SYDNEY GIRLS HIGH SCHOOL

LABOR OMNIA VINCIT

YEAR 12 4 UNIT MATHEMATICS

March 1999

Time Allowed: 90 minutes.

Instructions:

- There are sixteen (16) questions.
- Attempt all questions
- Questions are <u>not</u> of equal value
- All necessary working should be shown. Marks may be deducted for careless or badly arranged work.
- Write on one side of the paper only

Name: ____________________

Class: ____________________

Marks

Question 1 — 8

If $z = \sqrt{3} + i$

a) find i) $\arg z$

ii) $|z|$

iii) $\bar{z}$

b) show on the Argand Diagram

i) z

ii) $\bar{z}$

iii) iz

iv) $\dfrac{1}{z}$

v) $z + iz$

Question 2 — 4

If $z = 4 - 3i$

a) Show that i) $z.\bar{z}$ is real

ii) $z + \bar{z}$ is real

b) Find a and b if $\dfrac{1}{z} = a + ib$

Question 3 — 4

A represents the complex number $z = 2 + 2i$,

B represents a 60^o anticlockwise rotation of A, and,

C represents a 90^o clockwise rotation of A.

Write down the complex numbers represented by B and C

Question 4 — 3

If $z = -\sqrt{3} + i$

i) express z in mod-arg form

ii) express z^6 in the form $a + ib$

Question 5 — 6

Sketch the region common to

i) $1 \le R(z) \le 3$ and $\dfrac{\pi}{6} \le \arg z \le \dfrac{\pi}{3}$

ii) $0 \le \arg(z+4) \le \dfrac{2\pi}{3}$ and $|z+4| \le 4$

Question 6 — 6

a) Express $\sqrt{24 - 70i}$ in the form $a + ib$, where a and b are real and $a > 0$.

b) Hence, or otherwise, solve $z^2 - (1-i)z - 6 + 17i = 0$

Question 7 — 7

a) i) Find the locus of z if $|z - 4i| = 2|z + 1|$

ii) Describe this locus.

b) Determine the locus of w , if $|z| = 2$ and $w = \dfrac{z-1}{z}$

Question 8 — 4

Given $|z+1| = 1$, find the maximum and minimum of $\arg(z+3)$

Question 9 8

Find the locus of z if $w = \frac{z-2i}{z+2}$ is

i) purely real

ii) purely imaginary

Sketch the locus in each case.

Question 10 4

Find and sketch the locus of z where $\arg\left(\frac{-z}{i}\right) = \arg\left(\frac{1}{z}\right)$

Question 11 6

Given $z = \cos\theta + i\sin\theta$, express $\cos^4\theta$ in terms of $\cos 4\theta$ and $\cos 2\theta$.

Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{2}} \cos^4\theta . d\theta$

Question 12 4

The points A, B, C, and D are the vertices of the quadrilateral ABCD. They represent the complex numbers z_1, z_2, z_3, z_4 respectively such that

$$\arg(z_1 - z_4) = \arg(z_2 - z_3) \text{ and } |z_4 - z_3| = |z_1 - z_2|.$$

Describe the quadrilateral ABCD, including a specific name, justifying your answer with appropriate reasons and illustrations.

Question 13 10

Find the seven seventh roots of 1, ie solve $z^7 - 1 = 0$

i) Plot the roots on the Argand Diagram.

ii) If $w = cis\frac{2\pi}{7}$, show that

$1 + w + w^2 + w^3 + w^4 + w^5 + w^6 = 0$

iii) Factorise $z^7 - 1$ into real quadratic factors.

Question 14 8

a) If the polynomial $P(x)$ has a root at $x = a$ of multiplicity m, show that the derived polynomial $P'(x)$ has a root at $x = a$ of multiplicity m-1.

b) A polynomial $P(x) = 8x^4 - 20x^3 - 18x^2 + 81x - 54$ is known to have a triple root. Find the roots of this equation and sketch the curve, showing the essential features.

Question 15 6

Given that $(2-i)$ is a zero of $2x^3 + mx^2 + nx + 15$, determine m and n, where m and n are real. Hence, factorise $2x^3 + mx^2 + nx + 15$ over the real field.

