# SYDNEY GIRLS HIGH SCHOOL

2006 HSC Assessment Task 1

November 28, 2005

MATHEMATICS Extension 2

Year 12

Time allowed: 90 minutes

**Topics: Curve Sketching, Circular Motion**

DIRECTIONS TO CANDIDATES:

- Attempt all questions
- Questions are not of equal value
- There are 13 questions
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work
- Board approved calculators may be used
- Write on one side of the paper only
- Use g = 10 $ms^{-2}$ in all questions requiring a numerical value of g.

Question 1. MARKS

Sketch the following curves, showing any important features

$a) y = \log_3 x, \quad b) x = \sqrt{4-y^2}, \quad c) y = -|x-2|$ [6]

Question2.

Sketch the following polynomials

$a) y = (x-1)(x-2)(x-3)^2, \quad b) y = (1-x)(x-2)^2(x-3)^2$ [6]

Question 3.

Sketch the following curves showing any asymptotes whether they be vertical, horizontal or inclined.

$a)(x-2)(y+3)=1, \quad b) x^2 - y^2 = 4, \quad c) y = \dfrac{1}{(x-1)^2}, \quad d) y = \dfrac{2x^2-3x}{x-1}$ [8]

Question 4.

Sketch the following piecemeal function showing any points of discontinuity

$$y = \begin{cases} (x+1)^2 & \text{for } x \le 0 \\ \sqrt{1-x^2} & \text{for } 0 < x < 1 \\ 2^{x-1} & \text{for } x \ge 1 \end{cases}$$ [4]

$x^2 + y^2 = 1$

$x = 1, \quad y = 1$

Question 5.

Find the centre and then sketch the following curves:

$a) x^2 + y^2 - 8x + 6y = 0, \quad b) x^2 + 4y^2 + 2x - 3 = 0$ [6]

Question 6.

a) Sketch the curve $f(x) = 4x - x^2$ on axes with the same horizontal and vertical scale

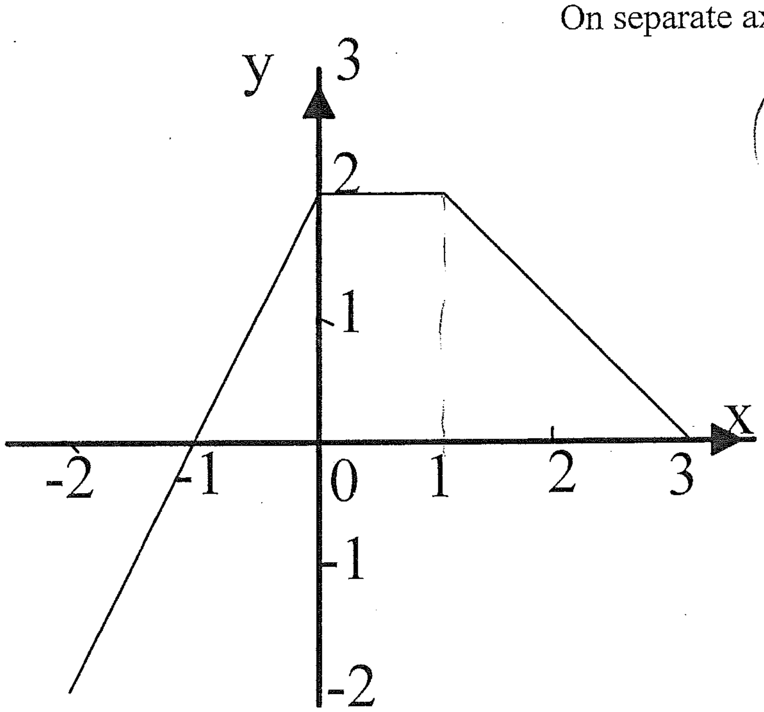
b) Determine the largest possible domain including $x = 3$ for which $f(x) = 4x - x^2$ has an inverse function

c) Find the equation of the inverse function $f^{-1}(x)$

d) State any common points between $f^{-1}(x)$ and $f(x)$ [8]

Question 7.

