SYDNEY GIRLS' HIGH SCHOOL

2004

MATHEMATICS

YEAR 11

YEARLY EXAMINATION

Time allowed: 90 minutes

DIRECTIONS TO CANDIDATES

- There are four (4) questions.
- Attempt ALL questions.
- Questions are of equal value.
- Start each question on a new page. Write on one side of the paper only.
- Show all necessary working. Marks may be deducted for careless or badly arranged work.
- Diagrams are NOT drawn to scale
- Board-approved calculators may be used.

QUESTION ONE (25 marks)

(a) Evaluate $\dfrac{8 \cdot 9 \times 4 \cdot 5}{(1 \cdot 07)^3}$ correct to 3 significant figures

(b) The mass of 1 atom of oxygen is $2 \cdot 7 \times 10^{-23}$ g.
Find the mass of 8×10^{29} atoms of oxygen.
Give your answer in scientific notation.

(c) Factorise and simplify fully $\dfrac{3x^2 - 12}{10 - 5x}$

(d) Solve (i) $\dfrac{x}{2} - \dfrac{x+1}{5} = 4$

(ii) $3^{5y-4} = 81$

(e) Express as a fraction in simplest form $0.\dot{2}$

(f) Solve $|5 - 2x| > 9$

(g) Solve simultaneously

$$xy = 3$$
$$x + y = 4$$

(h) If $x = \sqrt{2} - 1$ find $x + \dfrac{1}{x}$.

(i) Find the integers a and b such that $(a + 3\sqrt{2})^2 = b + 12\sqrt{2}$

(j) The cost of a football ticket is $24 plus 10% G.S.T.

(i) Find the cost of one ticket

If a person buys five (5) tickets they receive a 10% discount off the total price of the five (5) tickets.

(ii) Calculate the amount paid for the five (5) tickets.

QUESTION TWO (25 marks)

(a) If

$$f(x) = \begin{cases} -2 & \text{if } x \le -5 \\ 0 & \text{if } -5 < x < 2 \\ 2x & \text{if } x \ge 2 \end{cases}$$

find (i) $f(-6) + f(1) + f(6)$

(ii) $f(a^2 + 2)$

(b) Sketch the following functions and state the domain and range for each

(i) $y = \sqrt{25 - x^2}$

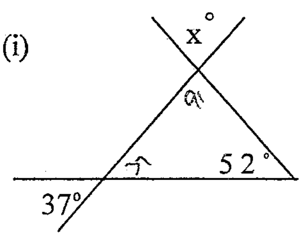
(ii) $y = \dfrac{2}{x - 3}$

(iii) $y = x^2 - 2x - 8$

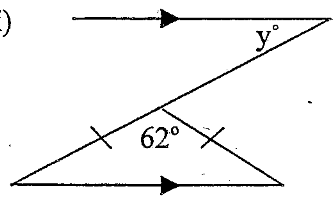
(iv) $y = |2x - 3|$

(c) Find the value of the pronumeral in each case.

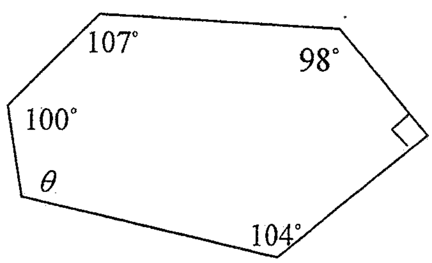
(i)



(ii)



(iii)



(d) (i) On a number plane sketch neatly the region satisfied by

$y \ge 2$

$y \le -2x + 4$

and $x \ge 0$

(ii) What is the area of this shaded region?

QUESTION THREE (25marks)

(a) Write down the exact value of

(i) $\tan 225^{\circ}$

(ii) $\cos 315^{\circ}$

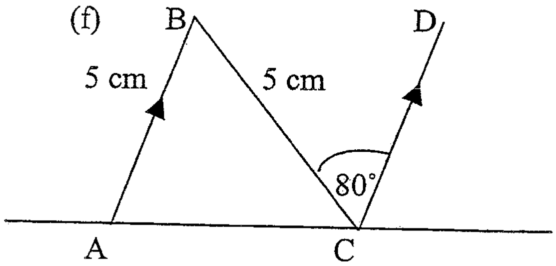
(b) Solve $2\cos\theta = 1$ for $0^{\circ} \leq \theta \leq 360^{\circ}$