Question 16 12

Given α, β, γ are the roots of $x^3 + 2x^2 - 3x + 4 = 0$

a) find

i) $\alpha^2 + \beta^2 + \gamma^2$

ii) $\alpha^3 + \beta^3 + \gamma^3$

iii) $\alpha^4 + \beta^4 + \gamma^4$

b) find the equation whose roots are

i) $\alpha + 2, \beta + 2, \gamma + 2$

ii) $\alpha^2, \beta^2, \gamma^2$

iii) $\frac{\alpha\beta}{\gamma}, \frac{\alpha\gamma}{\beta}, \frac{\beta\gamma}{\alpha}$

End of Exam

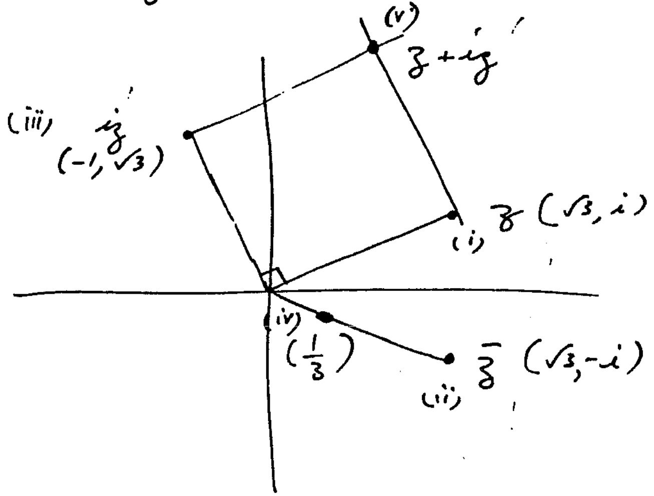
① $z = \sqrt{3} + i$

ⓐ (i) $\tan\theta = \frac{1}{\sqrt{3}} = \frac{\pi}{6}$. $\therefore \arg z = \frac{\pi}{6}$

(ii) $|z| = \sqrt{x^2 + y^2} = \sqrt{(\sqrt{3})^2 + 1^2} = \underline{2}$

(iii) $\bar{z} = \sqrt{3} - i$

ⓑ.



$\frac{1}{z} = \frac{1}{\sqrt{3}+i} \cdot \frac{\sqrt{3}-i}{\sqrt{3}-i} = \frac{1}{4}(\sqrt{3} - i)$

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②. $z = 4 - 3i$

(i) $z \cdot \bar{z} = (4-3i)(4+3i) = 16 + 9 = 25$

∴ Real

(ii) $z + \bar{z} = (4-3i) + (4+3i) = 8$

∴ Real

$\frac{1}{z} = a + ib = \frac{1}{4-3i} \cdot \frac{4+3i}{4+3i}$

$= \frac{4+3i}{25}$

$\therefore a = \frac{4}{25} \quad b = \frac{3}{25}$

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③ $z = 2 + 2i$

60° rotation anticlockwise is $(\frac{1}{2} + \frac{\sqrt{3}}{2} i)$.

∴ Co-ords of B. $(2+2i)(\frac{1}{2} + \frac{\sqrt{3}}{2} i)$

$z_1 = (1 - \sqrt{3}) + i(1 + \sqrt{3})$ 2

90° rotation clockwise is $-i$

$\therefore z_2 = 2 - 2i$

Co-ords of C $(2, -2i)$ 2

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④ (i) $z = -\sqrt{3} + i$ $\tan\theta = \frac{1}{-\sqrt{3}}$

$|z| = \sqrt{(\sqrt{3})^2 + 1^2} = 2$. $\therefore \arg z = \frac{5\pi}{6}$.

$\therefore z = 2 \,\mathrm{cis}\, \frac{5\pi}{6}$.

(ii) Now $z^6 = (2 \,\mathrm{cis}\, \frac{5\pi}{6})^6 = 2^6 \,\mathrm{cis}\, 5\pi$ (By De Moivre's Thm.)

