SYDNEY GIRLS HIGH SCHOOL

MATHEMATICS

4 Unit

Year 12

March 2000

Time Allowed: 90 minutes

Instructions:

- There are nine (9) questions
- Attempt all questions
- Questions are <u>not</u> of equal value
- All necessary working should be shown. Marks may be deducted for careless or badly arranged work
- Write on one side of the paper only.

Name: ....

Question 1

(4) a) Factorise the following quadratics into linear complex factors

(i) x^2+2x+2

(ii) $4x^2+4x+2$

(8) b) If the complex number z is equal to $-2i$, find in cartesian form, and plot on an Argand Diagram, the following complex numbers.

(i) $\bar{z}$

(ii $z+\bar{z}$

(iii) $z\bar{z}$

(iv) $z+i\bar{z}$

Question 2.

(6) a) (i) Find the solution to $\sqrt{16+30i}$

(ii) Solve for complex z if: $z^2-z-4=i(z+7)$

(6) b) If w is a cube root of 1, simplify

(i) $1+2w+w^2$

(ii) $\dfrac{1+2w+3w^2}{2+3w+w^2}$

(iii) $(2+2w^2)^4$

Question 3.

(6) a) (i) Given de Moivres Theorem, $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ is true for positive integers, show that it is also true for negative integers.

(ii) If $z = (1 - i\sqrt{3})$, express z^{-4} in the simplest cartesian form.

(3) b) Sketch the region in the Argand diagram consisting of those points z for which $|\arg z| < \frac{\pi}{3}, z + \bar{z} < 4, and |z| \geq 2$

(3) c) Draw the locus of z given $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$

Question 4.

Consider the equation $z^5 = -1$

(5) a) Solve for complex z and plot the solutions on the Argand Diagram

(3) b) Express the factors of $z^5 + 1$ over:

(i) the complex field
(ii) the real field
(iii) the rational field

(4) c) Factorise $z^4 - z^3 + z^2 - z + 1$ over the real field and hence or otherwise show that $\cos\frac{\pi}{5}\cos\frac{3\pi}{5} = -\frac{1}{4}$

Question 5.

(4) a) Let $z_1 = 4 + 8i$, $z_2 = -4 - 8i$

Show that the locus specified by $|z - z_1| = 3|z - z_2|$ is a circle. Give its centre and radius.

(3) b) Describe the locus of the complex number w, where $w = \frac{z-4}{z}$ and $|z| = 2$

(6) c) Sketch $|z - 2i| = 1$

Find the maximum and minimum value of

(i) $\arg z$
(ii) $|z|$

Question 6.

(3) a) Using De Moivre's Theorem, express $\sin 3\theta$ and $\cos 3\theta$ in terms of $\sin\theta$ and $\cos\theta$

(3) b) Hence, express $\tan 3\theta$ in terms of $\tan\theta$

(3) c) Solve $\tan 3\theta = 1$ for $0 < \theta < \pi$

(3) d) Hence, explain why $\tan\frac{\pi}{12}$, $\tan\frac{5\pi}{12}$ and $\tan\frac{3\pi}{4}$ are roots of $x^3 - 3x^2 - 3x + 1 = 0$ hence evaluate $\tan\frac{\pi}{12} + \tan\frac{5\pi}{12} + \tan\frac{3\pi}{4}$

Question 7.

(8) a) (i) Express $\sin^6\theta$ in the form of $A \overset{\cos}{\cancel{\sin}}\, 6\theta + B \overset{\cos}{\cancel{\sin}}\, 4\theta + C \overset{\cos}{\cancel{\sin}}\, 2\theta + D$

(ii) Hence, or otherwise, find $\int \sin^6\theta\, d\theta$

(4) b) OABC is a rhombus where $< AOC = 60^o$.

Given that A represents the complex number $2 + i$, give the complex numbers represented by points B, C.

Question 8.