The function y = f(x) is shown




On separate axes sketch:

a) $y = \{f(x)\}^2$

b) $y = \dfrac{1}{f(x)}$

c) $y = f|x|$

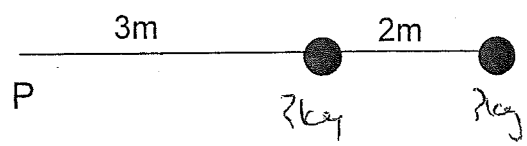
d) $y = \sqrt{f(x)}$ $y^2 = f(x)$

e) $y = 2^{f(x)}$

[10]

Question 8.

A 5 metre long piece of string with two 3 kg masses attached is swung in a horizontal plane around a point P. If the masses are attached 3 metres from P and at the end of the string, what is the maximum angular speed (in radians per second) that the string can be rotated if it has a breaking strain of 600 Newtons




[6]

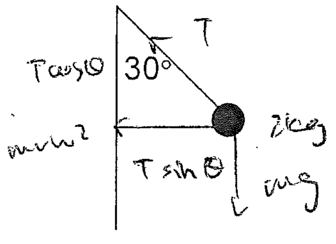
Question 9.

A particle of mass 2 kg is rotating in a conical pendulum with Angle at the vertex $30^0$. If the particle rotates at 20 rpm, find

a) the tension in the string
b) the radius of motion

$20 \times \frac{2\pi}{60}$

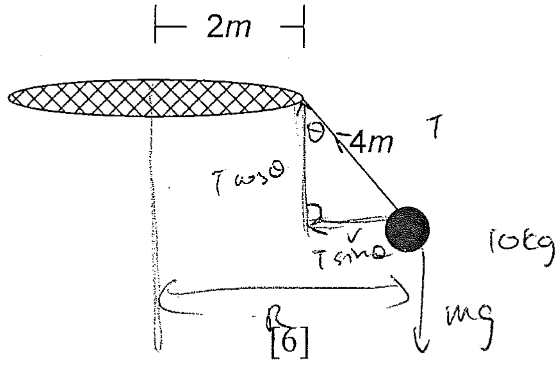
$\omega = \frac{2\pi}{3}$




[6]

Question 10.

A 4 metre piece of string is attached to the edge of a disc of radius 2 metres. If a 10kg mass is attached to the other end of the string, how fast must the system rotate (in radians/second) if the string is to be angled at $30^0$ to the vertical




[6]

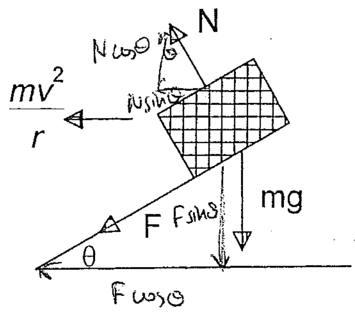
$mr\omega^2$

Question 11.

A road is banked at an angle $\theta$ as shown in the adjacent diagram.

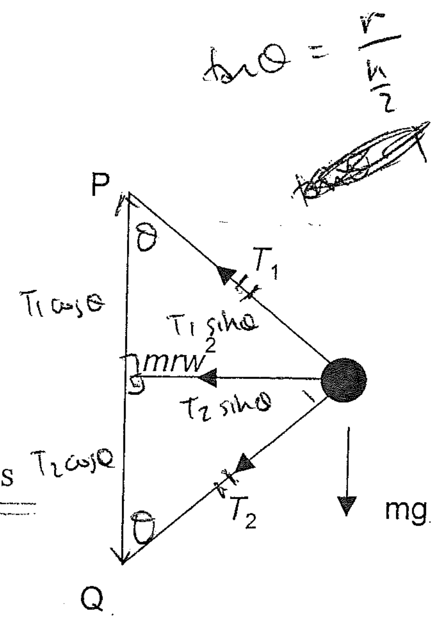
a) If the force of circular motion is given as $F = \frac{mv^2}{r}$, and the force due to gravity as mg, resolve the frictional force F and reaction force N in the horizontal and vertical directions and hence find an expression for F

b) A bus, turning a corner causes the same frictional force down the slope when it travels at 90 km/hr as it does up the slope at 18 km/hr. If the radius of the corner is 260 metres, find (to the nearest degree) the angle at which the road is banked.




