Sydney Girls' High School

LABOR OMNIA VINCIT

2006
MATHEMATICS

YEAR 11
YEARLY EXAMINATION

Time Allowed: 90 minutes

TOPICS: Algebra, Geometry, Functions, Trigonometry, and the Linear Function.

Directions to Candidates

- There are four (4) questions.
- Attempt ALL questions.
- Questions are of equal value.
- Start each question on a new page.
- Write on one side of the paper only.
- Show all necessary working. Marks will be deducted for careless or badly arranged work.
- Diagrams are NOT drawn to scale.
- Board-approved calculators may be used.

Total: 100 marks

QUESTION 1 (25 marks) — Marks

a) Calculate $\dfrac{4\cdot 23}{\sqrt{6\cdot 14-1\cdot 78}}$ correct to 2 decimal places. — 1

b) Simplify $|-5|-|8|$. — 1

c) The volume V of a sphere is given by $V=\dfrac{4}{3}\pi r^3$.
If a sphere has volume 5cm^3, find the radius correct to 2 decimal places. — 2

d) Solve $4(x-5)=3-2(x-1)$ — 2

e) Simplify $\dfrac{2}{3}-\dfrac{x-1}{4}$ — 3

f) If $f(x)=3x^2-5x+4$ and $g(x)=2x+10$ find $f(1)-g(-1)$ — 2

g) Solve the following inequality and graph the solution on a number line — 3
$$|x+1|\le 3$$

h) (i) Rationalise the denominator of $\dfrac{2}{2-\sqrt{3}}$. — 3

(ii) Find integers a and b such that $\dfrac{2}{2-\sqrt{3}}=a+\sqrt{b}$. — 2

i) Solve the pair of simultaneous equations — 3
$$4x-y=3$$
$$10x+3y=2$$

j) Express $\dfrac{x+1}{x^2-x}-\dfrac{x-1}{x^2+x}$ as a fraction in its lowest terms. — 3

QUESTION 2 (25 marks) — Marks

a) Factorise $2x^2+3x-2$ — 1

b) Find the exact value of $\sin\theta$ if $\tan\theta=\dfrac{3}{7}$ and $\cos\theta<0$. — 3

c) Solve the equation $2\cos x=\sqrt{3}$, where $0\le x\le 360^\circ$ — 3

d) Find the equation of the straight line through (1, 4) parallel to $3x+2y-6=0$ — 3

e) Simplify $\dfrac{\sin x}{1+\cos x}+\dfrac{\sin x}{1-\cos x}$ — 4

f) Find the equation of the straight line through (3,-2) that passes through the intersection of the lines $5x+2y-13=0$ and $x-3y+11=0$. — 3

g) (i) Sketch the graph of $y=x^2-6$ showing x and y intercepts. — 3

(ii) On the same graph sketch the graph of $y=|x|$. — 1

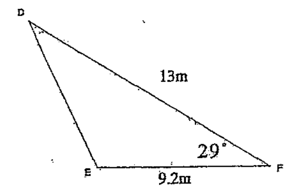
(iii) Find the x co-ordinates of the two points where the graphs intersect. — 2

(iv) Hence, solve the inequality $x^2-6\le|x|$. — 2

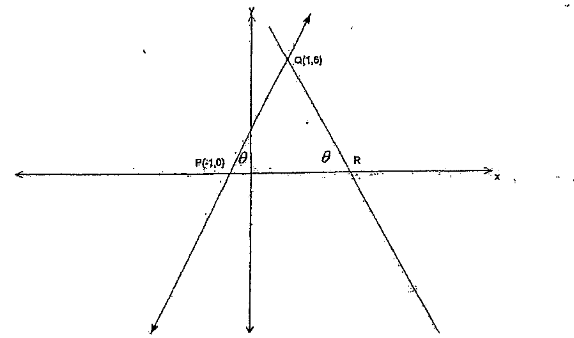
QUESTION 3 (25 marks) — Marks

a) Find the distance between the points (1, 3) and (-5, 0). Express your answer in simplest surd form. — 2

b) Find the area of ΔDEF — 3



c)



In the diagram above P and Q have co-ordinates (-1, 0) and (1, 6) and $\angle QPR=\angle QRP=\theta$.

Copy the diagram onto your answer sheet.