(6) a) Two roots of a polynomial with rational coefficients are, $(1-\sqrt{2})$ and $(2+i)$. Find the monic polynomial with the lowest degree that satisfies these roots, and express in the form $ax^n + bx^{n-1} + \ldots$

(6) b) Given that $x^3 + px^2 + qx + r = 0$ has roots α, β, γ

(i) Find the equation whose roots are $\alpha^2, \beta^2, \gamma^2$

(ii) Evaluate $\alpha^3 + \beta^3 + \gamma^3$

Question 9.

(4) a) Solve $x^4 - 6x^3 + 12x^2 - 10x + 3 = 0$ given that the equation has a root of multiplicity 3

(4) b) The remainder when $x^3 + px + q$ is divided by $(x-2)(x+3)$ is $2x + 1$ Find p and q.

(5) c) w is a cube root of 1. Let α and β be real numbers. Find in its simplest form the cubic equation whose roots are $\alpha + \beta$, $\alpha w + \beta w^{-1}$, $\alpha w^2 + \beta w^{-2}$

2000 March 4U Task.

Question 1

a) i) $x^2+2x+2=0$ (say)

$x = \dfrac{-2 \pm \sqrt{4-8}}{2} = \dfrac{-2 \pm \sqrt{-4}}{2}$

$= \dfrac{-2 \pm 2i}{2} \quad \therefore x = -1 \pm i$

$x^2+2x+2 = [x-(-1+i)][x-(-1-i)]$
$= (x+1-i)(x+1+i)$

OR $x^2+2x+2 = x^2+2x+1+1$
$= (x+1)^2+1$
$= (x+1)^2 - i^2$
$= (x+1+i)(x+1-i)$

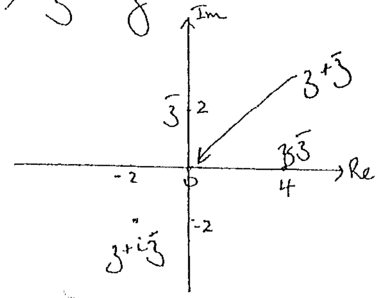
ii) $4x^2+4x+2 = 4x^2+4x+1+1$
$= (2x+1)^2 - i^2$
$= (2x+1+i)(2x+1-i)$

b) $z = -2i \qquad i\bar{z} = -2$

i) $\bar{z} = 2i$ ii) $z+\bar{z} = -2i+2i = 0$

iii) $z\bar{z} = -2i \cdot 2i = 4$

iv) $z + i\bar{z} = -2i - 2$



Question 2

a) i) let $z = \sqrt{16+30i}$

$z = x+iy$

$\therefore (x+iy)^2 = 16+30i$

$x^2-y^2 = 16$

and $2ixy = 30i \Rightarrow 2xy = 30$

now $(x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2$
$= 16^2 + 30^2$

$\therefore x^2+y^2 = 34 \quad (x^2+y^2>0)$

$\therefore x^2+y^2 = 34$ (1)
$x^2-y^2 = 16$ (2)
$2x^2 = 50$
$x = \pm 5$
$y = \pm 3$