[12]

Question 12
A particle is attached by means of two equal strings to P and Q in the same vertical line. The particle describes a horizontal circle with constant angular speed $w$.



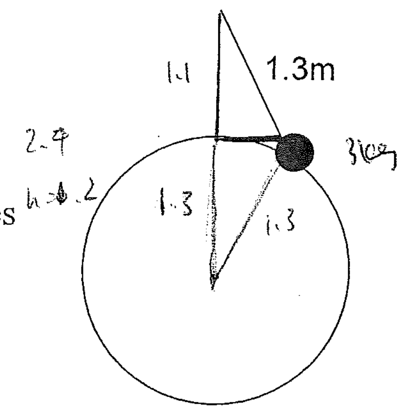

a) Prove that in order for the strings to remain taut, $w > \sqrt{\frac{2g}{h}}$ where h is the distance PQ

b) If $w = 2\sqrt{\frac{2g}{h}}$, find the ratio of the tensions in the two strings

[12]

Question 13.

From a point 1.1 metres above the top of a sphere a 1.3 metre length of string has a 3kg mass attached and that mass is rotating around the surface of the sphere at 2 rad/sec. $\omega$




a) ~~Copy~~ the adjacent diagram and show the where the forces of tension (T), normal (N), gravity (mg) and circular motion ($mrw^2$) lie.

b) If the sphere has a radius (also) of 1.3 metres, determine the force(N) exerted by the sphere on the mass

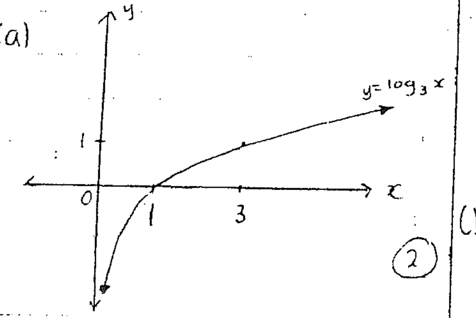
[10]

⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙ end of paper ⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙

HSC Extension 2 2006
Assessment Task 1.

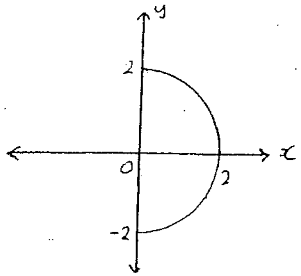
## QUESTION 1 (6 marks)

(a)



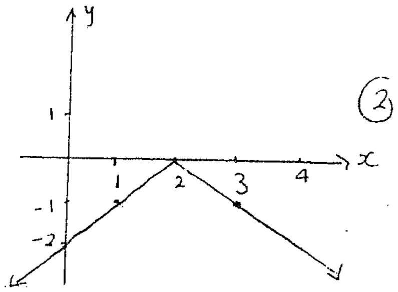

(2)

(b) $x = \sqrt{4 - y^2}$




(2)

(c) $y = -|x-2|$

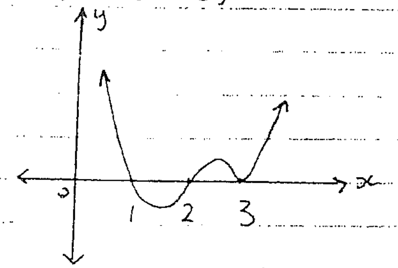



(2)

## QUESTION 2 (6 marks)

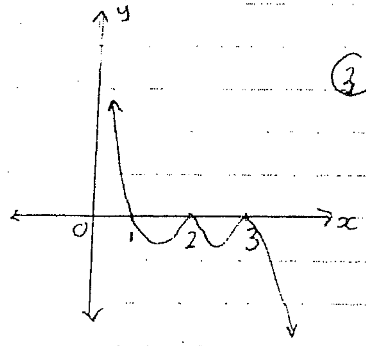
$y = (x-1)(x-2)(x-3)^2$

(a)