(i) Find the co-ordinates of the midpoint of PQ. — 2

(ii) Show that PQ has equation $y=3x+3$. — 3

(iii) Show that $\tan\theta=3$ *hence* find θ correct to the nearest minute — 2

(iv) Find the co-ordinates of R — 2

(v) Show that the gradient of QR is -3 — 2

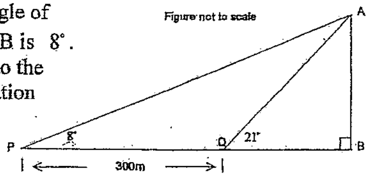
(vi) Show that the equation of QR is $3x+y-9=0$ — 3

(vii) Find the perpendicular distance from P to QR — 3

(viii) On your diagram, shade the region satisfying both the inequalities: $y\le 3x+3$ and $3x+y-9\ge 0$ — 3

QUESTION 4 (25 marks) Marks

a) From position P, Anne finds that the angle of elevation of the top A of a rock pillar AB is 8°. After walking 300m towards the pillar to the point Q she finds that the angle of elevation of A is 21°.



(i) Copy the diagram onto your answer sheet.

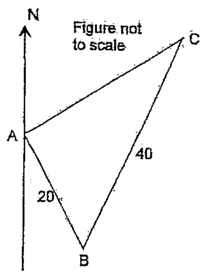
(ii) Find the size of $\angle PAQ$. 2

(iii) Calculate the length of AQ to the nearest metre. 3

(iv) Find the height of the rock pillar to the nearest metre. 3

b) Two geologists on a large flat mining claim drive 20km from point A on a bearing of 150°T to point B.

They then drive 40km on a bearing of 020°T to point C.



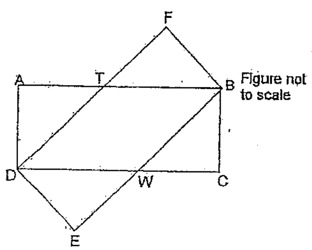
(i) Copy the diagram onto your answer sheet showing information given.

(ii) Show that $\angle ABC = 50^\circ$ giving reasons. 2

(iii) Find the distance of point C from point A. 3

c) Prove that $\dfrac{\tan x}{\sec x - 1} = \dfrac{\sec x + 1}{\tan x}$ 3

d) ABCD and DFBE are two congruent rectangles with sides 3 and 7 units.



(i) Prove $\Delta ADT \equiv \Delta FBT$ 3

(ii) Show that $AT = \dfrac{20}{7}$ 3

(iii) Find the area of DWBT 3

THE END

YEAR 11 – YEARLY EXAMINATION 2006

stion 1: (25 marks)

a) $\boxed{2 \cdot 03}$ ✓ (1)

b) $5 - 8 = \boxed{-3}$ ✓ (1)

c) $V = \frac{4}{3}\pi r^3$

$5 = \frac{4}{3}\pi r^3$ ✓

$r = \sqrt[3]{1 \cdot 19}$ (2)

$r = \boxed{1 \cdot 06}$ ✓

d) $4x - 20 = 3 - 2x + 2$

$6x = 25$ ✓ (2)

$\boxed{x = 4\frac{1}{6}}$ ✓

e) $\frac{8 - 3(x-1)}{12}$ ✓ (3)

$\frac{8 - 3x + 3}{12} = \boxed{\frac{11 - 3x}{12}}$ ✓ ✓

f) $f(1) = 3 - 5 + 4$ (2)

$= 2$

$g(-1) = -2 + 10$ ✓

$= 8$

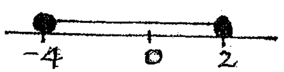
$\therefore f(1) - g(-1) = 2 - 8$

$= \boxed{-6}$ ✓

g) $|x+1| \le 3$ (3)

$-3 \le x + 1 \le 3$

$\boxed{-4 \le x \le 2}$ ✓



✓

h) i) $\frac{2}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{4 + 2\sqrt{3}}{4 - 3}$ (3)

$= \boxed{4 + 2\sqrt{3}}$ ✓

ii) $4 + 2\sqrt{3} = 4 + \sqrt{12}$ ✓ (2)

$\therefore \boxed{a = 4, \quad b = 12}$ ✓

i) $4x - y = 3 \quad \ldots (1)$

$10x + 3y = 2 \quad \ldots (2)$

From (1) $y = 4x - 3$

$10x + 3(4x - 3) = 2$

$10x + 12x - 9 = 2$

$22x = 11$

$x = \frac{1}{2}$ ✓ (3)

$y = 4(\frac{1}{2}) - 3$

$y = 2 - 3$

$y = -1$ ✓

$\therefore \boxed{x = \frac{1}{2}, \quad y = -1}$ ✓

j) $\frac{x+1}{x^2 - x} - \frac{x-1}{x^2 + x}$

$\frac{(x+1)^2 - (x-1)^2}{x(x-1)(x+1)}$ (3)

