Sydney Girls High School

MATHEMATICS EXTENSION 2

2010 HSC Assessment Task 1
November 2009

Time Allowed - 90 minutes
(plus 5 minutes reading time)

Topics: Circular Motion (Exercises 1-8) , Curve Sketching (Exercises 1-8)

General Instructions:

- There are TEN (10) Questions which are NOT of equal value.
- Questions do not necessarily appear in order of difficulty.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write your student number clearly at the top of each question and clearly number each question.
- Use $g = 10 \text{ ms}^{-2}$

Total: 90 marks

QUESTION 1 (8 Marks)

MARKS

Sketch the following on separate number planes, showing important features :

a. $y = (x+1)^2(x-2)(x-4)^3$ — 2

b. $x = \sqrt{4-y^2}$ — 2

c. $\frac{x^2}{16} + \frac{y^2}{25} = 1$ — 2

d. $x^2 - y^2 = 4$ — 2

QUESTION 2 (12 Marks)

MARKS

a. Given $F(x) = 1 - x^2$, sketch :

i. $y = F(x)$ — 1

ii. $y = -F(x)$ — 1

iii. $y = |F(x)|$ — 2

iv. $y = \{F(x)\}^2$ — 2

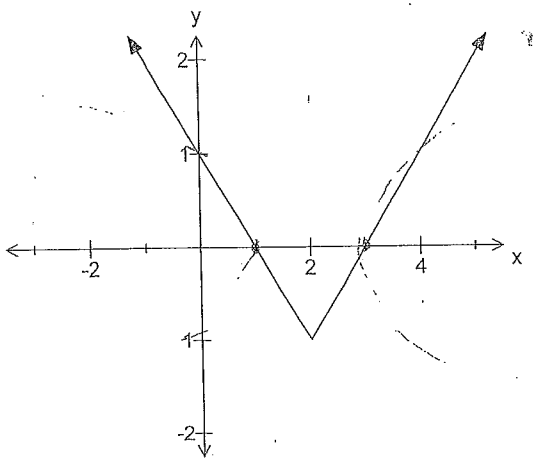
b. For the following, sketch $y = f(x)$, $y = f^{-1}(x)$ and $y = x$ on the same number plane. Show any key points, including all intersection points.

i. $f(x) = x^3$ — 3

ii. $f(x) = 2^x$ — 3

QUESTION 3 (7 Marks)

MARKS



Copy the function $y = f(x)$ as shown above and sketch on the same number plane :

a. $y = f(|x|)$ — 2

b. $y = \dfrac{1}{f(x)}$ — 2

c. $y^2 = f(x)$ — 2

d. $y = f(x-2)$ — 1

QUESTION 4 (10 Marks)

MARKS

Sketch the following on separate number planes, showing important features :

a. $y = \dfrac{3x+4}{x+2}$ — 2

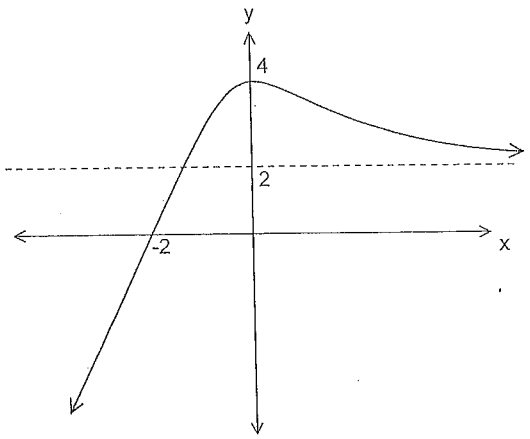
b. $25y^2 - 9x^2 = 225$ — 2

c. $y = x - \dfrac{2}{x+1}$ — 3

d. $y = \dfrac{3}{(x+2)(x-1)^3}$ — 3

QUESTION 5 (8 Marks)

MARKS



Copy the function $y = f(x)$ as shown above and sketch on the same number plane :

a. $y = f(-x)$ 1

b. $y = 2^{f(x)}$ 2

c. $y = -\sqrt{f(x)}$ 2

d. $y = f^{-1}(x)$ 2

e. $|y| = f(x)$ 1

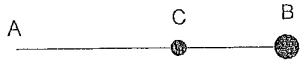
QUESTION 6 (8 Marks)

MARKS

A 3 metre piece of string AB has a mass of 5 kg attached at point B. The string is rotated in a horizontal circle about A and breaks as soon as it exceeds a speed of rotation of 45 revolutions per minute.

a. Find the maximum possible tension in the string. 3

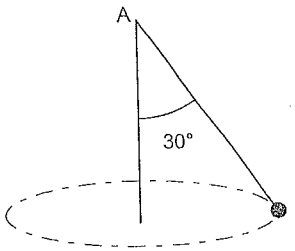
b. The mass at B is replaced by a 3 kg mass and an additional 1 kg mass is attached to the string at C, 2 metres from A (as shown below). Find the new maximum number of revolutions per minute that the string can be rotated. 5



QUESTION 7 (8 Marks)

MARKS

A mass is rotating in circular motion as a conical pendulum as shown in the diagram.