(3)

(b) $y = (1-x)(x-2)^2(x-3)^2$



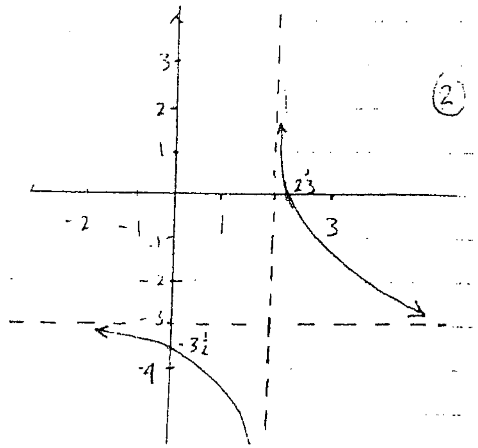

(3)

## QUESTION 3 (8 marks)

(a) $(x-2)(y+3) = 1$

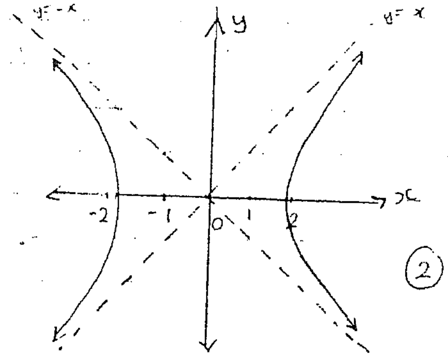
Vertical asymptote: $x = 2$

Horizontal asymptote: $y = -3$




(2)

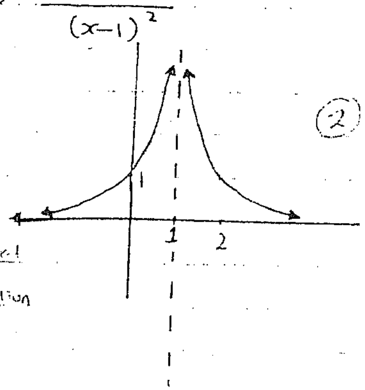
3(b) $x^2 - y^2 = 4$




(2)

3(c) $y = \dfrac{1}{(x-1)^2}$

vertical asympt.
$x = 1$
horiz. asympt
$y = 0$
even no. of factors
asympt approached
from same direction
y intercept $y = 1$




(2)

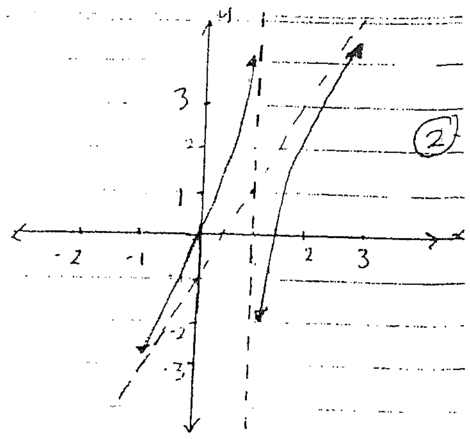
(d) $y = \dfrac{2x^2 - 3x}{x-1}$

$$\begin{array}{r} 2x - 1 \\ x-1 \overline{)\, 2x^2 - 3x} \\ -\ 2x^2 - 2x \\ \hline -1x + 0 \\ -\ -1x + 1 \\ \hline -1 \end{array}$$

$\therefore\ y = (2x-1) - \dfrac{1}{x-1}$

oblique asympt. $y = 2x - 1$

Vertical asympt: $x = 1$

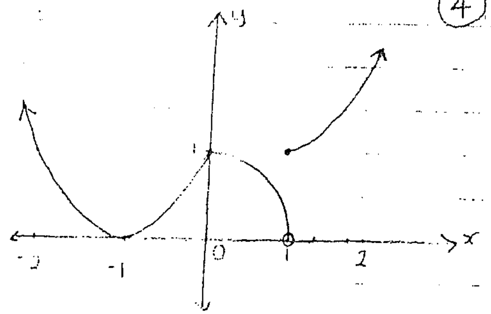



(2)

