b) A parabola $y=5+2x-x^2$ is given (7)

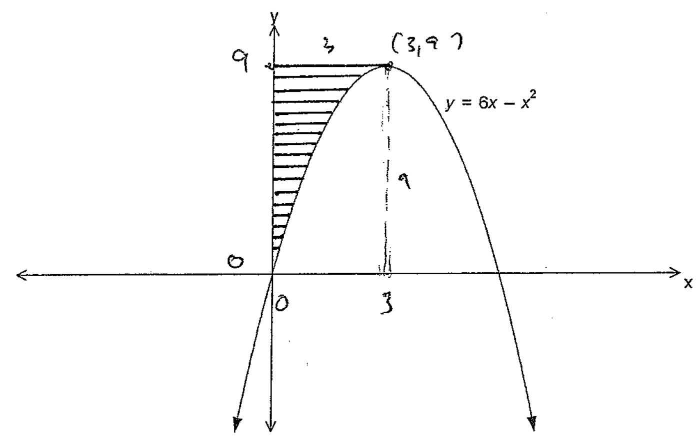
i) Find the coordinates of the vertex
ii) Find the coordinates of the focus
iii) Find the equation of the directrix
iv) Find the area under the curve between $x=-1$ and $x=1$

c) Use Simpson's rule with three ordinates to find an approximate value for

$\int_1^5\frac{1}{x^3+x}dx$ (correct to 2 decimal places) (3)

d) (4)

i) find the coordinates of the vertex
ii) Hence or otherwise, find the shaded area.



QUESTION FOUR

a) A and B are the points (5,-4) and (-3,2) respectively. The Point $P(x, y)$ moves So that $\angle APB = 90^\circ$. Find the locus of point P. (3)

b)

i) Find $\dfrac{d}{dx}\{x^2 - \log_e(x^2+1)\}$ (2)

ii) Hence or otherwise evaluate $\int_{-1}^{1} \dfrac{x^3}{x^2+1}\,dx$ (2)

0

c)

i) Show that $\dfrac{x+5}{x+3} = 1 + \dfrac{2}{x+3}$ (3)

ii) Hence find $\int \dfrac{x+5}{x+3}\,dx$

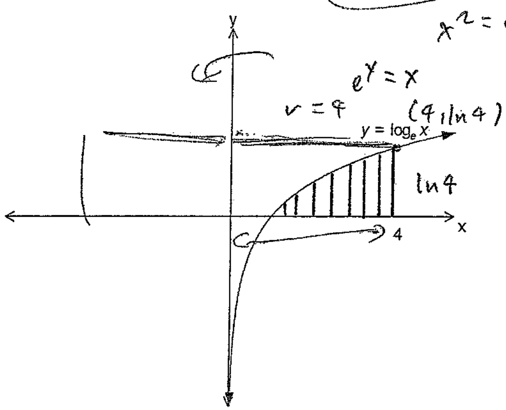
d) Find $\int \dfrac{e^x}{5+e^x}\,dx$ (1)

e) Find the area bounded by $y = x^2 - 6x$ and $y = 3x$ (3)

f) Find the volume of the solid formed when the area between the curve and the x-axis and the line $x = 4$ is rotated around the y-axis.

$x^2 = e^{2y}$ (3)

$x = 4,\ y = \ln 4$



THE END

HSC Assessment Task 1 2006

Exten. 1

1)

a) $y' = 6x\,e^{3x^2+5}$ (1)

b) $\int_1^2 x - 2 + \frac{1}{x}\,dx$

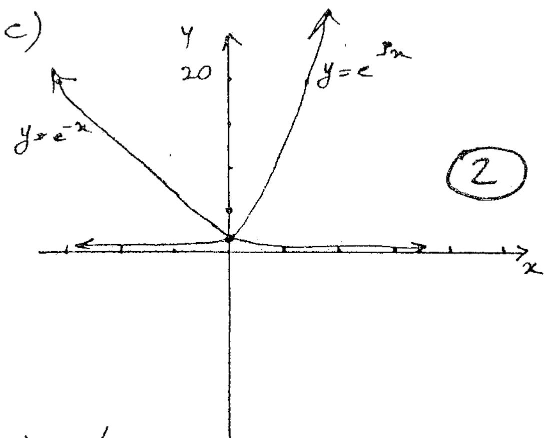
$= \left[\frac{x^2}{2} - 2x + \log_e x\right]_1^2$

$= 2 - 4 + \log_e 2 - (\frac{1}{2} - 2 + \log_e 1)$

$= -2 + \log_e 2 - (-1\frac{1}{2})$

$= \log_e 2 - \frac{1}{2}$ (2)

c)



(2)

d) $y' = 3\log_e x + \frac{1}{x}\times 3x$

$= 3\log_e x + 3$

$= 3(\log_e x + 1)$ (2)

e) $\log_a 12 = \log_a(3\times 4)$

$= \log_a 3 + \log_a 4$

$= \log_a 3 + 2\log_a 2$

$= 1.4 + 2\times 0.8$

$=$ [illegible] (2)

f)