$z^6 = 64 \,\mathrm{cis}\, \pi = \underline{-64 + 0i} = \underline{64}$

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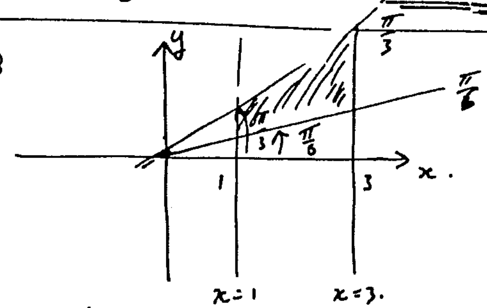
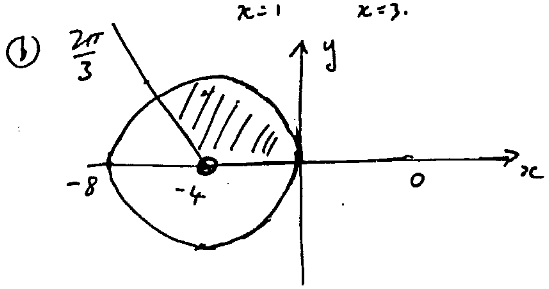
⑤ ⓐ



ⓑ



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⑥ (i) $\sqrt{24-70i} = a+ib$

$24-70i = a^2-b^2+2abi$

$\therefore a^2-b^2 = 24 \quad ab = -35.$

$\therefore a = 7 \quad$ or $\quad a = -7$

3 $\quad b = -5 \quad\quad b = 5.$

but $a > 0.$

$\therefore \underline{7-5i}$

(ii) $z^2-(1-i)z-6+17i = 0$

$$z = \frac{1-i \pm \sqrt{(1-i)^2-4(-6+17i)}}{2}$$

$$z = \frac{1-i \pm \sqrt{1-2i-1+24-68i}}{2} = \frac{1-i \pm \sqrt{24-70i}}{2}.$$

$$\therefore z = \frac{(1-i) \pm (7-5i)}{2}$$

$\underline{z = 4-3i}$ or $\underline{z = -3+2i}$

Q7 ⓐ $|z-4i| = 2|z+1|$

$x^2+(y-4)^2 = 2^2\{(x+1)^2+y\}^2$

$x^2+y^2-8y+16 = 4\{x^2+2x+1+y^2\}.$

3 $\quad 0 = 3x^2+3y^2+8x+8y-12.$

$x^2+y^2+\frac{8}{3}x+\frac{8}{3}y-4 = 0.$

$(x+\frac{4}{3})^2+(y+\frac{4}{3})^2 = 4+\frac{16}{9}+\frac{16}{9} = \frac{68}{9}$

Circle: Centre $(\frac{-4}{3}, \frac{-4}{3})$

Radius $\frac{\sqrt{68}}{3}$ units.