## QUESTION 4 (4 marks)

$$y = \begin{cases} (x+1)^2 & \text{for } x \le 0 \\ \sqrt{1-x^2} & \text{for } 0 < x < 1 \\ 2^{x-1} & \text{for } x \geq 1 \end{cases}$$

(4)




## QUESTION 5 (6 marks)

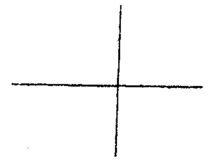
(a) $x^2 + y^2 - 8x + 6y = 0$

$x^2 - 8x + (-4)^2 + y^2 + 6y + (+3)^2 = 16 + 9$

$(x-4)^2 + (y+3)^2 = 25$

Circle with centre $(4,3)$

and radius $= 5$ units

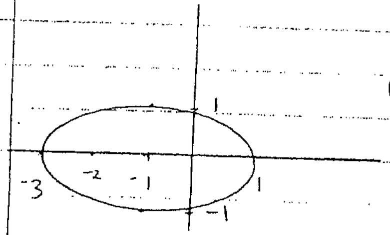


Q5(b) $x^2+2x+(1)^2+4y^2=3+1^2$

$$(x+1)^2+4y^2=4$$

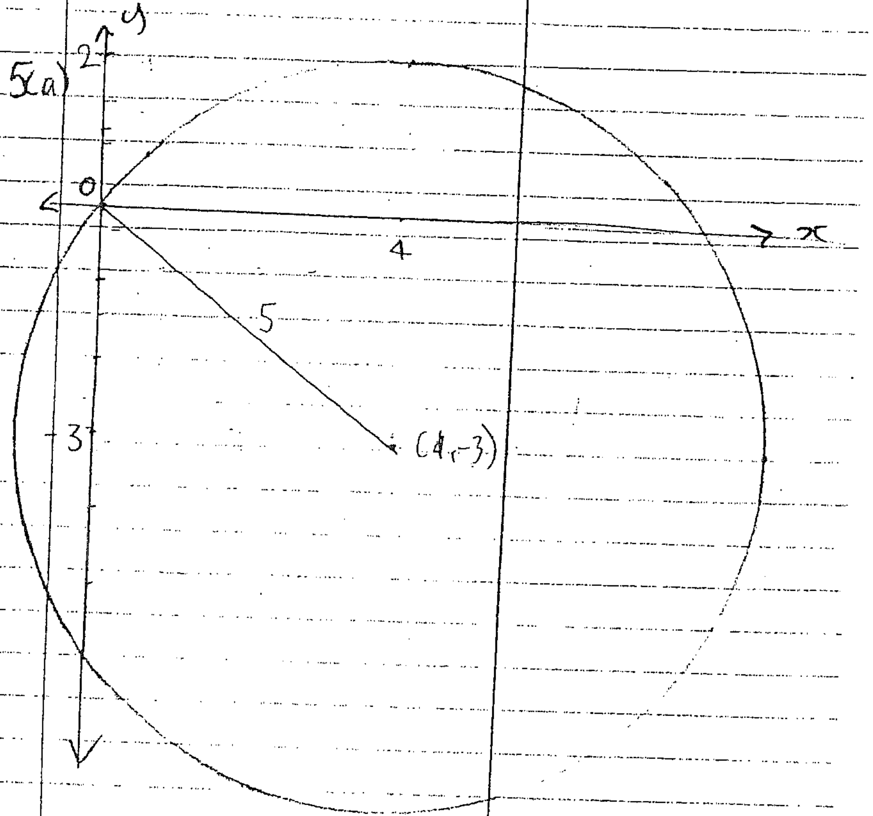
$$\frac{(x+1)^2}{4}+y^2=1$$

Ellipse with centre $(-1,0)$



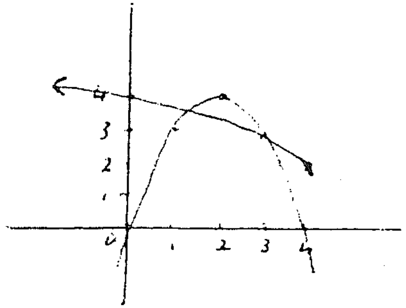

(3)

