Sydney Girls' High School

LABOR OMNIA VINCIT

2005
MATHEMATICS EXTENSION 1

YEAR 11
YEARLY EXAMINATION

Time Allowed: 75 minutes

TOPICS: Calculus, Probability, Sequences & Series, Quadratic Equations and Harder 2U.

Directions to Candidates

- There are four (4) questions.
- Attempt ALL questions.
- Questions are of equal value.
- Start each question on a new page.
- Write on one side of the paper only.
- Show all necessary working. Marks will be deducted for careless or badly arranged work.
- Diagrams are NOT drawn to scale.
- Board-approved calculators may be used.

Total: 80 marks

QUESTION 1 (20 marks)

Marks

a) If $x^2 - 4x + 2 = 0$, find the value of

(i) $\alpha + \beta$ 1

(ii) $\alpha\beta$ 1

(iii) $\alpha^2 + \beta^2$ 2

(iv) $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$ 1

b) Solve for x if: $\dfrac{1}{x - 1} \leq 2 \quad (x \neq 1)$ 4

c) There are five black pens and three red pens in a pencil case.
If two pens are chosen at random, find the probability that both pens are of the same colour. 3

d) If $\dfrac{d^2y}{dx^2} = -6x$ and when $x = 1$, $\dfrac{dy}{dx} = 1$ and $y = 1$. Find y when $x = 0$. 4

e) If $n^2 \equiv a(n^2 + n) + b(n - 2) + c$ find the values of the constants a, b, and c. 4

QUESTION 2 (20 marks) Marks

a) Differentiate

(i) $y = 3x^5 - 4x + 1$ 1

(ii) $y = (x + 1)(2x - 3)^4$ 2

(iii) $y = \dfrac{x}{1 - x}$ 2

b) Find the value of k for which $x^2 + 6x - 3k = 0$ has roots where one is the reciprocal of the other. 3

c) In a class of 25 students, 4 study both Mathematics and Economics, 10 study Mathematics only and 9 study neither Mathematics nor Economics.
If a student is selected at random,
find the probability of the event that the student chosen

(i) studies Mathematics only 1
(ii) studies Economics only 1
(iii) does not study Mathematics. 1

d) Find the equation of the normal to the curve $y = x^2 + \dfrac{5}{x} - 2$ at the point where $x = 1$. 3

e) A road building company has to distribute a pile of sand along a straight road using only one truck.
If each truckload has to be deposited at 60 metre intervals,

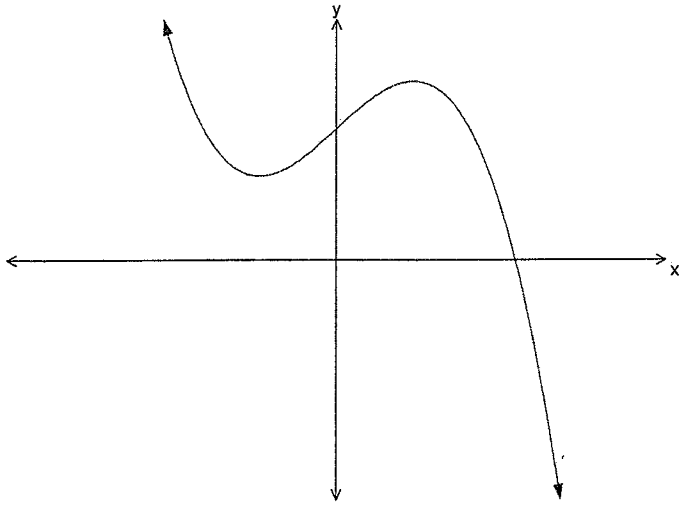
(i) Find how far the truck has travelled after delivering the n^{th} truckload and returned to the pile. 3

(ii) If Trudy the truck driver has a tea-break after travelling 7920 metres. How many truckloads has she distributed? 3

QUESTION 3 (20 marks) Marks

a) For what values of p is the equation $x^2 = (p+2)x + (2-3p)$ positive definite? 3

b) The diagram shows the graph of a certain function f(x)



(i) Copy diagram onto your examination paper

(ii) On the *same* set of axes, draw a sketch of the derivative $f'(x)$ of the function. 2

c) Monica buys five tickets in a raffle in which 20 tickets are sold.
Three different tickets are to be drawn out for first, second and third prizes.

