SYDNEY GIRLS HIGH SCHOOL

LABOR OMNIA VINCIT

YEAR 11 MATHEMATICS

YEARLY EXAMINATION

SEPTEMBER 2005

Time allowed: 90 minutes

Topics: Chapters 1-10 (J&C)

Instructions:

- There are Four (4) questions. Questions are of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work.
- Start each question on a new page. Write on one side of the paper only.

Name: ....

QUESTION ONE

a) Simplify $\sqrt{45}-3(\sqrt{5}-\sqrt{9})$ (2)

b) Harvey Ross increases the cost price of an ipod by 35% and gives a particular customer discount of 8%. If the customer paid \$378.95, find the cost price of the ipod for Harvey ? Answer to the nearest dollar. (3)

c) Express $0.1\dot{3}$ as a fraction (2)

d) If $a=6.45$, $b=5.76$ and $c=21.9$, evaluate $\dfrac{a^2+\sqrt{b}}{c}$ correct to 3 significant figures. (2)

e) A number is added to its reciprocal and the answer $\dfrac{58}{21}$. Write down an equation and use it to find the number. (3)

f) Solve for x

$$x^2-8x=0$$ (2)

g) If $f(x)=9x^2-6x+2$, evaluate $f(2)+f(-3)$ (2)

h) Express $\dfrac{12\sin 60^\circ}{\cos 45^\circ}$ in simplest exact form (2)

i) Solve $|2x-1|<7$ (2)

QUESTION TWO

a) The line $x+y=1$ meets the circle $x^2+y^2=9$ at A and B. (4)

i) Sketch this information on a clear diagram

ii) Shade the region defined by $x^2+y^2\leq 9$ and $x+y\geq 1$

b) Solve $\frac{n-6}{2}+\frac{3n}{4}=n-1$ (3)

c) Draw a separate sketch of the following , showing all the main features

a) $y=|2x|-3$ (2)

b) $y=8x-4x^2$ (2)

d) If $\sec\theta=\frac{8}{5}$ and θ is acute, find the exact form of $\text{cosec}\theta$ and $\tan\theta$. (3)

e) Find the exact value of

i) $\sin 330^\circ$ (2)

ii) $\tan 510^\circ$ (2)

f) Express $\frac{a^{-2}+b^{-2}}{a^2+b^2}$ as a single fraction in its lowest terms. (2)

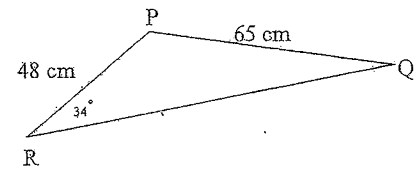
QUESTION THREE

a) In the diagram shown $\angle P$ is an obtuse angle.

i) Find the size of $\angle Q$ to the nearest minute (2)

ii) Find the size of $\angle P$ (1)

iii) Find the length of RQ (2)



b) State the domain of the following

$$f(x)=\sqrt{x-3}-\sqrt{5-x}$$ (2)

c) State the domain and the range of $y=-\sqrt{4-x^2}$ (2)

d) Determine whether the following function is odd, even or neither. (3)

$f(x)=7x^2-6x-4$

e) Find the value of x if 4^x equals one quarter of 2^{88}. (2)

f) Solve the following equations for $0^\circ\leq\theta\leq 360^\circ$, to the nearest degree (6)

i) $\tan\theta=1.4$

ii) $8\cos^3\theta-1=0$

iii) $2\sin\theta=3\cos\theta$

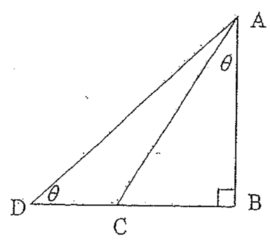
QUESTION FOUR

a) Simplify $\dfrac{\operatorname{cosec}^2 x - \cot^2 x}{\cos^2 x}$ (2)

b) In the diagram, AD = 4AB and $\angle ADC = \angle BAC$

i) Find the size of θ (2)

ii) If DC is 5cm, find the length of AB (2)



c) The points A, B and C are equally spaced on the circumference of a circle of radius x. Find the area of the triangle ABC in terms of x. (2)

d) Show that $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$ (2)

e) The vertices of the triangle ABC are the points A(0,0), B(0,2) and C(3,-1).

i) Draw a sketch diagram of the triangle. (1)

ii) The point K on BC is such that AK is perpendicular to BC. Find the coordinates of K, and show the point K on your diagram (2)

iii) Find the area of the triangle ABC (1)

f) Find the equation of the straight line which passes through the point of intersection of $3x - y + 4 = 0$ and $x + 2y + 3 = 0$ and is parallel to $3y - x + 2 = 0$. (3)

g) Eliminate θ from the following equations (3)

$$\begin{cases} x = 5\sin^2\theta \\ y = 4\cos\theta \end{cases}$$

--- End of Exam ---

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1)a) $3\sqrt{5} - 3\sqrt{5} + 9$