$\therefore z = 5+3i$ or $z = -5-3i$

ii) $z^2 - z - 4 = iz + 7i$

$z^2 - z(1+i) - (4+7i) = 0$

$z = \dfrac{(1+i) \pm \sqrt{(1+i)^2 + 4(4+7i)}}{2}$

$= \dfrac{(1+i) \pm \sqrt{2i+16+28i}}{2}$

$= \dfrac{(1+i) \pm \sqrt{16+30i}}{2}$

$= \dfrac{(1+i) \pm (5+3i)}{2}$

$\therefore z = \dfrac{6+4i}{2}$ or $\dfrac{-4-2i}{2}$

$= 3+2i, \; -2-i$

b) $w^3 = 1$ and $1+w+w^2 = 0$

i) $1+2w+w^2 = 1+2w+(-1-w)$
$= w$

ii) $\dfrac{1+2w+3w^2}{2+3w+w^2} = \dfrac{w(1+2w+3w^2)}{w(2+3w+w^2)}$

$= \dfrac{w(1+2w+3w^2)}{2w+3w^2+w^3} = w$

iii) $2^4(1+w^2)^4 = 16(-w)^4 = 16w^4$
$= 16w \cdot w^3 = 16w$

Question 3

a) i) $(\cos\theta + i\sin\theta)^n$
$= \cos n\theta + i\sin n\theta$

let $n = -m$ where m is positive

$(\cos\theta + i\sin\theta)^{-m} = \cos -m\theta + i\sin -m\theta$

LHS : $\dfrac{1}{(\cos\theta+i\sin\theta)^m} = \dfrac{1}{\cos m\theta + i\sin m\theta}$

$= \dfrac{\cos m\theta - i\sin m\theta}{\cos^2 m\theta + \sin^2 m\theta}$

$= \cos m\theta - i\sin m\theta$

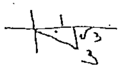
Also : $\cos -m\theta = \cos m\theta$ (even fn)
$\sin -m\theta = -\sin m\theta$ (odd fn)

$\therefore$ RHS $= \cos m\theta - i\sin m\theta$
$=$ LHS

ii) $z = 1 - i\sqrt{3} \qquad |z| = \sqrt{1+3} = 2$

$\arg z = \theta$ where $\tan\theta = -\sqrt{3}$

$\therefore \arg z = -\dfrac{\pi}{3}$



$\therefore z^{-4} = \left[2\left(\cos -\dfrac{\pi}{3} + i\sin -\dfrac{\pi}{3}\right)\right]^{-4}$

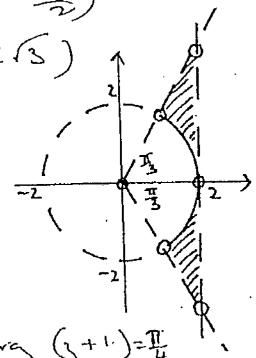
$= \dfrac{1}{2^4}\left(\cos + \dfrac{4\pi}{3} + i\sin \dfrac{4\pi}{3}\right)$

$= \dfrac{1}{16}\left(-\dfrac{1}{2} - i\dfrac{\sqrt{3}}{2}\right)$

$= \dfrac{-1}{32}(1 + i\sqrt{3})$

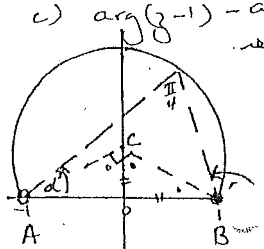
b)

$z + \bar{z} < 4$
$2x < 4$
$x < 2$



c) $\arg(z-1) - \arg(z+1) = \dfrac{\pi}{4}$

i.e. $\theta - \alpha = \dfrac{\pi}{4}$



Upper arc of circle with chord AB — centre $(0,1)$ radius $\sqrt{2}$ with $(-1,0)$ excluded.

Question 4

$z^5 = -1$

let $z = r(\cos\theta + i\sin\theta)$

$z^5 = r(\cos 5\theta + i\sin 5\theta)$ (by De M th)

$-1 = 1(\cos\pi + i\sin\pi) \quad \therefore r$

$\therefore \cos 5\theta = \cos(\pi + 2k\pi)$

$5\theta = \pi + 2k\pi$

$\theta = \dfrac{\pi + 2k\pi}{5}$

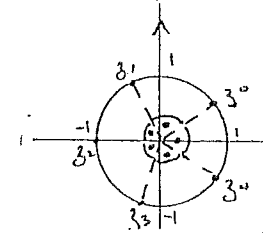
$\therefore z_0 = \cos\dfrac{\pi}{5} + i\sin\dfrac{\pi}{5}$