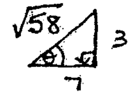
$\frac{\cancel{x^2} + 2x + \cancel{1} - \cancel{x^2} + 2x - \cancel{1}}{x(x-1)(x+1)}$

$= \boxed{\frac{4}{(x-1)(x+1)}}$

Question 2 (25 marks)

a) $(2x - 1)(x + 2)$ (1)

b) $\tan\theta = \frac{3}{7}$



S A
T C

(3)

$\therefore \sin\theta = -\frac{3}{\sqrt{58}}$

c) $2\cos x = \sqrt{3}$ (3)

$\cos x = \frac{\sqrt{3}}{2}$

S A ✓
T C ✓

$x = 30^\circ$ or 330°

d) $3x + 2y - 6 = 0$

$2y = -3x + 6$

$y = -\frac{3}{2}x + 2$

$m_1 = m_2 = -\frac{3}{2}$ (3)

Equation of line

$m = -\frac{3}{2} \qquad P(1, 4)$

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{3}{2}(x - 1)$

$2y - 8 = -3x + 3$

$3x + 2y - 11 = 0$

e) $\frac{\sin x(1 - \cos x) + \sin x(1 + \cos x)}{(1 + \cos x)(1 - \cos x)}$ (4)

$\frac{\sin x - \sin x\cancel{\cos x} + \sin x + \sin x\cancel{\cos x}}{1 - \cos^2 x}$

$\frac{2\sin x}{\sin^2 x} = \frac{2}{\sin x}$

f) $5x + 2y - 13 + k(x - 3y + 11) = 0$

$5(3) + 2(-2) - 13 + k(3 - 3(-2) + 11) = 0$

$15 - 4 - 13 + k(20) = 0$

$20k = 2$

$k = \frac{1}{10}$

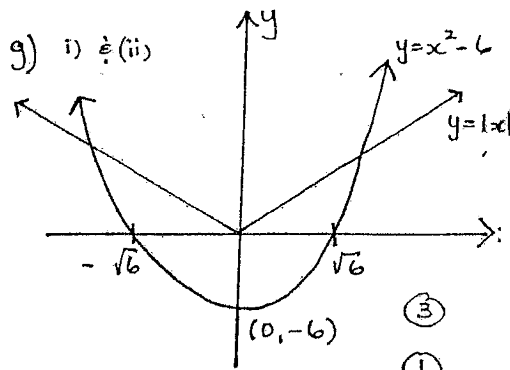
$5x + 2y - 13 + \frac{1}{10}(x - 3y + 11) = 0$

$50x + 20y - 130 + x - 3y + 11 = 0$

$51x + 17y - 119 = 0$

$3x + y - 7 = 0$ (3)

g) i) & (ii)



(3)

(1)

(iii) $x^2 - 6 = x$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$ (2)

$x = 3$ or -2

however $y = |x| \ \therefore y \ge 0$

$\therefore$ only solution $x = 3$

and since function is symmetrical $x = -3$

$\therefore x = 3$ or $x = -3$.

(iv) $x^2 - 6 \le |x|$ (2)

$-3 \le x \le 3$

Question 3 (25 marks)

a) P (1,3) and Q(−5,0)

distance $PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-5-1)^2+(0-3)^2}$

$= \sqrt{(-6)^2+(-3)^2}$

(2) $= \sqrt{36+9} = \sqrt{45}$

$= 3\sqrt{5}$

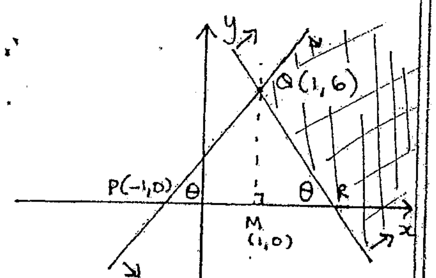
b) Area $= \frac{1}{2}de \sin F$

$= \frac{1}{2}(9.2)(13)\sin 29^\circ$

(3)

$= 28.99\ m^2$

c)



i) Midpoint $PQ \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-1+1}{2}, \frac{0+6}{2}\right)$

$= (0,3)$ (2)

(ii) Gradient $PQ = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{6-0}{1+1} = \frac{6}{2}$

$m = 3$

Equation $y - y_1 = m(x - x_1)$

$y - 6 = 3(x-1)$

$y - 6 = 3x - 3$

$y = 3x + 3$ (3)

(iii) since gradient $m = 3$

$\therefore \tan\theta = m$

$\tan\theta = 3$ (2)

hence $\theta = 71^\circ 34'$

(iv) Since ΔPQR is isosceles, draw ~~⊥~~ from Q.