The mass of 5 kg is attached to A by a light string of length 2 m and the angle between the string and the vertex is $30°$.

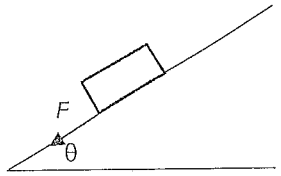
a. Find the tension in the string. 4

b. Find the speed that the mass is moving. Give your answer in m/s. 4

QUESTION 8 (10 Marks)

MARKS

A section of a car-racing track is being constructed and is to be banked at an angle of θ as shown below. A car of mass m kg is travelling around the track at v m/s.



a. Copy the diagram and mark in the normal force N and the gravitational force. 1

b. Resolve the forces vertically and horizontally. 2

c. Hence, find an expression for the : 4

i. lateral force F in terms of m, v, r, g and θ.

ii. normal force N in terms of m, v, r, g and θ.

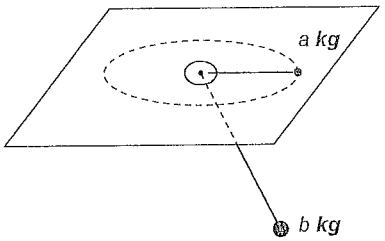
d. The road is constructed around a curve of radius 150 m to allow for an optimum speed of 120 km/h. Calculate the angle (answer to the nearest minute) at which the track should be banked. 3

QUESTION 9 (7 Marks)

MARKS

Two masses are connected by a light inelastic string. One mass of a kg is rotating in circular motion on a smooth horizontal table with angular velocity w rad/s. The other mass of b kg is at the other end of the string, rotating in a conical pendulum below the table with angular velocity $2w$ rad/s. 7

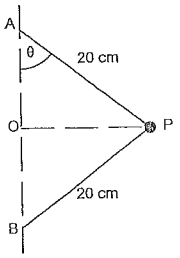
If $a:b=5:2$, find what fraction of the string lies above the table.



QUESTION 10 (12 Marks)

MARKS

A particle, P, of mass m, is attached by two strings, each of length 20 cm, to two fixed points, A and B, which are 24 cm apart and lie on a vertical line as shown in the diagram.



The particle P moves with constant speed, v m/s, in a horizontal circle about the midpoint of AB so that both pieces of string experience a tension.

The tension in AP is T_1 and the tension in BP is T_2. The acceleration due to gravity is g ms^{-2}.

a. Copy the diagram and mark on it all the forces acting on P. 2

b. Find the length of the radius of motion OP. 1

c. Resolve the forces on P horizontally and vertically. 2

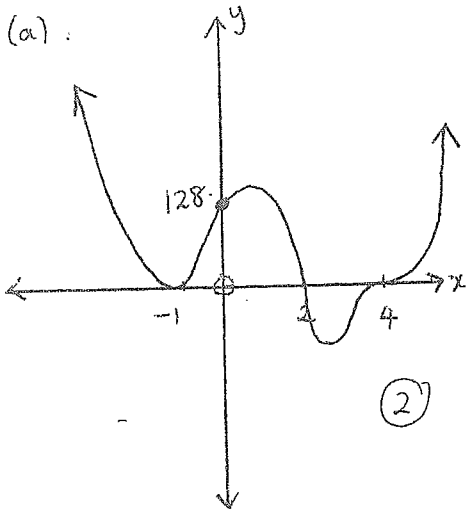
d. Find the tension in each part of the string in terms of m, v and g. 4

e. Show that $v > \dfrac{4\sqrt{3g}}{15}$. 3

2010 (Nov '09) - Extension 2 - TASK 1 : SOLUTIONS

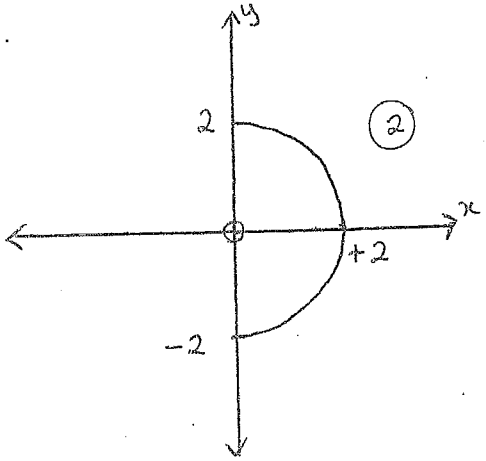
Question 1

(a).