$z_1 = \cos\dfrac{3\pi}{5} + i\sin\dfrac{3\pi}{5}$

$z_2 = \cos\dfrac{5\pi}{5} + i\sin\dfrac{5\pi}{5} = -$

$z_3 = \cos\dfrac{7\pi}{5} + i\sin\dfrac{7\pi}{5} = \bar{z}_1$

$z_4 = \cos\dfrac{9\pi}{5} + i\sin\dfrac{9\pi}{5} = \bar{z}_0$



pts equally spaced around unit cir

b) $z^5+1 = (z+1)(z-z_0)(z-z_4)(z-z_1)(z-z_3)$
$= (z+1)[(z-z_0)(z-\bar{z}_0)][(z-z_1)(z-\bar{z}_1)]$
$= (z+1)[z^2 - z(z_0+\bar{z}_0) + z_0\bar{z}_0][z^2 - z(z_1+\bar{z}_1) + z$
$= (z+1)(z^2 - 2\cos\frac{\pi}{5}z + 1)(z^2 - 2\cos\frac{3\pi}{5}z$
$= (z+1)(z^4 - z^3 + z^2 - z + 1)$ by long division

c) $z^4 - z^3 + z^2 - z + 1$
$= (z^2 - 2\cos\frac{\pi}{5}z + 1)(z^2 - 2\cos\frac{3\pi}{5}$

let $z^2 = -1$ i.e. $z = i$

$(-1)^2 - (-1)i + -1 - i + 1$
$= (-2\cos\frac{\pi}{5}i)(-2\cos\frac{3\pi}{5}i$

$\cancel{1+i}\,\cancel{-1-i} + 1 = 4\cos\frac{\pi}{5}\cos$

$\therefore \cos\dfrac{\pi}{5}\cos\dfrac{3\pi}{5} = \dfrac{1}{-4}$

or equate coeffs of z^2 in (*)

$1 = 1 + 4\cos\frac{\pi}{5}\cos\frac{3\pi}{5} + 1$

$1 = 2 + 4\cos\frac{\pi}{5}\cos\frac{3\pi}{5}$

$\cos\dfrac{\pi}{5}\cos\dfrac{3\pi}{5} = \dfrac{-1}{4}$

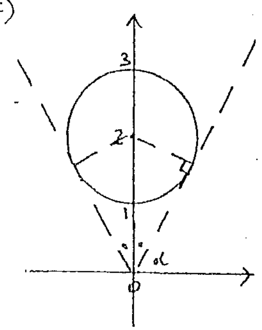
Question ~

ii) $|z-(4+8i)| = |(x-4)+i(y-8)|$
$= \sqrt{(x-4)^2+(y-8)^2}$
$|z-(-4-8i)| = |(x+4)+i(y+8)|$
$= \sqrt{(x+4)^2+(y+8)^2}$

now:
$(x-4)^2+(y-8)^2 = 9\left[(x+4)^2+(y+8)^2\right]$
$x^2-8x+16+y^2-16y+64$
$= 9x^2+72x+144+9y^2+144y+576$
$\therefore 8x^2+80x+8y^2+160y = 80-720$
$x^2+10x+y^2+20y = -80$
$(x+5)^2+(y+10)^2 = -80+25+100$
$= 45$
which represents a circle with centre $(-5,-10)$ rad $= 3\sqrt{5}$ u

b) $w = \frac{z-4}{z}$
$wz = z-4$
$4 = z-wz$
$= z(1-w)$
$|4| = |z||1-w|$
$\therefore 2|w-1| = 4 \Rightarrow |w-1| = 2$
circle, centre $(1,0)$ rad $= 2$ u

c)



$\alpha = 90-30°$
$= 60$
min $\arg z = 60°$
max $\arg z = 120°$
min $|z| = 1$
max $|z| = 3$