Q7 ⓑ $w = \frac{z-1}{z}$

$wz = z-1$

$1 = z-wz = z(1-w)$

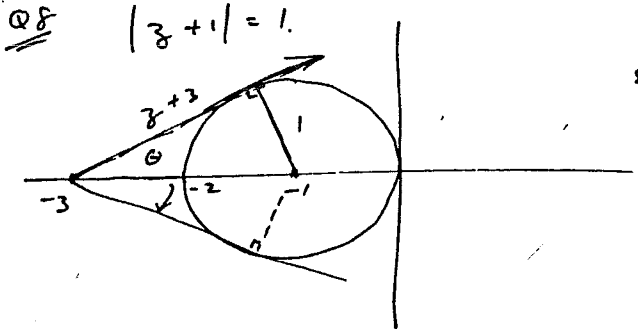
$\therefore z = \frac{1}{1-w}$

3 $\quad |z| = \left|\frac{1}{1-w}\right|$

$\therefore 2 = \left|\frac{1}{w-1}\right| \Rightarrow |w-1| = \frac{1}{2}.$

$\therefore$ Locus of w is a circle centre $(1,0)$ Radius $\frac{1}{2}$

Q8 $|z+1| = 1.$



$\sin\theta = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{6}.$

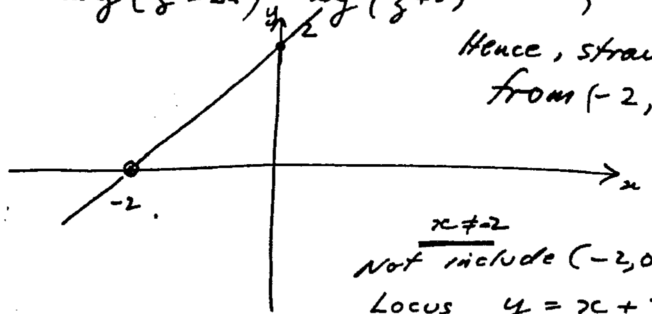
$\therefore$ Max arg is $\frac{\pi}{6}$

Min arg is $\frac{-\pi}{6}$

9. (i) $w = \dfrac{z-2i}{z+2}$ and w is purely real.

then: $\arg\left(\dfrac{z-2i}{z+2}\right) = \pm\pi, 0$

$\arg(z-2i) - \arg(z+2) = \pm\pi, 0$



Hence, straight line from $(-2,0)$ to $(0,2)$

$x \neq -2$

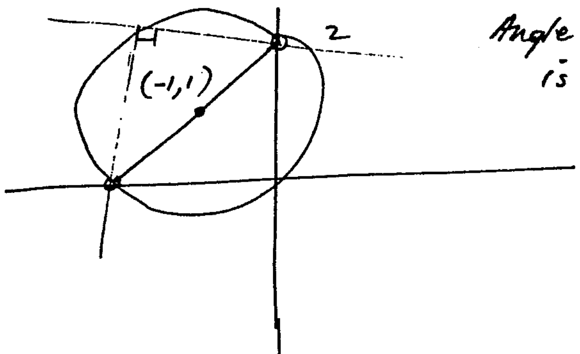
Not include $(-2,0)$.

Locus $y = x+2$

(ii) $w = \dfrac{z-2i}{z+2}$ and w is purely imaginary.

then $\arg\left(\dfrac{z-2i}{z+2}\right) = \pm\dfrac{\pi}{2}$.

then $\arg(z-2i) - \arg(z+2) = \pm\dfrac{\pi}{2}$.



Angle in semi-circle is a right angle:

Locus: Circle Centre $(-1,1)$
Radius $\sqrt{2}$ Units

or $(x+1)^2 + (y-1)^2 = 2$

Q9

$w = \dfrac{z-2i}{z+2}$

$\therefore w = \dfrac{z-2i}{z+2} \times \dfrac{\bar{z}+2}{\bar{z}+2}$ $\qquad z = x+iy$

$w = \dfrac{z\cdot\bar{z} - 2i\bar{z} + 2z - 4i}{(z+2)(\bar{z}+2)}$ $\qquad \bar{z} = x - iy.$

$w = \dfrac{x^2+y^2 - 2i(x-iy) + 2(x+iy) - 4i}{(z+2)(\bar{z}+2)}$.

$w = \dfrac{x^2+y^2 - 2ix - 2y + 2x + 2iy - 4i}{(z+2)(\bar{z}+2)}$.

(i) Now if w is purely real.

$-2ix + 2iy - 4i = 0.$ $\quad$ Exclude. $(-2,0)$

i.e. $2y = 2x+4$

$y = x+2$

(ii) Now if w is purely imaginary.

$x^2+y^2+2x-2y = 0.$

$(x^2+2x+1) + (y^2-2y+1) = 2.$

$(x+1)^2 + (y-1)^2 = 2.$

$\therefore$ Circle: $\left.\begin{array}{l}\text{Centre } (-1,1) \\ \text{Radius } \sqrt{2}.\end{array}\right\}.$

Exclude $(-2,0)$ and $(0,2)$
(end points of diameter)

(10). $\arg\left(\frac{-z}{i}\right) = \arg\left(\frac{1}{z}\right)$.

$\therefore \arg(-z) - \arg(i) = \arg 1 - \arg z$.