Using a tree diagram find the probability that Monica

(i) wins all three prizes 1

(ii) does not win a prize 1

(iii) wins at least one prize 1

(iv) wins exactly one prize. 2

d) Given the series $\frac{1}{2x} + \frac{1}{4x^2} + \frac{1}{8x^3} + \ldots$

(i) Find the limiting sum in terms of x 3

(ii) Hence, find the value of x when the limiting sum is $\frac{5}{7}$. 2

e) Prove by Mathematical Induction that

$$1^2 + 3^2 + 5^2 + 7^2 + \ldots + (2n-1)^2 = \frac{n}{3}(2n-1)(2n+1)$$ 5

QUESTION 4 (20 marks)

Marks

a) Solve for x: $x^2 + \frac{112}{x^2} = 23$ 4

b) Given $y = 1 + 3x - x^3$

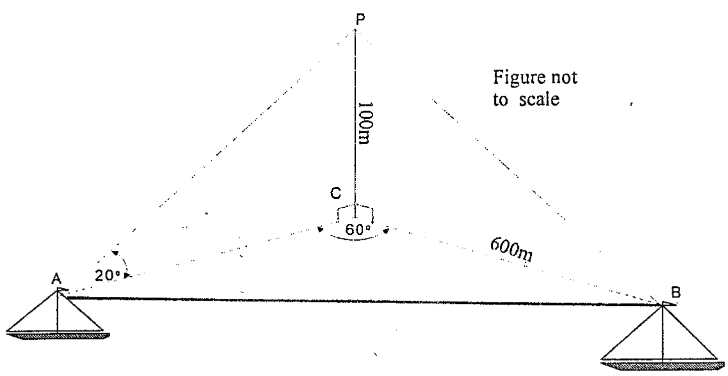
(i) Find the stationary points and determine their nature. 4

(ii) Find the point(s) of inflexion. 2

(iii) Sketch the curve for $-2 \le x \le 3$. 1

(iv) What is the minimum value of y for $-2 \le x \le 3$? 1

c)



Two yachts A and B subtend an angle of 60° at the base C of a cliff.
From yacht A the angle of elevation of the point P, 100 metres vertically above C, is 20°.
Yacht B is 600metres from C.

(i) Calculate the length AC (correct to 1 decimal place). 2

(ii) Calculate the distance between the two yachts to the nearest metre. 2

d) Michelle agrees to pay off her credit card debt of $10 000 by paying equal monthly instalments over 3 years. 4
If the interest rate of 18% p.a. is charged on the balance owing each month, what will be the monthly instalment.

THE END

SGHS 2005 Maths Ext 1 Trial Yearly Exam.

Question 1

-1-

$\frac{77}{80}$

a (i) $\alpha + \beta = \frac{-b}{a}$

$= 4$ ✓

(ii) $\alpha\beta = \frac{c}{a}$

$= 2$ ✓

(iii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 4^2 - 2(2)$ ✓

$= 16 - 4$

$= 12$ ✓

(iv) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$

$= \frac{12}{4}$ ✓

$= 3$ ✓

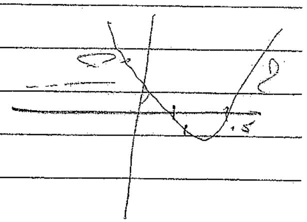
b) $\frac{1}{x-1} \leq 2$

$x - 1 \leq 2(x-1)^2$ ✓

$0 \leq 2(x-1)^2 - (x-1)$

$0 \leq (x-1)(2x-2-1)$

$0 \leq (x-1)(2x-3)$



$x \leq 1 \quad x \geq \frac{3}{2}$ ✓

Question 1

-2-

c) $P(\text{Same Sex}) = P(BB) + P(GG)$

$P(BB) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$

$= \frac{5}{14}$ ✓

$P(GG) = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56}$

$= \frac{3}{28}$ ✓

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$\therefore P(SS) = \frac{13}{28}$ ✓

d) $\frac{d^2y}{dx^2} = -6x \quad \therefore \frac{dy}{dx} = \frac{-6x^2}{2} + c$