Question ~

a) $(\cos\theta+i\sin\theta)^3 = \cos 3\theta+i\sin 3\theta$
$(\cos\theta+i\sin\theta)^3 = \cos^3\theta+3\cos^2\theta(i\sin\theta)$
$+3\cos\theta(i\sin\theta)^2+(i\sin\theta)^3$
$\therefore \cos 3\theta = \cos^3\theta-3\cos\theta\sin^2\theta$
$\sin 3\theta = 3\cos^2\theta\sin\theta-\sin^3\theta$

b) $\tan 3\theta = \frac{\sin 3\theta}{\cos 3\theta}$
$= \frac{3\cos^2\theta\sin\theta-\sin^3\theta}{\cos^3\theta-3\cos\theta\sin^2\theta}$
$= \frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$

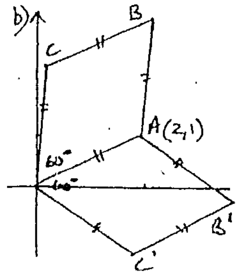
c) $\tan 3\theta = 1$
$3\theta = \frac{\pi}{4}, \pi+\frac{\pi}{4}, \frac{9\pi}{4},$
$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12},$

d) $\tan 3\theta = 1$
$\therefore \frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta} = 1$
$3\tan\theta-\tan^3\theta = 1-3\tan^2\theta$
$\tan^3\theta-3\tan^2\theta-3\tan\theta+1 = 0$
let $\tan\theta = x$
$x^3-3x^2-3x+1 = 0$
solutions are $x = \tan\theta$
ie $x = \tan\frac{\pi}{12}$,
$x = \tan\frac{5\pi}{12}$
$x = \tan\frac{3\pi}{4}$.
now sum of roots $= 3$
ie $\tan\frac{\pi}{12}+\tan\frac{5\pi}{12}+\tan\frac{3\pi}{4} = 3$

Question 7

a) $z+\frac{1}{z} = 2\cos\theta$
$z-\frac{1}{z} = 2i\sin\theta$
$z^n+\frac{1}{z^n} = 2\cos n\theta$
$\left(z-\frac{1}{z}\right)^6 = (2i\sin\theta)^6 = 64i^6\sin^6\theta$
$= 64(i^2)^3\sin^6\theta$
$= -64\sin^6\theta$
$\left(z-\frac{1}{z}\right)^6 = z^6-6z^5\cdot\frac{1}{z}+15z^4\cdot\frac{1}{z^2}$
$-20z^3\frac{1}{z^3}+15z^2\frac{1}{z^4}-6z\frac{1}{z^5}+\frac{1}{z^6}$
$-64\sin^6\theta = z^6-6z^4+15z^2-20$
$+\frac{15}{z^2}-\frac{6}{z^4}+\frac{1}{z^6}$
$= z^6+\frac{1}{z^6}-6\left(z^4+\frac{1}{z^4}\right)$
$+15\left(z^2+\frac{1}{z^2}\right)-20$
$\sin^6\theta = \frac{-1}{64}\big(2\cos 6\theta-12\cos 4\theta$
$+30\cos 2\theta-20\big)$
$\int\sin^6\theta\,d\theta = \frac{-1}{64}\int\big(2\cos 6\theta-12\cos 4\theta$
$+30\cos 2\theta-20\big)\,d\theta$
$= \frac{-1}{64}\left(\frac{\sin 6\theta}{3}-3\sin 4\theta+15\sin 2\theta-20\theta\right)$

b)



find C:
$(2+i)(\cos 60°+i\sin 60°)$
$= (2+i)\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$
$= 1+i\sqrt{3}+\frac{i}{2}-\frac{\sqrt{3}}{2}$
$\Rightarrow \left(1-\frac{\sqrt{3}}{2}\right)+i\left(\sqrt{3}+\frac{1}{2}\right)$
find B: A + C
$(2+i)+\left(1-\frac{\sqrt{3}}{2}+i\left(\sqrt{3}+\frac{1}{2}\right)\right)$
$\Rightarrow 3-\frac{\sqrt{3}}{2}+i\left(\frac{3}{2}+\sqrt{3}\right)$