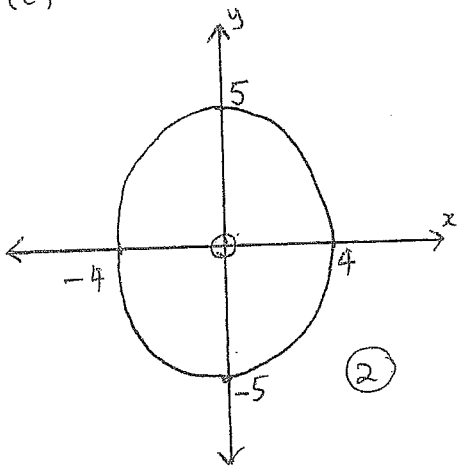
(2)

(b).



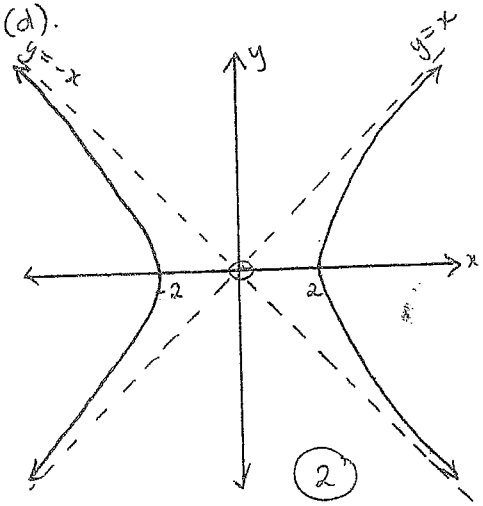
(2)

(c).



(2)

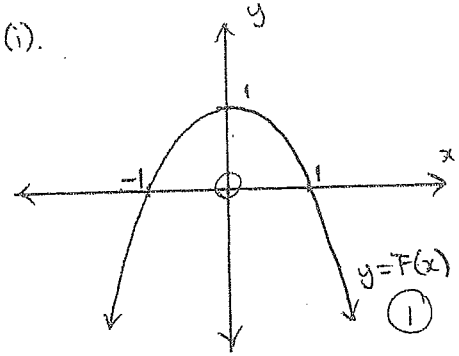
(d).



(2)

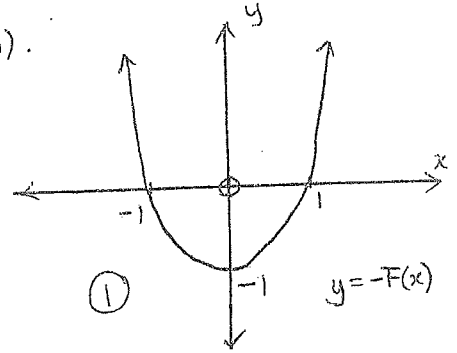
Question 2

(a). (i).



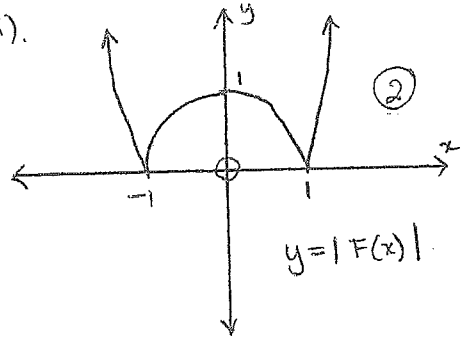
(1)

(ii).



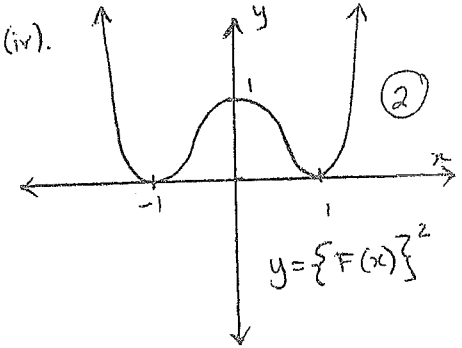
(1)

(iii).



