SYDNEY GIRLS H.S. — **Year Eleven Test 1**

February 2003

Instructions **Name**_____

Use separate paper

Show all necessary working

Question One (10 marks)

a) Evaluate $\dfrac{1}{\sqrt{5.3^2 - 1.7}}$ to 2 decimal places

b) Evaluate $|-5| - |7|$

c) Evaluate $8.3\times10^{15} \div 7.1\times10^{-3}$ and give your answer in scientific notation correct to 2 significant figures.

d) Write $2\sqrt{44}$ in simplest surd form.

e) Remove the grouping symbols: $(\sqrt{3}+4)(\sqrt{2}-\sqrt{5})$

f) Solve for x and plot your solution on a number line: $-5 < 2x + 7 \leq 11$

g) If $A = \dfrac{1}{2}h(a+b)$, find the value of a if $A = 100, h = 10$ and $b = 8$.

Question Two (10 marks)

a) Solve $6 - 2x \leq 14$ b) Solve $\left|\dfrac{2x-1}{2}\right| > 4$ c) Solve $|2x-5| = |x+2|$

d) Solve by completing the square $x^2 - 4x - 9 = 0$

Question Four (6 marks)

Solve the following;

a) $x^2 - 5x + 6 < 0$ b) $\dfrac{2}{x-4} > \dfrac{1}{3}$

Question Five (3 marks)

Show that $0.0\dot{5}$ is rational

Question Six (3 marks)

Simplify the following, expressing the answer with positive indices.

$$\left(x^{\frac{1}{2}} + y^{\frac{-1}{2}}\right)^2$$

Question Seven (6 marks)

a) Factorise $x^2 - 5x + 2xy - 10y$

b) Factorise $25 - (x-1)^2$

c) Factorise $8 + (2-x)^3$

PTO

Question Eight (5 marks)

a) Solve $\frac{x}{2}-\frac{x+1}{5}=1$

b) Solve the following pair of simultaneous equations.

$$\left.\begin{aligned} y &= x^3 \\ y &= x^2+6x \end{aligned}\right\}$$

Question Nine (5 marks)

a) Rationalise the denominator of $\frac{5}{2\sqrt{3}}$

b) Given $x=\sqrt{5}+2$, find b if $x+\frac{1}{x}=2\sqrt{b}$

Question Ten (9 marks)

a) Simplify $\frac{a-3}{a^3-27}$

b) Simplify $\frac{x-2}{6x^3+6}\times\frac{3x^2-3x+3}{x^2-4}$

c) Simplify $\frac{1}{x^2-4}+\frac{1}{x+2}-\frac{2}{x-2}$

Question Eleven (6 marks)

a) The sum of a number and its reciprocal is $2\frac{1}{20}$, form an equation and find the number.

b) The sum(S) of the first n counting numbers $1+2+3+.....+n$ is given by the formula $S=\frac{n}{2}(1+n)$, find the sum of the numbers from:

i.) 1 to 500

ii.) 200 to 500 inclusive.

Question 1.

a) $\frac{1}{5.137} = 0.195$ (Don't round)

$= 0.20$ (2. d.p). ✓

b) $5 - 7 = -2$. ✓

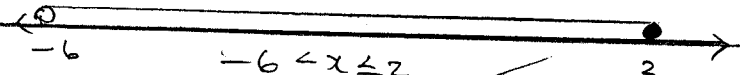
c) 1.2×10^{18} (2 sig figs) ✓

d) $4\sqrt{11}$ ✓

e) $\sqrt{6} - \sqrt{15} + 4\sqrt{2} - 4\sqrt{5}$ ✓

f) $-12 < 2x \leq 4$

$-6 < x \leq 2$. ✓



$-6 < x \leq 2$. ✓

g) $100 = \frac{1}{2}(10)(a+8)$

$100 = 5(a+8)$

$100 = 5a + 40$

$60 = 5a$

$a = 12$ ✓

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Question 2.