Question 8

a) roots $(1-\sqrt{2}),(2+i)$ $\therefore$ also $(2-i)$, ∴ coeffs are rat. also $(1+\sqrt{2})$
$\Rightarrow (x-1+\sqrt{2})(x-1-\sqrt{2})(x-2-i)(x-$
$= \left[(x-1)^2-\sqrt{2}^2\right]\left[(x-2)^2-i^2\right]$
$= (x^2-2x+1-2)(x^2-4x+4+1)$
$= (x^2-2x-1)(x^2-4x+5)$
$= x^4-4x^3+5x^2-2x^3+8x^2-10$
$-x^2+4x-5$
$= x^4-6x^3+12x^2-6x-5$
poly. is $x^4-6x^3+12x^2-6x-5$

b) $y = \alpha^2 \Rightarrow \alpha = y^{\frac{1}{2}}$ etc
$(y^{\frac{1}{2}})^3+p(y^{\frac{1}{2}})^2+q(y^{\frac{1}{2}})+r$
$y^{3/2}+py+qy^{\frac{1}{2}}+r = 0$
$y^{\frac{1}{2}}(y+q) = -py-r$
$y(y+q)^2 = (-py-r)^2$
$y(y^2+2qy+q^2) = p^2y^2+2pr$
$r^2 = y^3+y^2(2q-p^2)+y(q^2-2$
or $x^3+x^2(2q-p^2)+x(q^2-2rp)$

ii) $\alpha^3+p\alpha^2+q\alpha+r = 0$
$\beta^3+p\beta^2+q\beta+r = 0$
$\gamma^3+p\gamma^2+q\gamma+r = 0$
$\alpha^3+\beta^3+\gamma^3+p(\alpha^2+\beta^2+\gamma^2)$
$+q(\alpha+\beta+\gamma)+3r = 0$
$\alpha^3+\beta^3+\gamma^3 = -p(p^2-2q)$
$-q(-p)-3r$
$= -p^3+3pq-3r$

C' (in 4th quad): $(2+i)(\cos -60°+i$
$= (2+i)\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right) = \left(1+\frac{\sqrt{3}}{2}\right)+i($
B': $(2+i)+\left(1+\frac{\sqrt{3}}{2}+i\left(\frac{1}{2}-\right.\right.$
$= \left(3+\frac{\sqrt{3}}{2}\right)+i\left(\frac{3}{2}-\sqrt{3}\right)$

Question 9.

a) $P(x) = x^4 - 6x^3 + 12x^2 - 10x + 3$

$P'(x) = 4x^3 - 18x^2 + 24x - 10$

$P''(x) = 12x^2 - 36x + 24$

mult. 3 $\Rightarrow$ $P''(x) = 0$ gives the root

$12(x^2 - 3x + 2) = 0$

$12(x-2)(x-1) = 0$

$x = 2$ or 1

$P(2) = 16 - 48 + 24 - 20 + 3$
$\neq 0$

$P(1) = 1 - 6 + 12 - 10 + 3$
$= 0$

$\therefore$ $x = 1$ is the triple root

$P(x) = (x-1)^3 Q(x)$

$= (x-1)^3 (ax + b)$

equating coeff of x^4

$1 = a$

equating constants

$3 = -b$ $\therefore$ $b = -3$

$P(x) = (x-1)^3 (x-3) = 0$

$x = 1, 1, 1, 3$

b) $x^3 + px + q = (x-2)(x+3)Q(x) + 2x+1$

$x = 2$

$8 + 2p + q = 4+1$

$x = -3$

$-27 - 3p + q = -6 + 1$

$\therefore 2p + q = -3$ (1)

$-3p + q = 22$ (2)