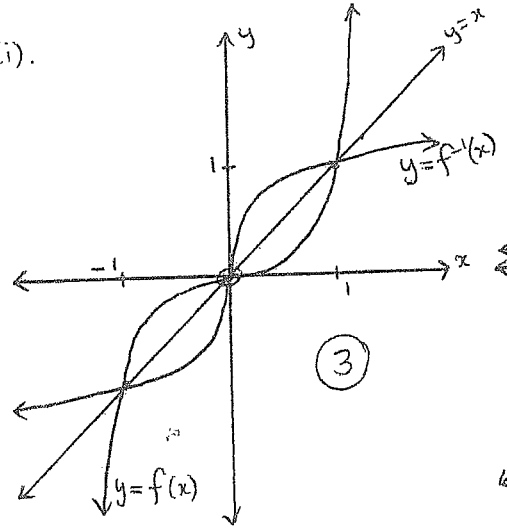
(2)

(iv).



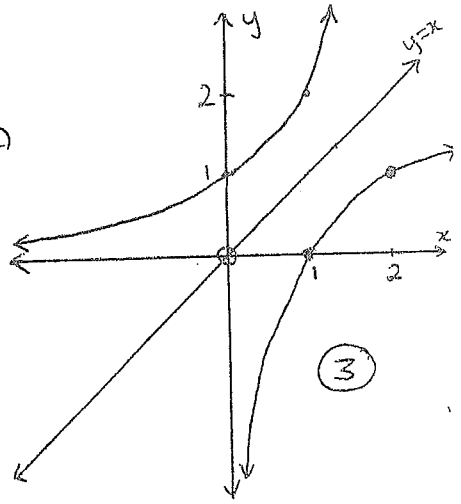
(2)

(b). (i).



$f(x) = x^3$

(3)

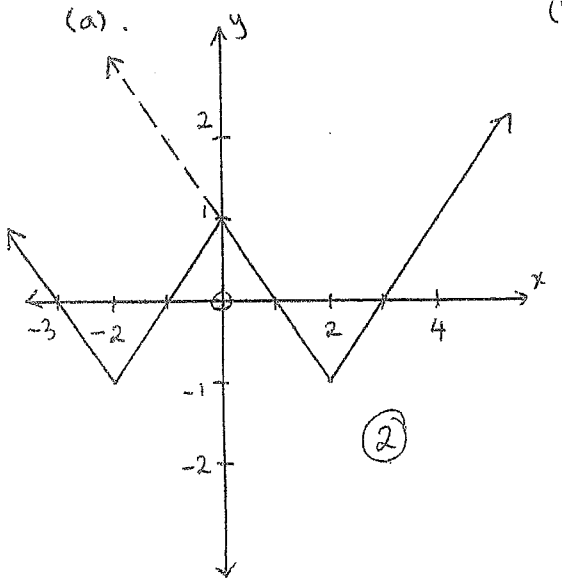


$f(x) = 2^x$

(3)

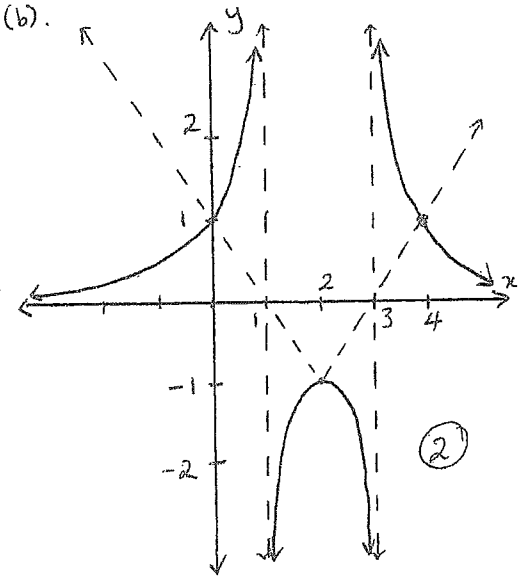
QUESTION 3

(a).



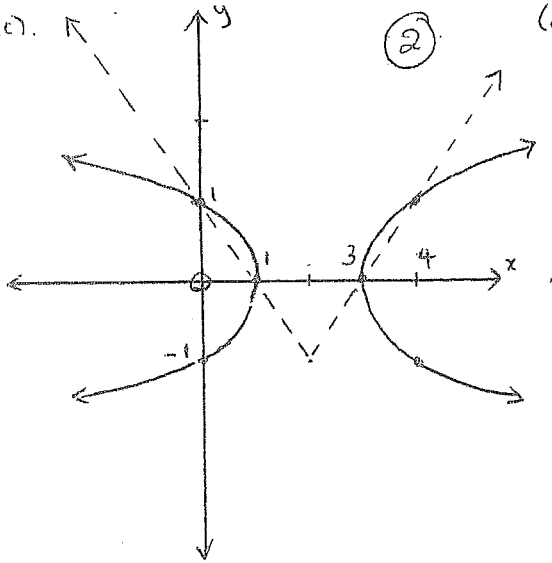
(2)

(b).