$= 9$

b) $\frac{135}{100}x \times \frac{92}{100} = 378.95$

$x = \$305.11$

c) Let $x = 0.1333...$

$10x = 1.333...$

$100x = 13.333...$

$90x = 12$

$x = \frac{2}{15}$

d) 2.01

e) $x + \frac{1}{x} = \frac{58}{21}$

$x^2 + 1 = \frac{58x}{21}$

$21x^2 - 58x + 21 = 0$

$x = \frac{3}{7}$ or $\frac{7}{3}$

f) $x(x-8) = 0$

$x = 0$ or 8

g) $f(2) = 9(4) - 6(2) + 2$

$= 26$

$f(-3) = 9(9) + 18 + 2$

$= 101$

$f(2) + f(-3) = 127$

h) $\dfrac{12 \times \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}} = \dfrac{6\sqrt{3}}{\frac{1}{\sqrt{2}}}$

$= 6\sqrt{6}$

i) $2x - 1 < 7$

or

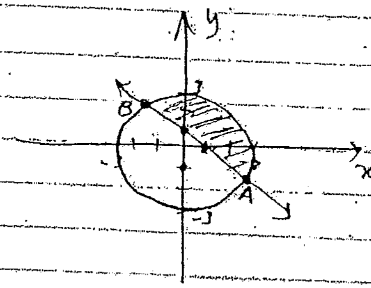
$2x - 1 > -7$

$2x < 8 \rightarrow x < 4$

$2x > -6 \rightarrow x > -3$

$-3 < x < 4$

Q2 a)



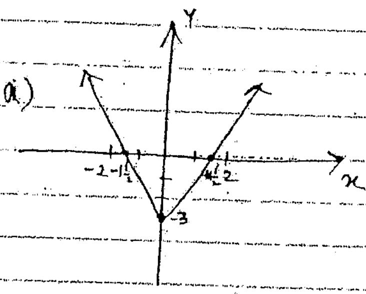
b) $4n - 24 + 6n = 8n - 8$

$2n = 16$

$n = 8$

c)

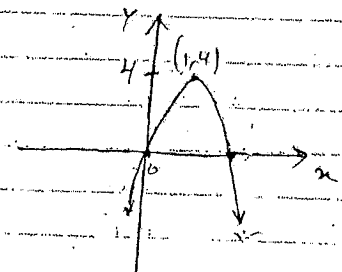
a)



b)

$y = 4x(2 - x)$

$x = 0$ or 2



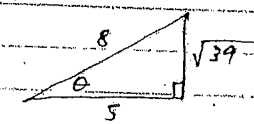
$x = 1 \rightarrow y = 4$

d)

$\sec\theta = \frac{8}{5}$

$\frac{1}{\cos\theta} = \frac{8}{5}$

$\cos\theta = \frac{5}{8}$



$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$= \frac{8}{\sqrt{39}}$

$\tan\theta = \frac{\sqrt{39}}{5}$

e) i) $\sin 330^\circ = -\sin 30^\circ$

$= -\frac{1}{2}$

ii) $\tan 510^\circ = \tan 150$

$= -\tan 30$

$= -\frac{1}{\sqrt{3}}$

f) $\dfrac{\frac{1}{a^2} + \frac{1}{b^2}}{a^2 + b^2} = \dfrac{\frac{b^2 + a^2}{a^2 b^2}}{a^2 + b^2}$

$= \frac{1}{a^2 b^2}$

Q3 a)

i) $\frac{\sin Q}{48} = \frac{\sin 34°}{65}$

$Q = 24°23'$

ii) $\angle P = 121°37'$

iii) $RQ^2 = 65^2 + 48^2 - 2 \times 65 \times 48 \cos 121°37'$

$RQ = 98.996$ cm

b) $x - 3 \geq 0$

$x \geq 3$

$5 - x \geq 0$

$-x \geq -5$

$x \leq 5$

Domain: $3 \leq x \leq 5$

c) Domain: $-2 \leq x \leq 2$

Range: $-2 \leq y \leq 0$

d) $f(x) = 7x^2 - 6x - 4$

$f(-x) = 7(-x)^2 - 6(-x) - 4$

$= 7x^2 + 6x - 4$

$-f(x) = -7x^2 + 6x + 4$

$f(x) \neq f(-x)$ ∴ not even

$-f(x) \neq f(-x)$ ∴ not odd

∴ neither.

e) $4^x = \frac{1}{4} \times 2^{88}$

$2^{2x} = 2^{-2} \times 2^{88}$

$2^{2x} = 2^{86}$

$2x = 86$

$\boxed{x = 43}$

f) i) $\theta = 54°, 234°$

ii) $\cos\theta = \frac{1}{2}$

$\theta = 60°, 300°$

iii) $2\tan\theta = 3$

$\tan\theta = \frac{3}{2}$

$\theta = 56°, 236°$

Q4)

a) $\frac{\frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}}{\cos^2 x}$

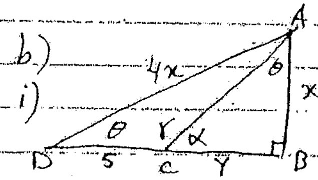
$= \frac{\frac{1 - \cos^2 x}{\sin^2 x}}{\cos^2 x}$