(c) Simplify $\dfrac{3^{m} \times 9^{m+1}}{27^{2m}}$

(d) Find the exact value of $\cos\theta$ given $\tan\theta = \dfrac{7}{9}$ and $\sin\theta < 0$

(e) Simplify $\tan\theta.\mathrm{cosec}\,\theta$

(f)



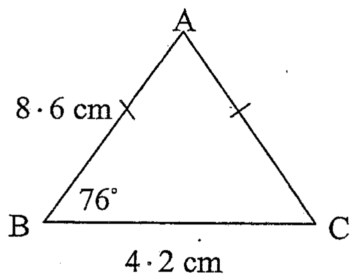
In the diagram, AB is parallel to CD, AB is 5cm, BC is 5cm and $\angle BCD = 80^{\circ}$.

Copy diagram onto your answer sheet

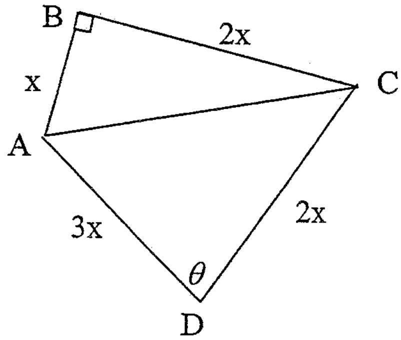
(i) Find $\angle BAC$, giving reasons for your answer.

(ii) Hence find the length of AC, correct to 2 decimal pla

(g) Find the area of the triangle ABC correct to 1 decimal place



(h) The diagram shows the quadrilateral ABCD with sides x cm, 2x cm, 2x cm and 3x cm. Side AB is perpendicular to BC and $\angle ADC = \theta$



Find an expression in terms of x for the length of AC.
Hence or otherwise, find the size of angle θ to the nearest degree

QUESTION FOUR (25marks)

(a) Show that $f(x) = x^3$ is an odd function

(b) On the same number plane sketch the following graphs for $0^\circ \leq x \leq 360^\circ$

(i) $y = \sin x$

(ii) $y = \operatorname{cosec} x$

(c) If $\sin\theta = \dfrac{\sqrt{5}}{3}$ and θ is obtuse

find (i) $\cos\theta$ and

(ii) $\tan\theta$ in simplest form.

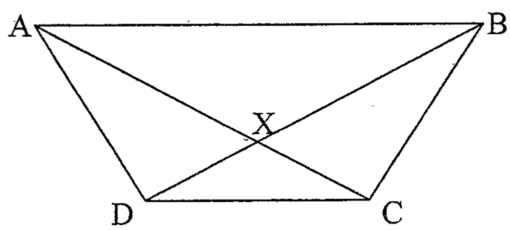
(d) Prove $\sin^4\theta - \cos^4\theta = 2\sin^2\theta - 1$

(e) ABCD is a trapezium in which AB is parallel to DC.
The diagonals intersect at X. AB = 12 cm, DC = 8 cm and AC = 10 cm.

Copy the diagram onto your answer sheet and clearly label the information given.

(i) Prove ΔAXB is similar to ΔCXD

(ii) Hence, find the length of AX.

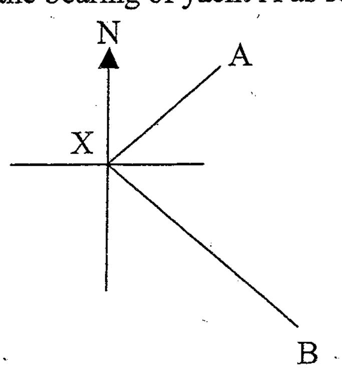


(f) Two yachts A and B sail in a straight line from a Buoy X.
A sails 10 km in the direction of 040° and
B sails 25 km in the direction of 150°.

(i) Copy the diagram onto your answer sheet and label the given information

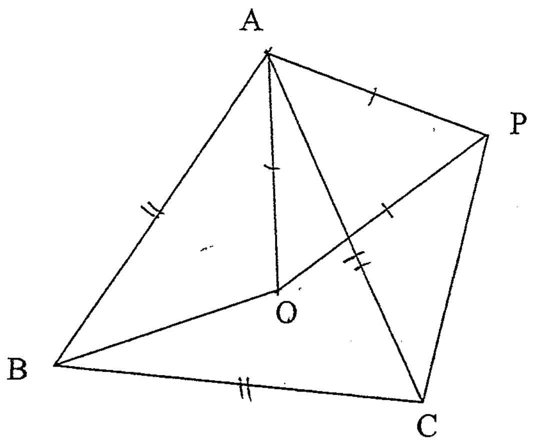
(ii) Find the size of $\angle AXB$

(iii) What is the bearing of yacht A as seen from yacht B?