a) $6 - 2x \leq 14$

$-2x \leq 8$

$x \geq -4$ ✓

b) $|2x-1| > 8$

$-(2x-1) > 8$ $\quad (2x-1) > 8$

$-2x + 1 > 8$ $\quad 2x > 9$

$-2x > 7$ $\quad x > 4\frac{1}{2}$

$x < -3\frac{1}{2}$

$\therefore x > 4\frac{1}{2}$ or $x < -3\frac{1}{2}$ ✓

c) $|2x-5| = |x+2|$

$-(2x-5) = (x+2)$ $\qquad (2x-5) = (x+2)$

$-2x+5 = x+2$ $\qquad 2x-5 = x+2$

$-3x = -3$ $\qquad x = 7$

$x = 1$

$\therefore x = 1$ or 7. ///

d) $x^2 - 4x - 9 = 0$

$x^2 - 4x = 9$

$x^2 - 4x + 4 = 9 + 4$.

$(x-2)^2 = 13$

$x = 2 \pm \sqrt{13}$ ///

$= 5.6$ or -1.6 (2.d.p)

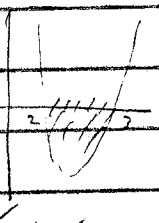
10

Question Four

a) $x^2 - 5x + 6 < 0$.

$(x-3)(x-2) < 0$.

$\therefore 2 < x < 3$ ///



b) $\frac{2(x-4)^2}{(x-4)} > \frac{(x-4)^2}{3}$

$2(x-4) > \frac{x^2-8x+16}{3}$

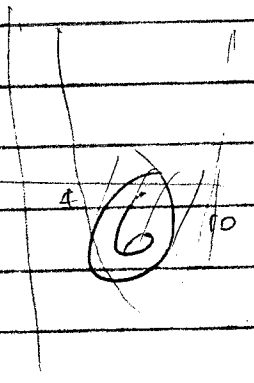
$6(x-4) > x^2 - 8x + 16$

$6x - 24 > x^2 - 8x + 16$

$x^2 - 14x + 40 < 0$.

$(x-10)(x-4) < 0$

$\therefore 4 < x < 10$



6

Question 5.

$x = 0.05'$ ... ①

① × 100

$100x = 5.5'$ ... ②

① × 1000 ...

$1000x = 55.5$ ... ③

③ − ②

$1000x - 100x = .50$

$900x = 50$

$x = \frac{50}{900}$

$= \frac{1}{18}$ ✓ (3)

∴ rational since '18' is ~~positive~~ integer

Question 6.

$\left(x^{\frac{1}{2}} + y^{-\frac{1}{2}}\right)^2$

$\boxed{(a+b)^2 = a^2 + 2ab + b^2}$

$(x^{1/2} + y^{-1/2})^2 = (x^{1/2})^2 + 2(x^{1/2})(y^{-1/2}) + (y^{-1/2})^2$

$= x + \frac{x^{1/2}}{y^{1/2}} + \frac{1}{y}$ ✓ (3)

Question 7.

a) $x^2 - 5x + 2xy - 10y$.

$x(x-5) + 2y(x-5)$

$= (x+2y)(x-5)$ //

b) $25 - (x-1)^2$

$a^2 - b^2 = (a+b)(a-b)$ Let $5 = a$

Let $+(x-1) = b$.

$(5+(x-1))(5-(x-1))$ //

$(4+x)(6-x)$

c) ~~$8 + (2x)$~~

$8 + (2-x)^3$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ Let $a = 2$

Let $b = 2-x$

~~$x + 2x$~~ $(2+(2-x))(2^2 - 2(2-x) + (2-x)^2)$

$(4-2x)(4 - 4 + 2x + (4 - 4x + x^2))$

$(4-2x)(2x + 4 - 4x + x^2)$

$(4-2x)(x^2 - 2x + 4)$ //

5/6

Question 8.