$\left[4x - \frac{2x^2}{2}\right]_0^a = 4$

$\left[4x - x^2\right]_0^a = 4$

$4a - a^2 = 4$

$a^2 - 4a + 4 = 0$

$(a-2)^2 = 0$

$\boxed{a = 2}$ (2)

g) $V = \pi\int_{-4}^{4}\frac{16-x^2}{4}\,dx$

$= \frac{1}{4}\pi\left[16x - \frac{x^3}{3}\right]_{-4}^{4}$

$= \frac{1}{4}\pi\left[64 - \frac{64}{3} - (-64 + \frac{64}{3})\right]$

$= \frac{1}{4}\pi(85\frac{1}{3})$

$= \frac{64}{3}\pi\ u^3$ (2)

h) i) $x^2 = x^2 - 12x + 36 + y^2$

$y^2 = 12x - 36$

$y^2 = 12(x-3)$ (2)

ii) Parabola vertex $(3, 0)$

Directrix $x = 0$ (1)

iii) $a = 3$

Q2.

a) i) $\int(6-\sqrt{x})^2\,dx$

$= \int(36 - 12\sqrt{x} + x)\,dx$

$= 36x - \frac{12x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^2}{2} + c$

$= 36x - 8\sqrt{x^3} + \frac{x^2}{2} + c$ (2)

ii) $\int\frac{4x}{1-5x^2}\,dx$

$= -\frac{4}{10}\int\frac{-10x}{1-5x^2}\,dx$

$= -\frac{2}{5}\log_e(1-5x^2) + c$ (2)

iii) $\frac{d}{dx}e^{6x^2+1} = 12x\,e^{6x^2+1}$

$\int 12x\,e^{6x^2+1}\,dx = e^{6x^2+1} + c$

$= \int 3x\,e^{6x^2+1}\,dx = \frac{1}{4}(e^{6x^2+1} + c)$ (3)

iv) $\int e^{-x} + e^{x}\,dx$

$= \frac{e^{-x}}{-1} + e^{x} + c$

$= -\frac{1}{e^{x}} + e^{x} + c$ (3)

b) i) $y = \frac{2x}{e^x+1}$

$y' = \frac{2(e^x+1) - e^x(2x)}{(e^x+1)^2}$

$= \frac{2e^x + 2 - 2xe^x}{(e^x+1)^2}$ (2

ii) $y' = 6(e^{4x}+2)^5\times 4e^{4x}$

$= 24e^{4x}(e^{4x}+2)$

c) $4x^2 + 4y^2 + 20x - 24y = -25$

$x^2 + y^2 + 5x - 6y = -\frac{25}{4}$

$(x+\frac{5}{2})^2 + (y-3)^2 = -\frac{25}{4} + 15\frac{1}{4}$

$(x+\frac{5}{2})^2 + (y-3)^2 = 9$

$C(-\frac{5}{2}, 3)$ $r = 3$ (3)

Q 3(a)(i) $x = 4 \longrightarrow y = 1$

$y = \frac{x^2}{16}$

$y' = \frac{2x}{16}$

$y' = \frac{x}{8}$

at $x = 4$

$m = \frac{1}{2}$ (2)

$y - 1 = \frac{1}{2}(x-4)$

$2y - 2 = x - 4$

$\boxed{x - 2y - 2 = 0}$ $y = \frac{1}{2}x - 1$

ii) $F(0,4)$

$y - 4 = \frac{1-4}{4-0}(x - 0)$

$y - 4 = -\frac{3}{4}(x)$

$4y - 16 = -3x$

$3x + 4y - 16 = 0$

$x^2 = 16y \longrightarrow (1)$

$3x + 4y - 16 = 0 \longrightarrow (2)$

From 2

$y = \frac{-3x+16}{4}$

$x^2 = 16\left(\frac{-3x+16}{4}\right)$

$x^2 = -12x + 64$

$x^2 + 12x - 64 = 0$

$x^2 + 16x - 4x - 64 = 0$

$x(x+16) - 4(x+16) = 0$

$(x-4)(x+16) = 0$

$x = 4$ or -16

$x = -16 \longrightarrow y = 16$ (3)

$R(-16, 16)$

b) i) $x^2 - 2x = -y + 5$

$x^2 - 2x + 1 = -y + 5 + 1$

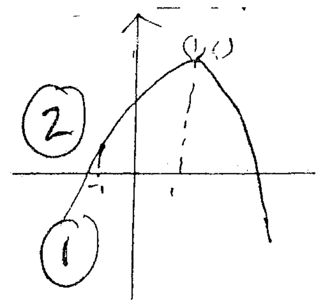
$(x-1)^2 = -(y - 6)$ (2)