QUESTION FOUR (continued)

(g)



In the figure triangles ACB and APO are equilateral.

Copy this diagram onto your answer sheet and label all information given.

(i) Explain why $\angle BAO = \angle PAC$.

(ii) Prove $\Delta AOB \equiv \Delta APC$

(iii) Hence, prove $OB = CP$.

THE END

Daniel Ong

SGHS Yr 11 2U Exam.

Question 1.

a) 32.7 (3 sig figs). ✓

b) $2.7 \times 10^{-23} \times 8 \times 10^{29} = 21.6 \times 10^{6}$

$= ~~2.161~~ 2.16 \times 10^{8}$ ✗ $= 2.16 \times 10^{7}$

c) $\frac{3x^2-12}{10-5x} = \frac{3(x^2-4)}{5(2-x)}$

$= \frac{3(x-2)(x+2)}{5(2-x)}$ Note: $\frac{x-2}{2-x} = -1$

d) i) $\frac{x}{2} - \frac{x-1}{5} = 4$

$\frac{5x-2x+2}{10} = 4$

$3x+2 = 40.$

$3x = ~~42~~ 38$

$x = ~~14~~ \frac{38}{3} = 12\frac{2}{3}$

ii) $3^{5y-4} = 81.$

$3^{5y-4} = 3^4$

$5y-4 = 4$ ✓

$5y = 8$

$y = 8/5.$ ✓

e) $0.\dot{2}$

$= x = 0.\dot{2}$

$10x = 2.\dot{2}$ ✓

$9x = 2$

$x = 2/9.$ ✓

f) $|5-2x| > 9$

$5-2x > 9$ or $-5+2x > 9$

$-4 > 2x$ $\quad$ $2x > 14$

$x < -2$ ✓ $\quad$ $x > 7$ ✓

g) $xy = 3$ ②

$x+y = 4.$ ①

$x = 4-y$

$y(4-y) = 3$

$4y - y^2 - 3 = 0$

$y^2 - 4y + 3 = 0.$

$(y-1)(y-3) = 0.$ ✓

$y = 1$ or $y = 3.$ ✓

$\therefore x = 3$ ✓ or $3x = 3$

$x = 1.$ ✓

h) $\frac{x}{1} - \frac{1}{x}$ $\Rightarrow$ Quicker if $(\sqrt{2}-1) + \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$

$\frac{x^2+1}{x}$ $\quad = \sqrt{2}-1+\sqrt{2}+1 = 2\sqrt{2}.$ ✓

$= 2 - 2\sqrt{2} + 1 + 1 \quad = 4 - 2\sqrt{2} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = 4\sqrt{2}+4-4-2\sqrt{2} \quad 2\sqrt{2}.$

i). $(a+3\sqrt{2})^2 = b+12\sqrt{2}$

$a^2 + 6a\sqrt{2} + 18 = b + 12\sqrt{2}$

$a^2+18=b \qquad 6a\sqrt{2} = 12\sqrt{2}$

$4 + 18 = b \qquad 6a = 12$

$\underline{b = 22} \qquad \underline{a=2}$ ✓

j). i). 24×1.1

$\underline{= \$26.40}$ ✓

ii) $26.4 \times 5 \times 0.9$

$\underline{= \$118.80}$ ✓

2. a) i). $f(-6)+f(1)+f(6)$.

$-2 + 0 + 12$

$\underline{= 10}$ ✓

ii). $f(a^2+2)$.

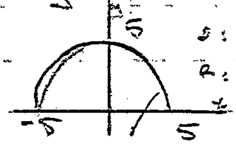
$a^2 \geq 0$. ✓

$\therefore f(a^2+2) \geq 2$.