5(a)




Q6

a)




1

b) Turning pt at (2, 4)

$\therefore$ Domain is $x \geq 2$ 1

c) Let $x = 4y - y^2$

$\therefore y^2 - 4y + 4 = 4 - x$

$(y-2)^2 = 4 - x$

$y - 2 = \pm\sqrt{4-x}$

$y - 2 = +\sqrt{4-x}$ if originally $x \geq 3$

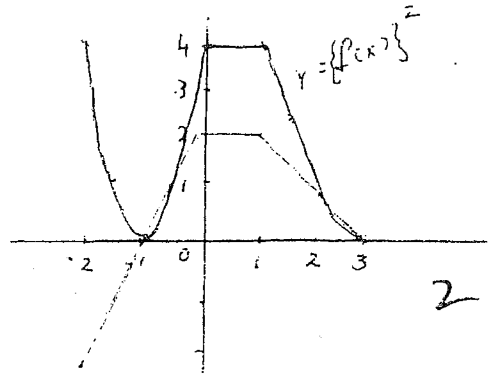
$\therefore y = 2 + \sqrt{4-x}$

4

d) The common point is (3,3)

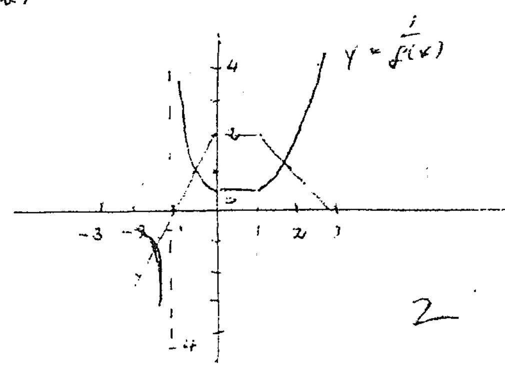
(from the diagram)

2



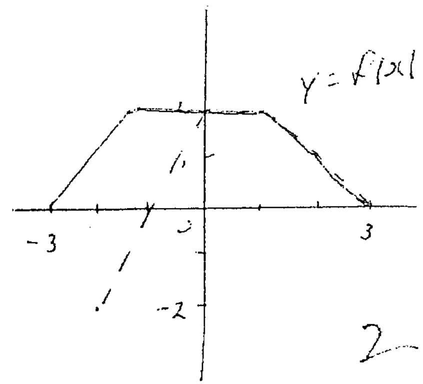

2

b)



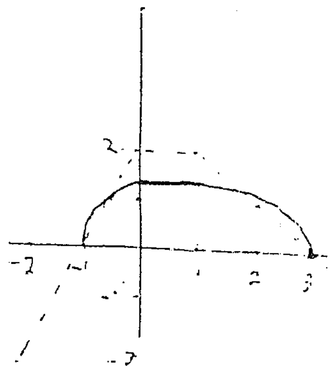

2

c)




2

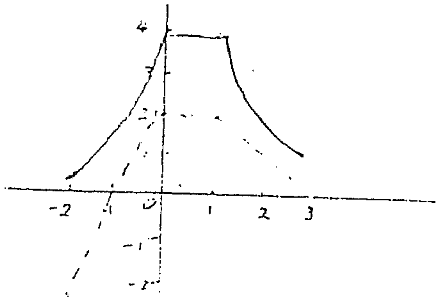
$f(x)$



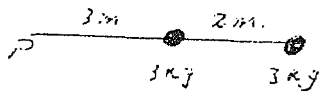

2

e) $y = 2^{f(x)}$

2




Q8,




$T = m_1 r_1 \omega^2 + m_1 r_2 \omega^2$

$600 = 3 \times 3 \times \omega^2 + 3 \times 5 \times \omega^2$

$600 = 9\omega^2 + 15\omega^2$

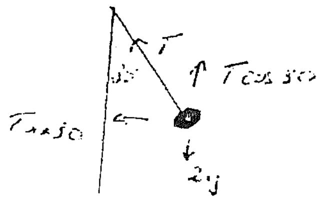
$600 = 24\omega^2$

$25 = \omega^2$

$\therefore \omega = 5$ rad/sec.