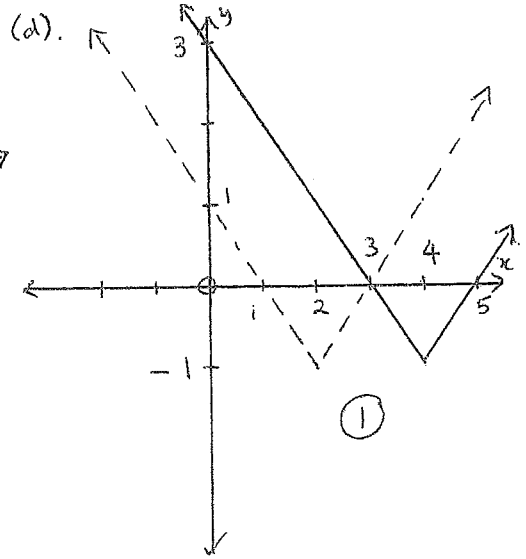
(2)

(c).



(2)

(d).

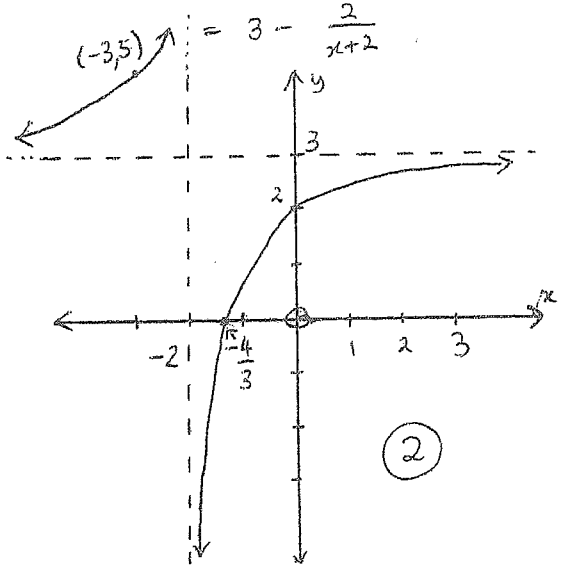


(1)

QUESTION 4

(a). $y = \frac{3x+6}{x+2} - \frac{2}{x+2}$

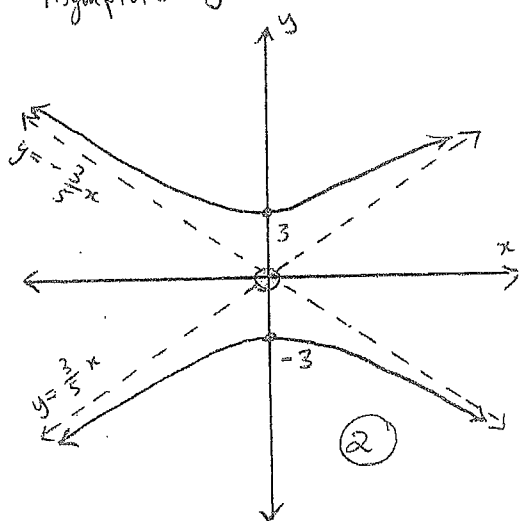
$= 3 - \frac{2}{x+2}$



(2)

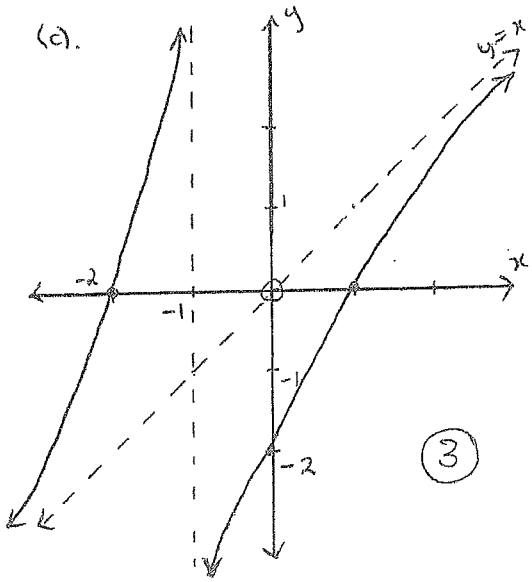
(b). $(5y+3x)(5y-3x) = 225$

Asymptotes $y = \pm \frac{3}{5}x$



(2)

(c).