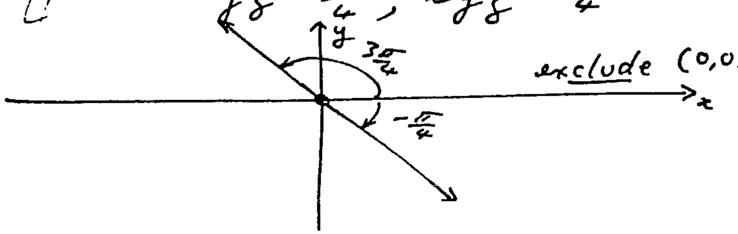
$\arg(-1.z) - \frac{\pi}{2} = 0 - \arg z$.

$\arg z + \arg(-1) + \arg z = \frac{\pi}{2}$.

$2\arg z \pm \pi = \frac{\pi}{2}$

$2\arg z = \frac{3\pi}{2}$ or $-\frac{\pi}{2}$

$\therefore \arg z = \frac{3\pi}{4}, \ \arg z = \frac{-\pi}{4}$



(11) $z = \cos\theta + i\sin\theta$.

$z + \bar{z} = \cos\theta + i\sin\theta + \cos\theta - i\sin\theta$.

$\therefore z + \bar{z} = 2\cos\theta. \quad \Rightarrow \left(\frac{1}{z} + z\right)$

By De Moivres Thm

$z^n = \cos n\theta + i\sin n\theta$.

$z^{-n} = \cos n\theta - i\sin n\theta$.

$\therefore z^n + \frac{1}{z^n} = 2\cos n\theta$.

$z^4 = (\cos\theta + i\sin\theta)^4$.

$\therefore (2\cos\theta)^4 = \left(z + \frac{1}{z}\right)^4$

$= z^4 + \frac{1}{z^4} + 4z^3.\frac{1}{z} + 6z^2.\frac{1}{z^2} + 4z.\frac{1}{z^3}$

$= z^4 + \frac{1}{z^4} + 4\left(z^2 + \frac{1}{z^2}\right) + 6$.

<u>Now:</u> $z^4 + \frac{1}{z^4} = 2\cos 4\theta$.

$z^2 + \frac{1}{z^2} = 2\cos 2\theta$.

$\therefore (2\cos\theta)^4 = 2\cos 4\theta + 8\cos 2\theta + 6$.

$\therefore 16\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6$.

$$\cos^4\theta = \frac{\cos 4\theta + 4\cos 2\theta + 3}{8}$$

(11) $I = \int_0^{\frac{\pi}{2}} \cos^4\theta . d\theta$.

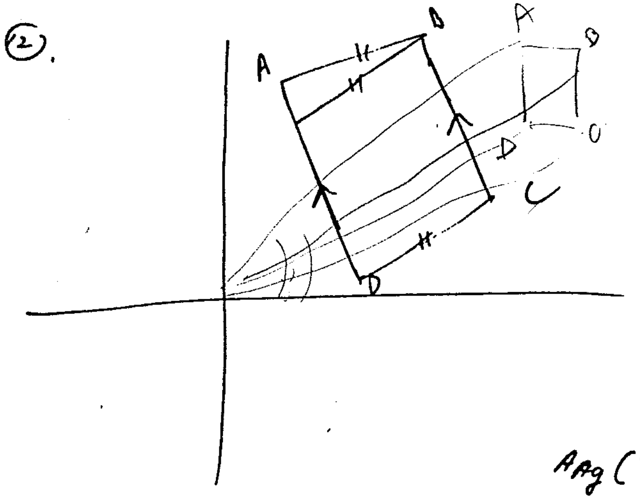
$I = \frac{1}{8}\int_0^{\frac{\pi}{2}} (\cos 4\theta + 4\cos 2\theta + 3.) . d\theta$.

$I = \frac{1}{8}\left[\frac{1}{4}.\sin 4\theta + 4.\frac{1}{2}.\sin 2\theta + 3\theta\right]_0^{\frac{\pi}{2}}$

$I = \frac{1}{8}\left[\frac{1}{4}.0 + 2.0. + 3.\frac{\pi}{2}\right] - [0 + 0 + 0]$

$I = \frac{3\pi}{16}$

(12).



arg z_1 = Arg z_n

$\operatorname{Arg}(z_1 - z_4) = \arg(z_2 - z_3)$ means $AD \parallel BC$.