a) $\frac{x}{2} - \frac{x+1}{5} = 1$

$5x - 2(x+1) = 10$

$5x - 2x - 2 = 10$

$3x = 12$

$x = 4$ ✓ 2

b) PTO

(7) 8b

$y = x^3$ ... ①

$y = x^2 + 6x$ ... ②

put ① into ②

$x^3 = x^2 + 6x$

$x^3 - x^2 - 6x = 0$

$x(x^2 - x - 6) = 0$

$x(x-3)(x+2) = 0$

$\therefore x = 0, 3 \text{ or } -2$ ... ③

$\rightarrow y = 0, 27 \text{ or } -8$

$\therefore$ intersect at points

$(0,0), (3,27), (-2,-8)$

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Question 9

a) $\frac{5}{2\sqrt{3}} \times \frac{2\sqrt{3}}{2\sqrt{3}} = \frac{10\sqrt{3}}{12}$

$= \frac{5\sqrt{3}}{6}$

b) $x = \sqrt{5} + 2$, find $x + \frac{1}{x} = 2\sqrt{b}$

$\sqrt{5} + 2 + \frac{1}{\sqrt{5}+2} = \ldots$

$\frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} + \frac{\sqrt{5}+2}{1}$

$\frac{\sqrt{5}-2}{5-4} + \frac{\sqrt{5}+2}{1}$

$= \frac{\sqrt{5}+2+\sqrt{5}-2}{1}$

$= \boxed{2\sqrt{5}}$

$\therefore b = 5$

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Question 10.

$$\frac{a-3}{a^3-27} = \frac{(a-3)}{(a-3)(a^2+3a+9)}$$

$$= \frac{1}{a^2+3a+9}$$

b) $$\frac{x-2}{6x^3+6} \times \frac{3x^2-3x+3}{x^2-4}$$

$$= \frac{x-2}{6(x^3+1)} \times \frac{3(x^2-x+1)}{(x-2)(x+2)}$$

$$= \frac{x^2-x+1}{(x+1)(x^2-x+1)(x+2)}$$

$$= \frac{1}{2(x+1)(x+2)}$$

c) $$\frac{1}{x^2-4} + \frac{1}{x+2} \quad \frac{2}{x-2}.$$

$$\frac{1+(x-2)-2(x+2)}{(x+2)(x-2)}$$

$$= \frac{1+x-2-2x-4}{(x+2)(x-2)}$$

$$= \frac{-x-5}{(x+2)(x-5)}$$

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Question 11

(a) $x + \frac{1}{x} = \frac{41}{20}$.

$\frac{x^2}{x} + \frac{1}{x} = \frac{41x}{20x}$

$\frac{20x^2+20}{20x} = \frac{41x}{20x}$

$\therefore 20x^2 + 20 = 41x$

$20x^2 - 41x + 20 = 0$.

$(5x - 4)(4x - 5) = 0$.

$\therefore x = \frac{4}{5}$ or $\frac{5}{4}$

~~$20x^2 - 41x = 0$.~~

~~$x^2 - \frac{21x}{20} + \frac{441}{1600} = \frac{441}{1600}$~~

~~$\left(x - \frac{21}{40}\right)^2 = 0$~~

~~$x = \frac{21}{40}$~~

~~$x(20x - 21) = 0$~~

~~$x = 0$ or $\frac{20}{21}$~~

~~$\therefore x = \frac{20}{21}$~~

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Question 11.

a) Let the no be 'x'

$$\frac{x^2}{x} + \frac{1}{x} = \frac{41}{20}$$

$$\frac{x^2+1}{x} = \frac{41x}{20}$$

$$20(x^2+1) = (41x)x \qquad 20(x^2+1) = 41$$

$$20x^2 + 20 = 41x^2 \qquad 20x^2 + 20 = 41$$

$$20 = 21x^2 \qquad 20x^2 = 21$$

$$x^2 = \frac{20}{21} \qquad x = \frac{21}{20}$$

$$x = \sqrt{\frac{20}{21}} \qquad x = \sqrt{\frac{21}{20}}$$

bi) $S = \frac{500}{2}(1+500)$

$= \frac{500 \times 501}{2}$

$= 125250.$ ✓

2

ii) $S = \frac{200}{2}(1+200)$

$= \frac{201 \times 200}{2}$

$= 21000$ (From 1-200).

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$S = 125250$ [500 | -500].

$125250 - 21000 = 104250$

$104250 + 200 = 109450.$