$= -3x^2 + c$ ✓

when $x = 1$, $\frac{dy}{dx} = 1$

$1 = -3 + c$

$c = 4$ ✓

$\therefore \frac{dy}{dx} = 4 - 3x^2$

$\therefore y = 4x - x^3 + c$

$1 = 4 - 1 + c$ ✓

$= 3 + c$

$c = -2$

$\therefore y = 4x - x^3 - 2$ ✓

when $x = 0$, $y = -2$

(e) $\therefore \; n^2 \equiv an^2 + (a+b)n + (c-2b)$ ✓

$\therefore \; a = 1$, $a+b = 0$ — $b = -1$, $c - 2b = 0$ — $c = -2$ ✓

$\therefore \; a = 1, \; b = -1, \; c = -2$ ✓

Question 2

a) (i) $y = 3x^5 - 4x + 1$

$y' = 15x^4 - 4$ ✓

(ii) $y = (x+1)(2x-3)^4$ $\quad u = x+1 \quad v = (2x-3)^4$

$u' = 1 \quad v' = 8(2x-3)^3$

$y' = (2x-3)^4 + 8(x+1)(2x-3)^3$ ✓ ⇒ factorise

$= (2x-3)^3\left[2x - 3 + 8x + 8\right] = (2x-3)^3(10x+5) = 5(2x-3)^3(2x+1)$

(iii) $y = \frac{x}{1-x}$ $\quad u = x \quad v = 1-x$

$u' = 1 \quad v' = -1$

$y' = \frac{1 - x + x}{(1-x)^2}$ ✓

$= \frac{1}{(1-x)^2}$ ✓

b) $x^2 + 6x - 3k = 0$

has roots α & $\frac{1}{\alpha}$

$\therefore \; \alpha + \frac{1}{\alpha} = -6$

$\alpha \cdot \frac{1}{\alpha} = -3k$

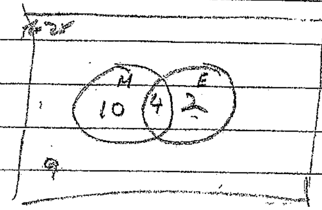
$1 = -3k$ ✓ $\rightarrow k = -\frac{1}{3}$

Question 2

c) (i) $\frac{10}{25}$ ✓ $= \frac{2}{5}$

(ii) $\frac{2}{25}$ ✓

(iii) $\frac{4}{25}$ ✓



d) $y = x^2 + 5(x)^{-1} - 2$

$y' = 2x - 5(x)^{-2}$

at $(1,4)$ $\; y' = -3$.

$\therefore \; m_{\perp} = \frac{1}{3}$ ✓

$y - 4 = \frac{1}{3}(x-1)$

$3y - 12 = x - 1$

$x - 3y + 11 = 0$ ✓

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e) (i) Let D_n be the distance travelled to delivery the nth load and back

$D_1 = 60 \times 2 = 120$

$D_2 = 2(60) \times 2 = 120(2)$ ✓

$D_3 = 3(60) \times 2 = 120(3)$ ✓

$\vdots$

$D_n = 60n \times 2$

$= 120n$

$\therefore \; S_n = \frac{n}{2}\left(240 + (120n - 120)\right)$ ✓

Question 2

e)(ii)

$$7920 = \frac{n}{2}(120 + 120n)$$

$$15840 = 120(n^2+n)$$

$$n^2 + n - 132 = 0$$

$$(n+12)(n-11) = 0$$

$$\therefore n = 11 \quad (n \geq 0)$$

$\therefore$ after 11 trips she ... a ...