$|z_4 - z_3| = |z_1 - z_2|$ means $DC = AB$.

<u>Now:</u> $AD \parallel BC$ and $DC = AB$.

$\therefore$ ABCD is an isosceles trapezium or a parallelogram.

(13) (i) $z^7 - 1 = 0$

Hence: $z^7 = 1 = \text{cis}\,0$ let $z = \cos\theta + i\sin\theta$

$z^7 = (\cos\theta + i\sin\theta)^7$

$z^7 = \text{cis}\,7\theta$

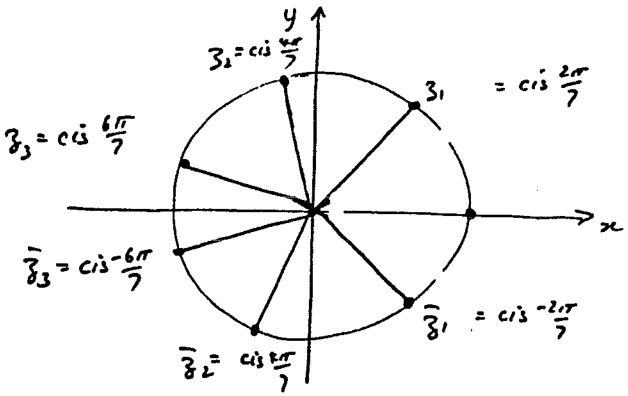
then $z^7 = \text{cis}(0 \pm 2k\pi)$

$\therefore z = \text{cis}\left(\frac{0 \pm 2k\pi}{7}\right)$ by De Moivre's Thm and $k = 0, 1, 2, 3, 4, 5, 6.$

$\therefore z = \text{cis}\,0,\ \text{cis}\,\frac{2\pi}{7},\ \text{cis}\,\frac{4\pi}{7},\ \text{cis}\,\frac{6\pi}{7}$

$\text{cis}\,\frac{-2\pi}{7},\ \text{cis}\,\frac{-4\pi}{7},\ \text{cis}\,\frac{-6\pi}{7}.$

(ii)



(iii) Now let $w = \text{cis}\,\frac{2\pi}{7}$

then $w^2 = \left(\text{cis}\,\frac{2\pi}{7}\right)^2 = \text{cis}\,\frac{4\pi}{7}$ by De Moivres Thm.

Hence $w^3 = \text{cis}\,\frac{6\pi}{7}$, $w^4 = \text{cis}\,\frac{8\pi}{7}$, $w^5 = \frac{10\pi}{7}$, $w^6 = \frac{12\pi}{7}$, $w^7 = \frac{14\pi}{7} = 2\pi.$

Now $z^7 = 1 \Rightarrow 1 - z^7 = 0$ and $z^7 = w^7$. when $z = w$.

$\therefore 1 - w^7 = (1 - w)(1 + w + w^2 + w^3 + w^4 + w^5 + w^6)$

But $(1 - w) \neq 0$ as $w \neq 1$

$\therefore (1 + w + w^2 + w^3 + w^4 + w^5 + w^6) = 0.$

OR $1 + w + w^2 + w^3 + w^4 + w^5 + w^6$ is a G.P. with $a = 1$, $r = w$.

Now $S_n = \frac{a(r^n - 1)}{r - 1} \Rightarrow S_7 = \frac{1.(w^7 - 1)}{w - 1}$ but $w^7 = 1$.

$\therefore S_7 = \frac{0}{w - 1} = 0.$

$\therefore 1 + w + w^2 + w^3 + w^4 + w^5 + w^6 = 0$

4/5 (iv) Now $z^7 - 1 = (z - 1)(z - z_1)(z - z_2)(z - z_3)(z - z_4)(z - z_5)(z - z_6)$

$= (z - 1)(z - w)(z - w^2)(z - w^3)(z - w^4)(z - w^5)(z - w^6)$

But w and w^6, w^2 and w^5, w^3 and w^4 are conjugate pairs.