$\therefore = 2(a^2+2)$

$= 2a^2+4$. ✓

b). i). $y=\sqrt{25-x^2}$

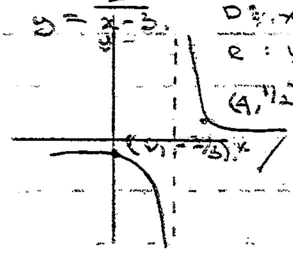


D: $-5 \leq x \leq 5$ ✓

R: $0 \leq y \leq 5$ ✓

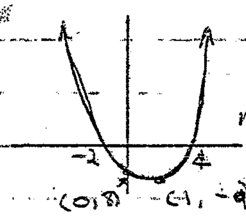
ii). $y = \frac{2}{x-3}$. D: $x \neq 3$. ✓

R: $y \neq 0$ ✓

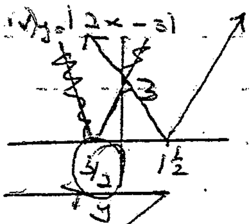


iii) $y = x^2-2x-8$. D: All real x ✓

$(x-4)(x+2)$. R: $y \geq -9$ ✓

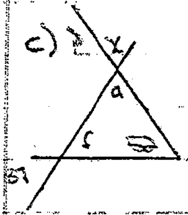


iv) $y = |2x-3|$



D: All real x ✓

R: $y \geq 0$. ✓

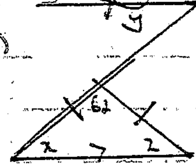
c) 

$f = 37$ (vert opp.)

$a = 91$ (Angle sum)

(vert. opp)

$\underline{\therefore x = 91}$ ✓

ii) 

$z = y$ (alternate ∠'s)

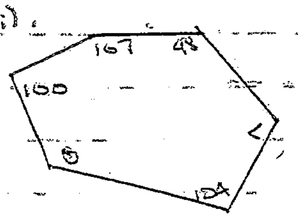
$x = z$ (base ∠'s of Isos △)

$\therefore y = z$

$\therefore 180 = 62 + 2y$ (Angle sum).

$118 = 2y$

$\underline{y = 59}$ ✓

iii). 

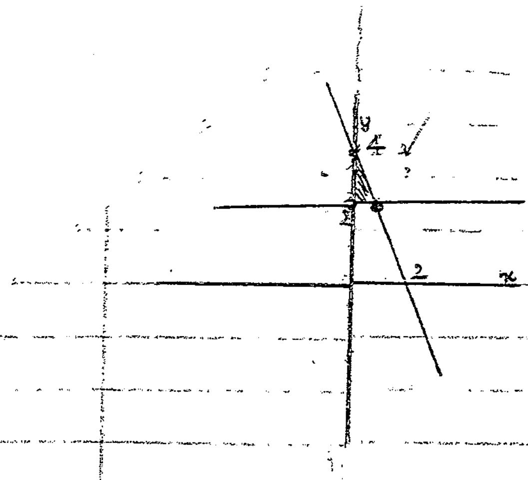
$n = 6$ sides

Angle sum $= (6-2) \times 180$

$= 720$. ✓

$\therefore \theta = 720 - 499$

$\underline{= 221}$. ✓



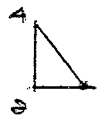
$y = 2$

$y = -2x + 4$

$2 = -2x + 4$

$-2 = -2x$

$x = 1$



Area = ½ bh

$= \frac{1}{2} \times 2 \times 1$

$= 1$ unit²

3. a) i) $\tan 225 = 1$

ii) $\cos 315 = \frac{1}{\sqrt{2}}$

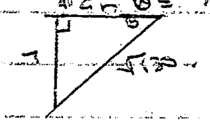
b) $\frac{3^m \times 9^{m+1}}{27^{2m}} = \frac{3^m \times 3^{2m+2}}{3^{6m}} = \frac{3^{3m} \times 3^2}{3^{6m}} = \frac{3^2}{3^{3m}} = 3^{2-3m}$

c) $2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\theta = 60, 300$

d) $\tan\theta = 7/9$



$\cos\theta = \frac{9}{\sqrt{130}}$

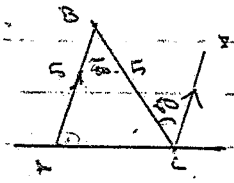
e) $\tan\theta \times \operatorname{cosec}\theta$

$= \frac{\sin\theta}{\cos\theta} \times \frac{1}{\sin\theta}$

$= \frac{1}{\cos\theta}$

$= \sec\theta$

f)



i) $A\hat{B}C = 80°$ (alternate ∠'s ∥ lines).