$V(1, 6)$

ii) $4a = 1$

$a = \frac{1}{4}$ (2)

$F(1, 5\frac{3}{4})$



iii) $y = 6\frac{1}{4}$ (1)

iv) $A = \int_{-1}^{1} 5 + 2x - x^2 \, dx$

$= \left[5x + \frac{2x^2}{2} - \frac{x^3}{3}\right]_{-1}^{1}$ (2)

$= \left[5x + x^2 - \frac{x^3}{3}\right]_{-1}^{1}$

$= (5 + 1 - \frac{1}{3}) - (-5 + 1 + \frac{1}{3})$

$= 9\frac{1}{3}\ u^2$

c)

$\int_1^5 \frac{1}{x^3 + x} dx \doteq \frac{4}{6}\left[\frac{1}{2} + 4 \times \frac{1}{30} + \frac{1}{130}\right]$

$= \frac{4}{6}\left(\frac{25}{39}\right)$

$= \frac{100}{234}$

$\doteq 0.43$ (3)

d) $y = 6x - x^2$

$-y = x^2 - 6x$

$-y + 9 = x^2 - 6x + 9$

$-y + 9 = (x - 3)^2$

$(x-3)^2 = -(y - 9)$

$V(3, 9)$

$A = 9 \times 3 - \int_0^3 6x - x^2 \, dx$

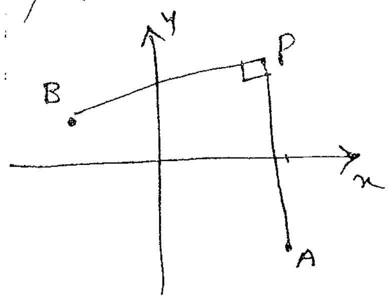
$= 27 - \left[\frac{6x^2}{2} - \frac{x^3}{3}\right]_0^3$ (4)

$= 27 - (27 - \frac{27}{3})$

$= 27 - 18$

$= 9\ u^2$

4) a)



$m_{AP} = \frac{y+4}{x-5}$

$m_{PB} = \frac{y-2}{x+3}$

$m_{AP} \times m_{PB} = -1$

$\frac{y+4}{x-5} \times \frac{y-2}{x+3} = -1$

$(y+4)(y-2) = -1(x-5)(x+3)$

$y^2+2y-8 = -1(x^2-2x-15)$

$y^2+2y-8 = -x^2+2x+15$

$y^2+2y+1+x^2-2x+1 = 23+2$

$(y+1)^2+(x-1)^2 = 25$ (3)

b)(i) $\frac{d}{dx}\left[x^2 - \log_e(x^2+1)\right]$

$= 2x - \frac{2x}{x^2+1}$

$= \frac{2x(x^2+1)-2x}{x^2+1}$

$= \frac{2x^3+2x-2x}{x^2+1}$

$= \frac{2x^3}{x^2+1}$ (2)

(ii) $\int \frac{2x^3}{x^2+1}\,dx$

$= x^2 - \log_e(x^2+1)$

$\therefore \int_{-1}^{1} \frac{x^3}{x^2+1}\,dx = \frac{1}{2}\left[x^2 - \log_e(x^2+1)\right]_{-1}^{1}$

$= \frac{1}{2}\left[1-\log_e 2 - (1-\log_e 2)\right]$

$= \frac{1}{2}[\,0\,]$

$= 0$ (2)

c)(i) $\frac{x+5}{x+3}$

$= \frac{x+3}{x+3} + \frac{2}{x+3}$

$= 1 + \frac{2}{x+3}$ (1)

ii) $\int \frac{x+5}{x+3}\,dx$

$= \int 1 + \frac{2}{x+3}\,dx$

$= x + 2\log_e(x+3) + c$ (2)

d) $\log_e(5+e^x) + c$ (1)

e) $y = x^2 - 6x$

$y = 3x$

$x^2 - 6x = 3x$

$x^2 - 9x = 0$

$x(x-9) = 0$

$x = 0$ or 9

$A = \int_0^9 x^2 - 6x - 3x\,dx$

$= \int_0^9 x^2 - 9x\,dx$

$= \left[\frac{x^3}{3} - \frac{9x^2}{2}\right]_0^9$

$= \left|\frac{9^3}{3} - \frac{9^3}{2}\right|$

$= 121\frac{1}{2}\ u^2$ (3)

f) $V = \pi r^2 h - \pi\int_0^{\ln 4} (e^y)^2\,dy$

$= \pi \times 4^2 \times \ln 4 - \pi\left[\frac{e^{2y}}{2}\right]_0^{\ln 4}$ (3)

$= 16\pi.\ln 4 - \pi\left[\frac{e^{2\ln 4} - e^0}{2}\right]$

$= 16\pi.\ln 4 - \pi\left(\frac{16-1}{2}\right)$

Ans: $16\pi.\ln 4 - \frac{15\pi}{2}\ u^3$