6

Q9




$\omega = 20$ rpm

$= \frac{20 \times 2\pi}{60}$

$= 2\pi/3$ rad/sec.

a) $T\cos 30 = mg$

$\therefore T \cdot \frac{\sqrt{3}}{2} = 2 \times 10$

$\therefore T = 40/\sqrt{3}$ N

3

b) $T\sin 30 = m r \omega^2$

$\therefore \frac{40}{\sqrt{3}} \times \frac{1}{2} = 2 \times r \times (2\pi/3)^2$

$20/\sqrt{3} = \frac{8\pi^2}{9} r$

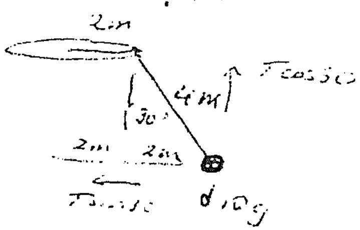
$\therefore \frac{180}{8\sqrt{3}\pi^2} = r$

$r = \frac{45}{2\sqrt{3}\pi^2}$ m. $= \frac{15\sqrt{3}}{2\pi^2}$ m.

$\doteqdot 1.32$ m

3

Q10




$T\cos 30 = mg$

$T\sin 30 = mR\omega^2$

where $R = 2 + 4\sin 30$
$= 4$

$\therefore \tan 30 = \frac{R\omega^2}{g}$

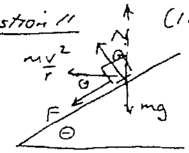
$\frac{10 \times 1}{4\sqrt{3}} = \omega^2$

$\therefore \omega = \sqrt{\frac{10}{4\sqrt{3}}}$

$= \sqrt{\frac{5}{2\sqrt{3}}}$ rad/sec.

$\doteqdot 1.2$ rad/sec.

Question 11 (12 marks)




(a) Vertically

1 $N\cos\theta = F\sin\theta + mg$. — ① $\times \sin\theta$

Horizontally

1 $N\sin\theta + F\cos\theta = m\frac{v^2}{r}$. — ② $\times \cos\theta$

$N\sin\theta\cos\theta = F\sin^2\theta + mg\sin\theta$

$N\sin\theta\cos\theta + F\cos^2\theta = m\frac{v^2}{r}\cos\theta$

$\therefore F\sin^2\theta + mg\sin\theta + F\cos^2\theta = m\frac{v^2}{r}\cos\theta$

3 $F = m\frac{v^2}{r}\cos\theta - mg\sin\theta$ *

(b) $90$ km/hr $\Rightarrow \frac{90.000}{3600}$ m/s $= 25$ m/s

1 $18$ km/hr $\Rightarrow$ $= 5$ m/s.

$r = 260$ m

1 Now $F_1 = -F_2$.

$\therefore m\frac{v_1^2}{r}\cos\theta - mg\sin\theta = -\left(\frac{mv_2^2}{r}\cos\theta - m\right.$

$\therefore \cancel{m}\left(\frac{v_1^2}{r}\cos\theta + \frac{v_2^2}{r}\cos\theta\right) = 2\cancel{m}g\sin\theta$

$\left(\frac{25^2}{260}\cos\theta + \frac{5^2}{260}\cos\theta\right) = 20\sin\theta$.

$\frac{650}{260}\cos\theta = 20\sin\theta$.

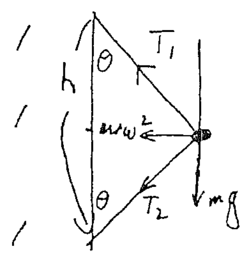
$\frac{650}{20 \times 260} = \tan\theta$.