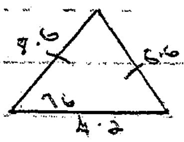
$BAC = 50°$ (Angle sum & base Angles of isos Δ).

ii) $\frac{5}{\sin 50} = \frac{AC}{\sin 80}$

$AC = \frac{5 \sin 80}{\sin 50}$

$= 6.43$

g)

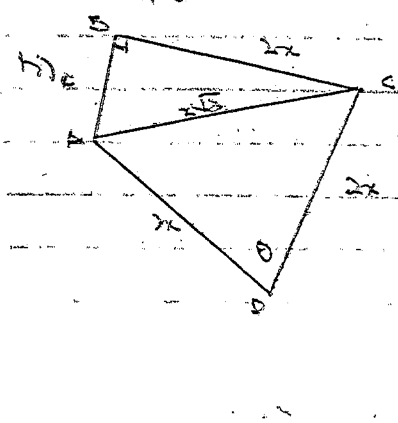


Area = ½ ab sin C

$= \frac{1}{2} \times 8.6 \times 4.2 \times \sin 76$

$= 17.5$ cm²

h)



i) $AC^2 = x^2 + 4x^2$

$= ~~3x~~ \; 5x^2$

$AC = \sqrt{5x^2}$ or $= x\sqrt{5}$

ii) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$= \frac{9 + 4 - 5}{12}$

$A = ~~[illegible]~~ \; 48° 11'$

4 a) $f(x) = x^3$

$f(-x) = (-x)^3$

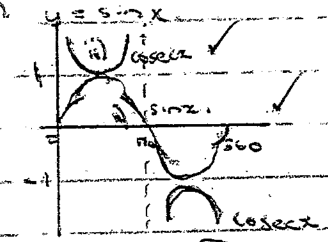
$= -x^3$ ✓

$-f(x) = -x^3$ ✓

$\therefore f(-x) = -f(x)$

$\therefore$ odd.

b) $y = \sin x$



c) $\sin\theta = \frac{\sqrt{5}}{3}$

2nd Quad; i) $\cos\theta = \frac{2}{3}$

$= -2/3$ ✓

ii) $\tan\theta = \frac{\sqrt{5}}{2}$

$= -\frac{\sqrt{5}}{2}$ ✓

d) $\sin^4\theta - \cos^4\theta = 2\sin^2\theta - 1$

$LHS = (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)$

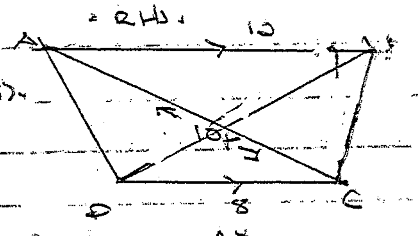
$= 1 \times [\sin^2\theta - (1 - \sin^2\theta)]$ ✓

$= -1 + 2\sin^2\theta$ ✓

$= 2\sin^2\theta - 1$ ✓

$= RHS$.

e) i)



i) In Δ's AXB & CXD

$A\hat{X}B = D\hat{X}C$ (vert opp) ✓

$A\hat{B}D = B\hat{D}C$ (alternate L's) ✓

$B\hat{A}C = A\hat{C}D$ (" ") ✓

$\therefore \Delta AXB \parallel\parallel \Delta CXD$. ✓

ii) $\frac{AB}{DC} = \frac{AX}{CX}$.

$\frac{12}{8} = \frac{AX}{BX}$ ✓ AX : BX

$1\frac{1}{2} : 1$.

Q 4(g) missing

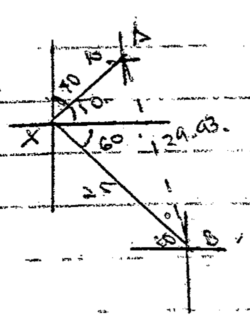
$\therefore AX + BX = 10$. ✓

$2\frac{1}{2} BX = 10$.

$BX = 4$. ✓

$AX = 6$.

f)



$\angle AXB = 110°$ ✓

$a^2 = b^2 + c^2 - 2bc\cos A$.

$= 100 + 625 - 500\cos 110$.

$a^2 =$ ~~846.01~~ 696.01

$a =$ ~~24.13~~ 26.38

$\frac{\sin A\hat{B}X}{10} = \frac{\sin 110}{\text{~~24.405~~} \; 26.38}$

$\sin A\hat{B}X = \frac{10\sin 110}{26.38}$ $\therefore A\hat{B}X = 18°18'$

Bearing is $348°18'$