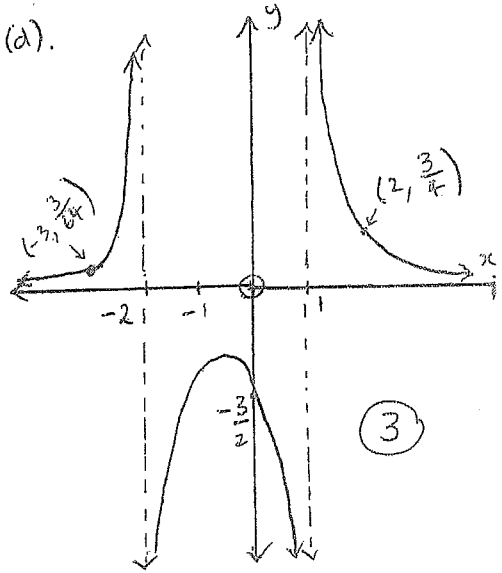


(3)

$y = x - \frac{2}{x+1}$

$= \frac{x^2+x-2}{x+1} = \frac{(x+2)(x-1)}{x+1}$

(d).

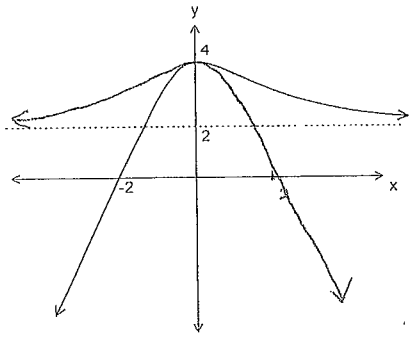


(3)

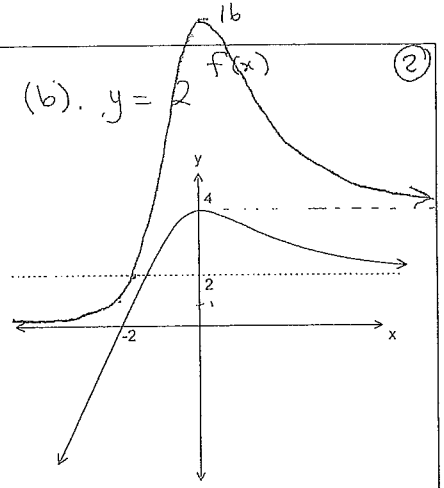
$y = \frac{3}{(x+2)(x-1)^3}$

QUESTION 5

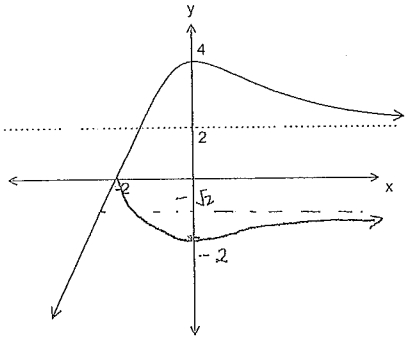
(a). $y = f(-x)$ ①



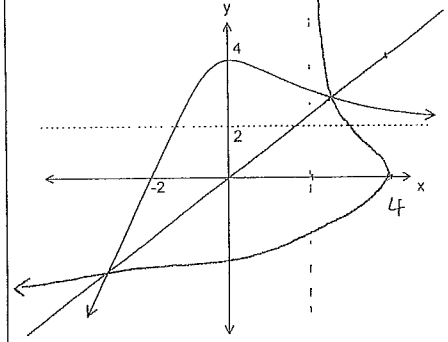
(b). $y = 2^{f(x)}$ ②



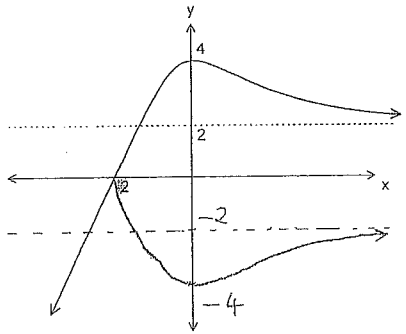
(c). $y = -\sqrt{f(x)}$ ②



(d). $y = f^{-1}(x)$ ②

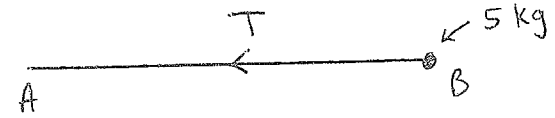


(e). $|y| = f(x)$ ①



QUESTION 6

(a).