$= \frac{\frac{\sin^2 x}{\sin^2 x}}{\cos^2 x}$

$= \frac{1}{\cos^2 x}$

$= \sec^2 x$

b) i)



$\sin\theta = \frac{x}{4x}$

$= \frac{1}{4}$

$\theta = 14°29'$

ii) $\alpha = 75°31'$

$\gamma = 104°29'$

$\tan\theta = \frac{x}{5+y}$

$\tan\alpha = \frac{x}{y}$

$\tan 14°29' = \frac{x}{5+y}$

$\tan 75°31' = \frac{x}{y}$

$x = 5\tan 14°29' + y\tan 14°29'$ ①

$x = y\tan 75°31'$ ②

$5\tan 14°29' + y\tan 14°29' = y\tan 75°31'$

$y(\tan 75°31' - \tan 14°29') = 5\tan 14°29'$

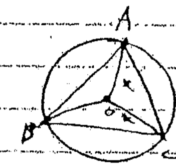
$y = \frac{5\tan 14°29'}{\tan 75°31' - \tan 14°29'}$

$y \doteq 0.357$

∴ $x = 0.357 \times \tan 75°31'$

$= 1.38$

c)



$\angle AOC = 120°$

$A\triangle ABC = 3 \times \frac{1}{2} \times x^2 \times \sin 120°$

$= \frac{3}{2} \times \frac{\sqrt{3}}{2} x^2$

$= \frac{3\sqrt{3}}{4} x^2$

d)

$LHS = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta$

$= 2\sin^2\theta + 2\cos^2\theta$

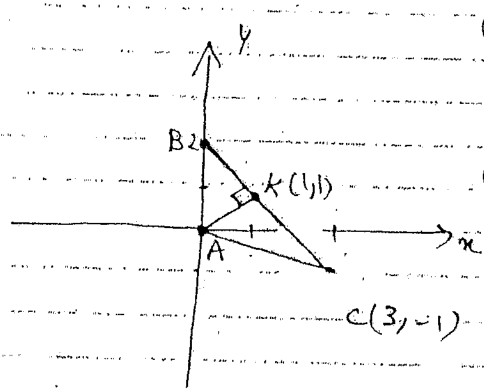
$= 2(\sin^2\theta + \cos^2\theta)$

$= 2 \times 1$

$= 2$

$= RHS$

e)



$m_{BC} = \frac{-1-2}{3-0}$

$= \frac{-3}{3}$

$= -1$

$m_{\perp} = 1$

For AK

$y - 0 = 1(x - 0)$

$y = x$

∴ BC

$y - 2 = -1(x - 0)$

$\boxed{y = -x + 2}$

$y = x$

$x = -x + 2$

$2x = 2$

$\boxed{x = 1}$

$\therefore \boxed{y = 1}$

$K(1, 1)$

iii) $AK = \sqrt{(1-0)^2 + (1-0)^2}$

$= \sqrt{2}$

$BC = \sqrt{(3-0)^2 + (-1-2)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$Area = \frac{1}{2} \times \sqrt{2} \times \sqrt{18}$

$= \frac{1}{2}\sqrt{36}$

f)

$3x - y + 4 + k(x + 2y + 3) = 0$

$3x - y + 4 + kx + 2ky + 3k = 0$

$x(3+k) - y(1+2k) + 4 + 3k = 0$

$y(1-2k) = x(3+k) + 4 + 3k$

$y = x\frac{(3+k)}{1-2k} + \frac{4+3k}{1-2k}$

$m_1 = \frac{3+k}{1-2k}$

$m_2 = \frac{1}{3}$

$m_1 = m_2$ for Parallel lines

$\therefore$

$\frac{3+k}{1-2k} = \frac{1}{3}$

$9 + 3k = 1 - 2k$

$5k = -8$

$k = \frac{-8}{5}$

$3x - y + 4 + \frac{8}{5}(x + 2y + 3) = 0$

$15x - 5y + 20 - 8x - 16y - 24 = 0$

$7x - 21y - 4 = 0$

For the other method pts of intersection is $\left(\frac{-11}{7}, -\frac{5}{7}\right)$

g) $\cos\theta = \frac{y}{4}$

$x = 5(1 - \cos^2\theta)$

$x = 5\left(1 - \left(\frac{y}{4}\right)^2\right)$

$x = 5 - \frac{5y^2}{16}$

$16x = 80 - 5y^2$

$\boxed{5y^2 + 16x - 80 = 0}$