$z^7 - 1 = (z - 1)(z - w)(z - \bar{w})(z - w^2)(z - \overline{w^2})(z - w^3)(z - \overline{w^3})$

Now $w + \bar{w} = 2\cos\frac{2\pi}{7}$

and $w.\bar{w} = +1.$

Hence, by applying conjugate pairs.

$z^7 - 1 = (z - 1)\left(z^2 - 2\cos\frac{2\pi}{7}z + 1\right)\left(z^2 - 2\cos\frac{4\pi}{7}z + 1\right)\left(z^2 - 2\cos\frac{6\pi}{7}z + 1\right)$

(14). (i) $\therefore P(x) = (x-a)^m \cdot Q(x)$

$$\frac{dP}{dx} = m\cdot(x-a)^{m-1}\cdot Q(x) + (x-a)^m\cdot Q'(x).$$

$$P'(x) = (x-a)^{m-1}\{m\cdot Q(x) + (x-a)\cdot Q'(x)\}.$$

$$P'(x) = (x-a)^{m-1}\{S(x)\}.$$

$\therefore$ If $x=a$ is a root of multiplicity m of $P(x)$
then $x=a$ is a root of multiplicity $(m-1)$ of $P'(x)$.

(ii) $P(x) = 8x^4 - 20x^3 - 18x^2 + 81x - 54$. (triple root)
$P'(x) = 32x^3 - 60x^2 - 36x + 81$ (double root)
$P''(x) = 96x^2 - 120x - 36$ (single root)
$P''(x) = 0 \Rightarrow$ $12(8x^2 - 10x + 3) = 0.$
$\therefore (4x+1)(2x-3) = 0.$
Now $x = \frac{3}{2}$ or $x = -\frac{1}{4}$
but $x = -\frac{1}{4}$ does not satisfy $P(x)$.

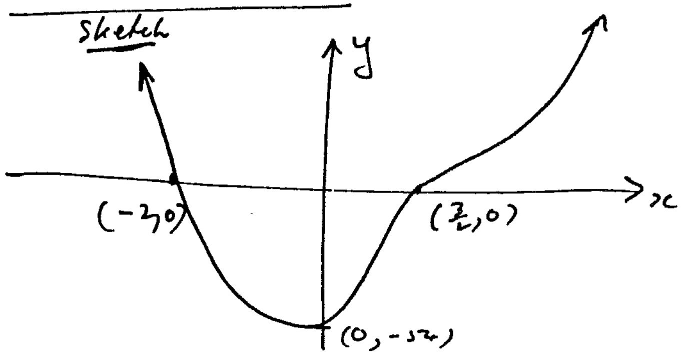
$\therefore$ triple root at $x = \frac{3}{2}$
$\therefore$ triple factor $(2x-3)$.

Now $\frac{-54}{8} = \left(\frac{3}{2}\right)^3 \cdot \alpha.$

$\frac{-54}{8} = \frac{27}{8}\cdot\alpha \Rightarrow \alpha = -2.$

$\therefore P(x) = (2x-3)^3(x+2)$

Sketch



Q15 $P(x) = 2x^3 + mx^2 + nx + 15$
and $x = 2-i$ is a zero
then $x = 2+i$ is a zero by the law of conjugate pairs.

Hence: $x^2 - 4x + 5$ is a factor.

Now:
$$\begin{array}{r|l} & 2x + 3 \\ \hline x^2-4x+5 & 2x^3 + mx^2 + nx + 15 \\ & 2x^3 - 8x^2 + 10x \\ \hline & (m+8)x^2 + (n-10)x + 15 \\ & k(x^2 - 4x + 5) \\ \hline & 0 \end{array}$$

$\therefore 5k = 15 \Rightarrow k = 3.$

$\therefore m + 8 - 3 = 0.$ $n - 10 + 12 = 0.$
$m = -5.$ $n = -2.$

$\therefore P(x) = 2x^3 - 5x^2 - 2x + 15$

$P(x) = (2x+3)(x^2 - 4x + 5)$

Q16 (a) $x^3+2x^2-3x+4=0$.

$$\left.\begin{aligned}\alpha+\beta+\gamma &= \frac{-2}{1} = -2\\ \alpha\beta+\beta\gamma+\gamma\alpha &= \frac{-3}{1} = -3\\ \alpha\beta\gamma &= \frac{-4}{1} = -4\end{aligned}\right\}$$

(i) $\alpha^2+\beta^2+\gamma^2$

$= (\alpha+\beta+\gamma)^2 - 2\sum\alpha\beta$.