$w = 45$ revs/min $\qquad r = 3$ m

$= \frac{45 \times 2\pi}{60}$ rad/s

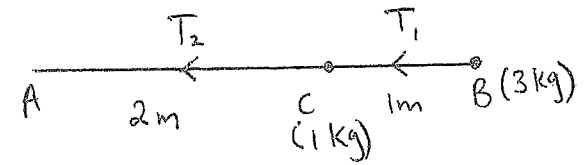
$= \frac{3\pi}{2}$ rad/s

$T = mrw^2$ ③

$= 5 \times 3 \times \left(\frac{3\pi}{2}\right)^2$

$= \frac{135\pi^2}{4}$ N

(b).



$T_1 = mrw^2$ $\qquad T_2 = 2 \times 1 \times w^2 + T_1$

$= 9w^2$ $\qquad = 11w^2$

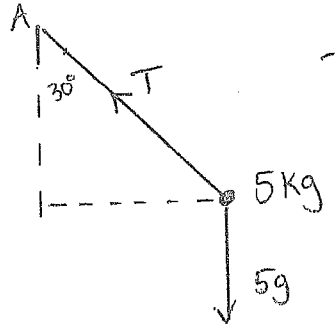
$11w^2 = \frac{135\pi^2}{4}$ ⑤

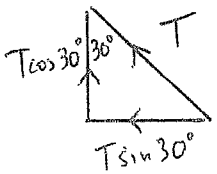
$\therefore w = \sqrt{\frac{135\pi^2}{44}}$ rad/s

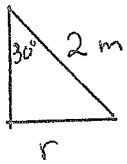
$= \sqrt{\frac{135\pi^2}{44}} \times \frac{60}{2\pi}$ rev/min.

$w \doteq 52.5$ rev/min.

QUESTION 7







(a). Vert. $T\cos 30 = 5g$ (4)

$$T = \frac{5g}{\cos 30} = \frac{100}{\sqrt{3}}$$

$$T \doteq 57.7\ N$$

(b). $T\sin 30 = \frac{mv^2}{r}$ $\quad \sin 30 = \frac{r}{2}$

$\therefore r = 1$

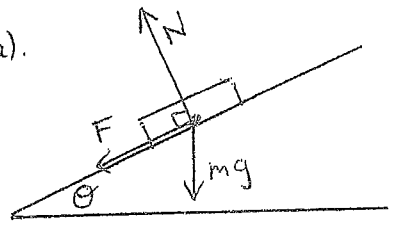
$$v^2 = \frac{rT\sin 30}{m}$$

$$= \frac{100}{\sqrt{3}} \times \frac{1}{2} \times \frac{1}{5} \quad (4)$$

$$= \frac{10}{\sqrt{3}} \quad \therefore v = \sqrt{\frac{10}{\sqrt{3}}} \doteq 2.40\ m/s$$

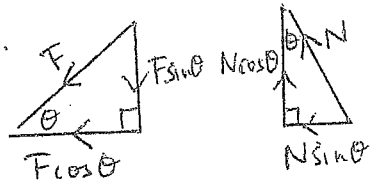
QUESTION 8

(a).



✓ ①

(b).



Hor. $N\sin\theta + F\cos\theta = \frac{mv^2}{r}$ ✓

Vert. $N\cos\theta - F\sin\theta - mg = 0$ (2)

i.e. $N\cos\theta - F\sin\theta = mg$ ✓

(c). $N\sin\theta\cos\theta + F\cos^2\theta = \frac{mv^2}{r}\cos\theta$ (1) ✓

$N\sin\theta\cos\theta - F\sin^2\theta = mg\sin\theta$ (2) ② ✓

(1) − (2) $\boxed{F = \frac{mv^2}{r}\cos\theta - mg\sin\theta}$ ✓

$N\sin^2\theta + F\sin\theta\cos\theta = \frac{mv^2}{r}\sin\theta$ (3) ✓

$N\cos^2\theta - F\sin\theta\cos\theta = mg\cos\theta$ (4) ② ✓

(3) + (4) $\boxed{N = \frac{mv^2}{r}\sin\theta + mg\cos\theta}$ ✓

(d). $r = 150$ m $\quad F = 0$ $\quad v = 120$ km/h $= 33\frac{1}{3}$ m/s ✓ $\quad \theta = ?$

$$F = \frac{mv^2}{r}\cos\theta - mg\sin\theta$$

$$0 = m\left(\frac{(33\frac{1}{3})^2}{150}\cos\theta - 10\sin\theta\right) \checkmark$$