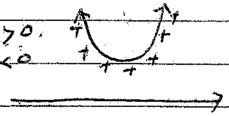
Question 3

a)

$$x^2 - (p+2)x + (2-3p) \geq 0$$

$a > 0$

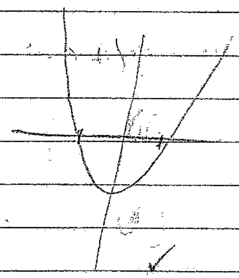
$\Delta < 0$



$$[-(p+2)]^2 - 4(2-3p) \geq 0 \quad <$$

$$p^2 + 4p + 4 - 8 + 12p \geq 0 \quad <$$

$$p^2 + 16p - 4 \geq 0$$



$$p = \frac{-16 \pm \sqrt{272}}{2}$$

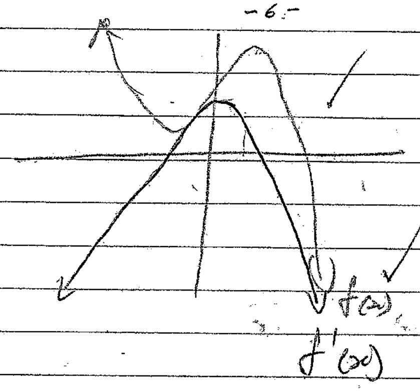
$$\approx 0.246 \text{ or } -16.246$$

$$-16.246 < p < 0.246$$

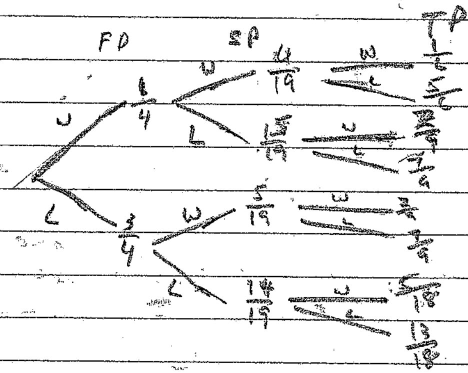
$\therefore$ ~~$p > 0.246$ or $p < -16.246$~~

b) (i) assume this diagram is copied.
Paper will not be wasted on account that I have copy of the original test paper.

b) (ii)



c) (i)



$$\therefore P(WWW) = \frac{1}{114}$$

(ii) $P(LLL) = \frac{91}{228}$

(iii) $P(\text{at least } 1) = \frac{137}{228}$

(iv) $P(\text{win only } 1) = P(WLL) + P(LWL) + P(LLW)$

$$= \frac{35}{228} + \frac{35}{228} + \frac{35}{228}$$

$$= \frac{35}{76}$$

Question 3

d (i) for series $a = \frac{1}{2x}$

$r = \frac{1}{2x}$

$\therefore \quad LS = \frac{a}{1-r}$

$= \frac{\frac{1}{2x}}{1-\frac{1}{2x}}$

$= \frac{\frac{1}{2x}}{\frac{2x-1}{2x}}$

$= \frac{1}{2x-1}$

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(ii) $\frac{1}{2x-1} = \frac{5}{7}$

$7 = 10x - 5$

$12 = 10x$

$x = 1.2$

e) Prove $P(1)$ is true.

$LHS: 1^2 = 1$

$RHS = \frac{1}{3}(1)(3) = 1$

$\therefore LHS = RHS \therefore P(1)$ is true

Assume $P(k)$ is true.

Prove $P(k+1)$ is true if $P(k)$ is true.

$P(k+1) = 1^2 + 3^2 + \dots + (2k-1)^2 + (2k+1)^2$

$= P(k) + (2k+1)^2$

$= \frac{k}{3}(2k+1)(2k-1) + (2k+1)^2$

$= (2n+1)\left[\frac{n}{3}(2n-1) + 2n+1\right]$

$= (2n+1)\left[\frac{2n^2}{3} - \frac{n}{3} + 2n+1\right]$

$= (2n+1)\left[\frac{2n^2}{3} - \frac{n}{3} + \frac{6n}{3} + 1\right]$

$= (2n+1)\left[\frac{2n^2+5n}{3} + 1\right]$

$= (2n+1)\left[\frac{2n^2+5n+3}{3}\right]$

$= 2n+1\left[\frac{(2n+3)(n+1)}{3}\right]$

$= \frac{k+1}{3}(2k+1)(2k+3)$

$= RHS.$

$\therefore$ By the principle of Mathematical Induction

$1^2 + 3^2 + 5^2 + 7^2 + \dots + (2n-1)^2 = \frac{n}{3}(2n+1)(2n-1)$

SGHS 2003 Maths Ext 1 Half Yearly Exam.