$= (-2)^2 - 2.-3. \quad = \underline{\underline{10}}$

(ii) $\alpha^3+\beta^3+\gamma^3$. Now: $x^3 = -2x^2+3x-4$.

$\therefore \quad \alpha^3 = -2\alpha^2+3\alpha-4$.

$\therefore \alpha^3+\beta^3+\gamma^3 = -2\sum\alpha^2 + 3\sum\alpha - 4\times 3$

$= -2.10 + 3.-2 - 12$

$= \underline{\underline{-38}}$

(iii) $\alpha^4+\beta^4+\gamma^4$ Now $x^4 = -2x^3+3x^2-4x$.

$\therefore \quad \alpha^4 = -2\alpha^3+3\alpha^2-4\alpha$.

$\therefore \alpha^4+\beta^4+\gamma^4 = -2\sum\alpha^3 + 3\sum\alpha^2 - 4\sum\alpha$

$= -2.(-38) + 3.(10) - 4(-2)$

$= 76 + 30 + 8 \quad = \underline{\underline{114}}$

(b) $P(x) = x^3+2x^2-3x+4$.

(i) Now Roots $\alpha+2, \beta+2, \gamma+2$

then $Y = x+2 \Rightarrow x = Y-2$.

Now $(Y-2)^3+2(Y-2)^2-3(Y-2)+4=0$.

$Y^3-6Y^2+12Y-8+2Y^2-8Y+8-3Y+6+4=0$.

$Y^3-4Y^2+Y+10=0$.

$\therefore \underline{P(x) = x^3-4x^2+x+10}$

Q16. (b) (ii) $P(x) = x^3+2x^2-3x+4$.

Now Roots $\alpha^2, \beta^2, \gamma^2$.

$\therefore \quad Y = x^2 \Rightarrow x = \sqrt{Y}$.

$(\sqrt{Y})^3 + 2(\sqrt{Y})^2 - 3(\sqrt{Y}) + 4 = 0$

$Y\sqrt{Y} + 2Y - 3\sqrt{Y} + 4 = 0$.

$\sqrt{Y}(Y-3) = -2Y-4$.

$Y(Y-3)^2 = (2Y+4)^2$

$Y(Y^2-6Y+9) = 4Y^2+16Y+16$.

$Y^3-6Y^2+9Y = 4Y^2+16Y+16$.

$Y^3-10Y^2-7Y-16=0$.

$\therefore \underline{P(x) = x^3-10x^2-7x-16}$

(iii) Roots: $\frac{\alpha\beta}{\gamma}, \frac{\alpha\gamma}{\beta}, \frac{\beta\gamma}{\alpha}$ Now $\alpha\beta\gamma = \underline{\underline{-4}}$

or $\frac{\alpha\beta\gamma}{\gamma^2}, \frac{\alpha\gamma\beta}{\beta^2}, \frac{\alpha\beta\gamma}{\alpha^2}$

or $\frac{-4}{\alpha^2}, \frac{-4}{\beta^2}, \frac{-4}{\gamma^2}$,

$\underline{\underline{\text{from}}}$ (ii) $\frac{-4}{x}, \frac{-4}{x}, \frac{-4}{x}$ $\therefore \quad Y = \frac{-4}{x}$.

$x = \frac{-4}{Y}$

$\therefore P(Y) = \left(\frac{-4}{Y}\right)^3 - 10\left(\frac{-4}{Y}\right)^2 - 7\left(\frac{-4}{Y}\right) - 16$

$= \frac{-64}{Y^3} - \frac{160}{Y^2} + \frac{28}{Y} - 16$.

$0 = -64 - 160Y + 28Y^2 - 16Y^3$.

$\therefore \quad 4Y^3 - 7Y^2 + 40Y + 16 = 0$.

$\therefore \underline{P(x) = 4x^3 - 7x^2 + 40x + 16}$