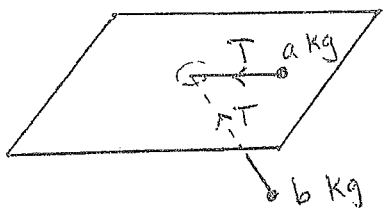
$$\therefore \tan\theta = \frac{(33\frac{1}{3})^2}{150 \times 10} \quad \therefore \theta \doteq 36°32' \quad (3)$$

✓

total 10

No half marks

QUESTION 9



a kg → ω rad/s

b kg → 2ω rad/s

$a : b = 5 : 2$

$\therefore \ 2a = 5b$

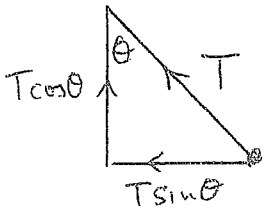
or $b = \frac{2a}{5}$ ✓

(a) kg

$T = mr\omega^2$

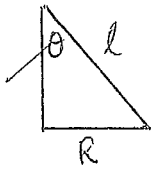
$= ar\omega^2$ ✓

(b) Kg



$T\sin\theta = mr\omega^2$ ✓

$T \times \frac{R}{\ell} = \frac{2a}{5} \times R \times (2\omega)^2$ ✓



$\frac{ar\omega^2}{\ell} = \frac{8a\omega^2}{5}$

$\therefore \ \frac{r}{\ell} = \frac{8}{5}$ ✓

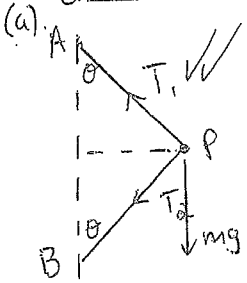
$\therefore \ \frac{8}{13}$ of string lies above the table ✓

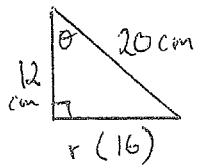
(7 marks)

Notes
- No half marks
- Methods of soln varied
  - eg - put $a=5$, $b=2$
  - Using $T_{below} = m\ell\omega^2$

QUESTION 10

(a).



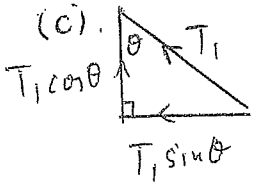


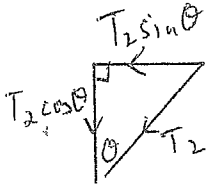
(b). $r = \sqrt{20^2 - 12^2}$ ✓

$\therefore \ r = 16$ cm or 0.16 m (1)

(2)

(c).





Hor.

$T_1\sin\theta + T_2\sin\theta = \frac{mv^2}{r}$ ✓

Vert.

$T_1\cos\theta - T_2\cos\theta - mg = 0$ ✓

i.e. $T_1\cos\theta - T_2\cos\theta = mg$ (2)

(d). $\sin\theta = \frac{16}{20}$ or $\frac{4}{5}$ ✓ $\cos\theta = \frac{12}{20}$ or $\frac{3}{5}$

$(T_1 + T_2)\sin\theta = \frac{mv^2}{0.16}$ | $(T_1 - T_2)\cos\theta = mg$

$T_1 + T_2 = \frac{125mv^2}{16}$ ✓ (1) | $T_1 - T_2 = \frac{5}{3}$ (2)

(1) + (2) $\quad 2T_1 = \frac{125mv^2}{16} + \frac{5mg}{3} \quad \therefore T_1 = \frac{125mv^2}{32} + \frac{5mg}{6}$ ✓

(1) − (2) $\quad 2T_2 = \frac{125mv^2}{16} - \frac{5mg}{3} \quad \therefore T_2 = \frac{125mv^2}{32} - \frac{5mg}{6}$ ✓

(4)

(e). $T_2 > 0$ ✓ $\quad \frac{125mv^2}{32} - \frac{5mg}{6} > 0$ ✓ No cfp a here

$\frac{125v^2}{32} > \frac{5g}{6}$

$v^2 > \frac{169}{75}$

$v > \sqrt{\frac{9}{75}}$

$v > \frac{4\sqrt{9}}{5\sqrt{3}}$ (3)

i.e. $v > \frac{4\sqrt{3g}}{15}$ ✓

- No half marks. Total 12 marks