$\therefore$ M has co-ordinates (1,0)

$\therefore$ PM = MR

$\therefore$ R has co-ordinates (3,0)

(2)

v) Gradient $QR = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{0-6}{3-1} = \frac{-6}{2}$ (2)

$= -3$

vi) Equation QR

$m = -3$, Q(1,6)

$y - y_1 = m(x - x_1)$

$y - 6 = -3(x-1)$

$y - 6 = -3x + 3$ (3)

$3x + y - 9 = 0$

vii) Perpendicular distance

$P = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

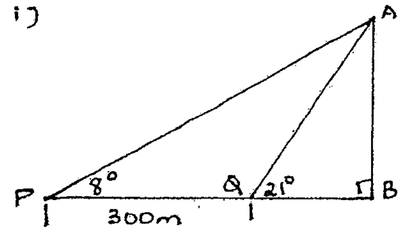
$= \left|\frac{3(-1)+0-9}{\sqrt{3^2+1^2}}\right|$ $\left[\begin{array}{l} 3x+y-9=0 \\ P(-1,0) \end{array}\right.$ (3)

$= \frac{|-12|}{\sqrt{10}}$

$= \frac{12}{\sqrt{10}}$ or $\frac{6\sqrt{10}}{5}$ units.

Question 4 (25 marks)

a) i)



ii) $\angle AQP = 180 - 21$ (straight angle)

$= 159^\circ$

$\therefore \angle PAQ = 180 - (159+8)$

$= 13^\circ$ (2)

iii) $\frac{300}{\sin 13^\circ} = \frac{AQ}{\sin 8^\circ}$

$\therefore AQ = \frac{300 \times \sin 8^\circ}{\sin 13^\circ}$

$= 185.6$ (3)

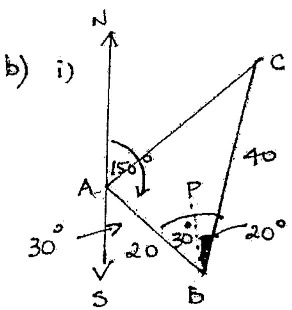
$= 186\,m$

iv) $\sin\theta = \frac{AB}{AQ}$

$186 \times \sin 21^\circ = AB$ (3)

$\therefore AB = 67\,m$

b) i)



(ii) $\angle NAB = 150^\circ$ (given)

(2) $\therefore S\hat{A}B = 30^\circ$ (straight angle)

$A\hat{B}P = 30^\circ$ (alternate)

$P\hat{B}C = 20^\circ$ (bearing given)

iii) $(AC)^2 = (AB)^2 + (BC)^2 - 2AB.BC.\cos A\hat{B}C$

$(AC)^2 = (20)^2 + (40)^2 - 2.(20)(40).\cos 50^\circ$

$(AC)^2 = 971.5398245$

$AC = \sqrt{971.5398245}$ (

$\doteq 31.169$

$= 31$ km (to nearest k

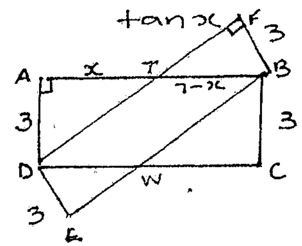
c) $L.H.S = \frac{\tan x}{\sec x - 1} \times \frac{(\sec x + 1)}{(\sec x + 1)}$

$= \frac{\tan x(\sec x + 1)}{\sec^2 x - 1}$

$= \frac{\cancel{\tan x}(\sec x + 1)}{\tan^{\cancel{2}} x}$

$= \frac{\sec x + 1}{\tan x} = R.H.S.$

d)



(i) Proof! In Δ's ADT and FBT

$\angle DAT = \angle BFT = 90^\circ$

(3) (angles of rectangle)

AD = FB = 3 units

(sides of rectangle − given)

$A\hat{T}D = F\hat{T}B$ (v. opposite angles)

$\therefore$ By AAS Test $\Delta ADT \equiv \Delta FBT$

(ii) $\therefore$ AT = x = FT and

DT = BT = 7 − x

corresponding sides of two congruent Δ's.

Question 4: (con't)

Applying Pythagoras' Theorem

$(AT)^2 + (AD)^2 = (DT)^2$

$x^2 + 3^2 = (7-x)^2$

$x^2 + 9 = 49 - 14x + x^2$

$14x = 40$

$x = \frac{40}{14}$

3

$x = \frac{20}{7}$

$\therefore AT = \frac{20}{7}$

(iii) Area DWBT =

area ABCD − 2area ADT

$= 7 \times 3 - 2 \times \left(\frac{1}{2} \times 3 \times \frac{20}{7}\right)$

$= 21 - 8\,{}^{4}/_{7}$

3

$= 12\,{}^{3}/_{7}$ units^2.