5

$\theta \doteqdot 7°$

Question 12 (12 marks)

(i)




*Vertically:*

$$T_1 \cos\theta = T_2 \cos\theta + mg.$$

*Horizontally:*

$$T_1 \sin\theta + T_2 \sin\theta = mr\omega^2.$$

For strings to remain taut

$$T_2 > 0$$

Take $T_2 = 0$ $\quad \therefore \ T_1 \cos\theta = mg.$

$$T_1 \sin\theta = mr\omega^2.$$

$$\therefore \tan\theta = \frac{r\omega^2}{g}.$$

$$\tan\theta = \frac{r}{\frac{1}{2}h}.$$

$$\Rightarrow \tan\theta = \frac{2r}{h} \qquad \therefore \frac{2r}{h} = \frac{r\omega^2}{g}.$$

$$\therefore \omega^2 = \frac{2g}{h}. \quad \Rightarrow \omega = \sqrt{\frac{2g}{h}}.$$

Now if $T_2 >$ then $\omega > \sqrt{\frac{2g}{h}}$

---

$\omega = 2\sqrt{\frac{2g}{h}}$     horizontally.

$$T_1 \sin\theta + T_2 \sin\theta = m.r. \times 4\left(\frac{2g}{h}\right)$$

Now $\tan\theta = \frac{2r}{h}$ $\quad T_1 \sin\theta + T_2 \sin\theta = m \tan\theta \times 4g.$

$$T_1 \cancel{\sin\theta} + T_2 \sin\theta = m.\frac{\cancel{\sin\theta}}{\cancel{\cos\theta}}.4g.$$

$$\therefore T_1 \cos\theta + T_2 \cos\theta = 4mg.$$

$$\therefore T_2 \cos\theta + mg + T_2 \cos\theta = 4mg.$$

$$2T_2 \cos\theta = 3mg.$$

$$T_2 \cos\theta = \frac{3}{2} mg.$$

$$T_1 \cos\theta = \frac{3}{2} mg + mg$$

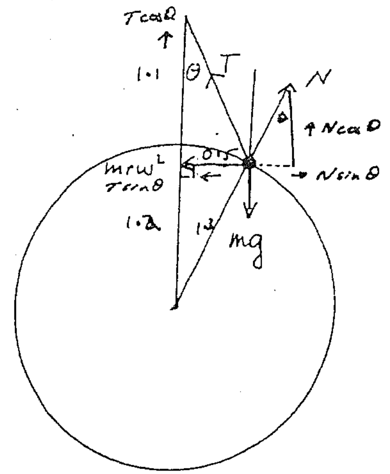
$$T_1 \cos\theta = \frac{5}{2} mg.$$

Now $T_1 \cancel{\cos\theta} : T_2 \cancel{\cos\theta} = \frac{5}{2} mg : \frac{3}{2} mg.$

$$\underline{T_1 : T_2 = 5 : 3}$$

Question 13 (10 marks)

(a)




$m = 3\,kg$

$\omega = 2\,rad/sec$

$r = \frac{1}{2}\,m.$

$\sin\theta = \frac{5}{13}$

$\cos\theta = \frac{12}{13}$

(b) *Vertically:*

$$T\cos\theta + N\cos\theta = mg.$$

$$\frac{12}{13}T + \frac{12}{13}N = 30$$

$$12T + 12N = 30 \times 13 \quad \Rightarrow \quad 12T = 30 \times 13 - 12N.$$

$$T = \frac{1}{12}\{30 \times 13 - 12N\}$$

*Horizontally:*

$$T\sin\theta - N\sin\theta = mr\omega^2$$

$$\frac{5}{13}T - \frac{5}{13}N = 3 \times \frac{1}{2} \times 4$$

$$5T - 5N = 6 \times 13 \quad \Rightarrow \quad 5T = 5N + 6 \times 13$$

$$T = \frac{1}{5}\{5N + 6 \times 13\}.$$

$$\therefore \frac{1}{12}\{30 \times 13 - 12N\} = \frac{1}{5}\{5N + 6 \times 13\}.$$

$$5\{30 \times 13 - 12N\} = 12\{5N + 6 \times 13\}.$$

$$1950 - 60N = 60N + 936$$

$$120N = 1014$$

$$N = \underline{8.45\ \text{Newtons}}$$