Question 4

a) $x^2 + \frac{112}{x^2} = 23$

$x^4 + 112 = 23x^2$

$x^4 - 23x^2 + 112 = 0$

let $u = x^2$

$u^2 - 23u + 112 = 0$ ✓

$(u-16)(u-7) = 0$

$\therefore x^2 = 16$ ✓ & $x^2 = 7$ ✓

$x = \pm 4$ $\quad x = \pm\sqrt{7}$

b) (i) $y = 1 + 3x - x^3$

$y' = 3 - 3x^2$

$y'' = -6x$

st pt at $y' = 0$ $\therefore$ $3 - 3x^2 = 0$

$3(1 - x^2) = 0$

$x = \pm 1$

at $(1, 3)$ ✓ $\quad$ at $(-1, -1)$

$y'' = -6$ $\quad$ $y'' = 6$ ✓

$\therefore$ concave down ✓ $\quad$ $\therefore$ concave up ✓

$\therefore$ max $\quad$ $\therefore$ min

(ii) P.O.I at $y'' = 0$ & change of concavity

$y'' = 0$ when $x = 0$

x	-0.5	0	0.5

✓ $\therefore$ change of concavity

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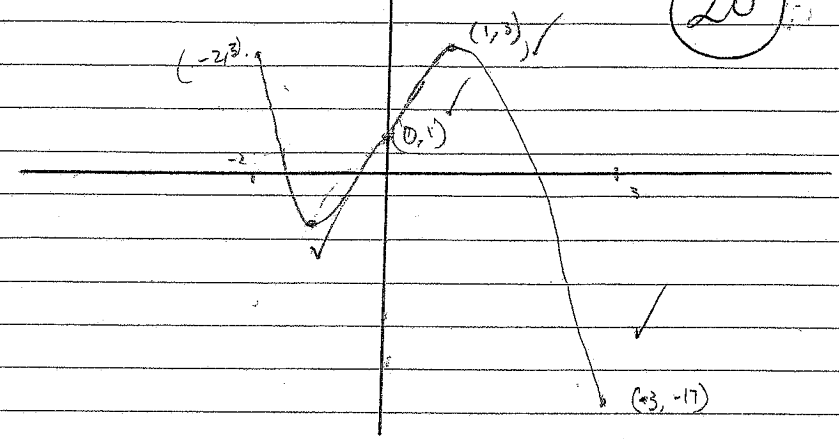
b) (iii)



(iv) $y = -17$ ✓ is the minimum y value

c) (i) $\tan 20° = \frac{100}{AC}$

$\frac{100}{\tan 20°} = AC$ ✓

$AC = 274.7$ m ✓ (1 d.p)

(ii) $AB^2 = AC^2 + 600^2 - 2(AC)(600)(\cos 60)$ ✓

$\approx 270,637.6765$

$\therefore AB = 520$ m. ✓

d) let A_n be the amount owing after the N^{th} payment

$A_1 = (10\ 000)(1.18) - M$

$A_2 = A_1(1.18) - M$

$= (10\ 000)(1.18)^2 - M(1 + 1.18)$ ✓

(d) $\therefore A_{36} = 10\,000(1.18)^{36} - M\left(\frac{1.18^{36}-1}{0.18}\right)$

$= 0$

$\therefore 10\,000\,(1.18)^{36} = M\left(\frac{1.18^{36}-1}{0.18}\right)$ ✓

$$M = \frac{10000\,(1.18)^{36}}{\left(\frac{1.18^{36}-1}{0.18}\right)}$$

$= \$1,804.66$ ✓