(1)-(2) $5p = -25$

$p = -5$

$q = 7$

c) $\omega^3 = 1$

$1 + \omega + \omega^2 = 0$

$\Sigma\alpha$

$= \alpha + \beta + \alpha\omega + \frac{\beta}{\omega} + \alpha\omega^2 + \frac{\beta}{\omega^2}$

$= \alpha(1 + \omega + \omega^2) + \beta\left(1 + \frac{1}{\omega} + \frac{1}{\omega^2}\right)$

$= 0 + \beta\left(\frac{\omega^2 + \omega + 1}{\omega^2}\right) = 0$

$\Sigma\alpha\beta$

$(\alpha+\beta)\left(\alpha\omega + \frac{\beta}{\omega}\right) + (\alpha+\beta)\left(\alpha\omega^2 + \frac{\beta}{\omega^2}\right)$

$+ \left(\alpha\omega + \frac{\beta}{\omega}\right)\left(\alpha\omega^2 + \frac{\beta}{\omega^2}\right)$

$= \alpha^2\omega + \frac{\alpha\beta}{\omega} + \alpha\beta\omega + \frac{\beta^2}{\omega}$

$+ \alpha^2\omega^2 + \frac{\alpha\beta}{\omega^2} + \alpha\beta\omega^2 + \frac{\beta^2}{\omega^2}$

$+ \alpha^2\omega^3 + \frac{\alpha\beta}{\omega} + \alpha\beta\omega + \frac{\beta^2}{\omega^3}$

$= \alpha^2(\omega + \omega^2 + \omega^3) + \alpha\beta\left(\frac{1}{\omega} + \frac{1}{\omega^2} + \frac{1}{\omega}\right)$

$+ \alpha\beta(\omega + \omega^2 + \omega) + \beta^2\left(\frac{1}{\omega} + \frac{1}{\omega^2} + \frac{1}{\omega^3}\right)$

$= \alpha^2(0) + \alpha\beta\left(\frac{2\omega + 1}{\omega^2}\right) + \alpha\beta(2\omega + \omega^2)$

$+ \beta^2\left(\frac{\omega^2 + \omega + 1}{\omega^3}\right)$

$= \alpha\beta(\omega^2 - 1 + 2\omega + \omega^2)$

$= \alpha\beta(2\omega^2 + 2\omega - 1)$

$= \alpha\beta(2(-1) - 1) = -3\alpha\beta$

$\Sigma\alpha\beta\gamma$

$(\alpha+\beta)\left(\alpha\omega + \frac{\beta}{\omega}\right)\left(\alpha\omega^2 + \frac{\beta}{\omega^2}\right)$

$= (\alpha+\beta)\left(\alpha^2\omega^3 + \frac{\alpha\beta}{\omega} + \alpha\beta\omega + \frac{\beta^2}{\omega^3}\right)$

$= \alpha^3\omega^3 + \frac{\alpha^2\beta}{\omega} + \alpha^2\beta\omega + \alpha\frac{\beta^2}{\omega^3}$

$+ \alpha^2\beta\omega^3 + \frac{\alpha\beta^2}{\omega} + \alpha\beta^2\omega + \frac{\beta^3}{\omega^3}$

$= \alpha^3 + \alpha^2\beta\left(\frac{1}{\omega} + \omega + \frac{1}{\omega^3}\right)$

$+ \alpha\beta^2\left(\frac{1}{\omega} + \omega + \frac{1}{\omega^3}\right) + \beta^3$

$= \alpha^3 + \beta^3$

NB: $\frac{\omega^2 + \omega^4 + 1}{\omega^3}$

$= \frac{\omega^2 + \omega + 1}{\omega^3}$

$= 0$

$\therefore$ equ: $x^3 - 0x^2 - 3\alpha\beta x - (\alpha^3 + \beta^3) = 0$ ie $x^3 - 3\alpha\beta \ldots + \beta^3) = 0$