Sydney Girls High School

MATHEMATICS

[March 2010]

YEAR 12
ASSESSMENT TASK 2

Topics: Probability, Sequences &Series and Applications of Calculus

General Instructions:

- Time allowed : 90 minutes + 5 minutes reading time
- There are FIVE (5) Questions which are of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each Question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write your student number clearly at the top of each question
- Clearly number each question.

Total: 90 marks

Question One (18 marks)

a) The second and fifth terms of a geometric sequence are -18 and $\frac{2}{3}$

i) Find the common ratio (2)

ii) Find the first term (1)

iii) Calculate the sum of the first 6 terms of the series , giving your answer in exact form. (2)

b) In a class of 25 students 18 like Twilight, 14 like Vampire Diaries and 2 like neither.

i) Draw a Venn diagram to show this information (1)

ii) If two people are chosen at random, what is the probability that both like Twilight only. (2)

c) The curve $y = 2x^3 + ax^2 - 3$ has a point of inflexion where $x = 1$, find the value of a. (2)

d) In a group there are 11 boys and 7 girls. Of these , there are 3 boys and 5 girls who have red hair.

i) If one person is selected at random , what is the probability that the person chosen :

a) is a boy (1)

b) has red hair (1)

c) is a girl who doesn't have red hair (1)

ii) If two people are chosen at random, what is the chance of the two people selected that:

a) both are girls (1)

b) neither has red hair (1)

c) both have red hair but only one is a girl (1)

d) at least one is a girl with red hair (2)

Question Two (18 marks)

a) The sum to n terms of an arithmetic series is given by $34n-n^2$. Find

i) the first term (1)
ii) The common difference (1)
iii) the n^{th} term (2)
iv) The value of n for which the first n^{th} term is negative (2)

b) i) The sixth term of an arithmetic series is 17 and the thirteenth term is 31. Find the common difference and the first term (3)
ii) Find the sum of the first 40 terms. (2)

c) Bella puts $500 in a trust fund for Reneesme. The trust receives interest, compounded annually, of 6%.

i) How much will there be in the trust after 50 years. Answer correct to the nearest dollar. (2)
ii) If each year after establishment of the trust fund Bella deposits an extra $500 which also earned interest at 6% p.a,
a) Show what the investment accumulates to after the 4th payment. (2)
b) How much will there be in the trust after 50 years. (3)

Question Three (18 marks)

a) For what values of x is the function $f(x)=-3x^2-12x+5$ decreasing? (3)

b) A heap of 8 cards consist of 2 Aces, 2 Kings, 2 Queens and 2 Jacks. If two cards are chosen at random from the heap, find the probability that they are 2 of a kind. (2)

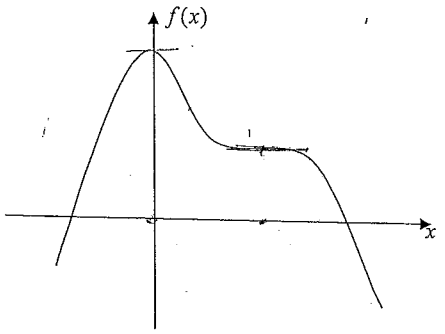
c) The first three terms of a series are $x-2, x+1$ and $3x-3$. Find the value of x if the sequence is:

i) Arithmetic (3)
ii) Geometric (3)

d) i) Write $0.\dot{3}\dot{7}$ as a geometric series. (1)
ii) Hence write $0.\dot{3}\dot{7}$ as fraction in its simplest form. (2)

e) For what values of x is the curve $y=2x^3+3x^2-12x+8$ concave upwards? (2)

f) Copy the following diagram in your answer sheet and draw the graph of $f'(x)$ (2)



Question Four (18 marks)

a) Consider $y = -x^3 + 12x^2 - 36x + 41$

i) Find any stationary points and determine their nature (5)

ii) Find any point of inflexion (3)

iii) Sketch the curve (2)

b) A car moves in a such a way that it covers 4m more in each second than it did in the previous second. If it covered 3m in the first second :

i) How far would it have travelled after 20 seconds (2)

ii) How long would it take the car to exceed 30m/s (2)

c) Find the primitive function of $2x^3 - \frac{2}{x^2}$ (2)

d) Given $f'(x) = \sqrt{x} + 2$, find $f(x)$ if $f(0) = 4$ (2)

Question Five (18 marks)

a) Find the difference between the limiting sum and the sum of the first 8 terms of the series 256+128+64+..... (3)

b) A couple wins $20 000, in a lottery. They decide to invest it all in an account which pays 10% per annum. Each year they withdraw $2500 to go on holidays

i) Show that after the third holiday the account will contain $20000 \times 1.1^3 - 2500(1 + 1.1 + 1.1^2)$ (3)

ii) Using trial and error or otherwise, find how many years they can continue to withdraw $2500 for their holiday? (3)

c) Draw a neat sketch of a continuous curve which has the following features. (3)

$f'(x) < 0$ for $0 \le x < 3$

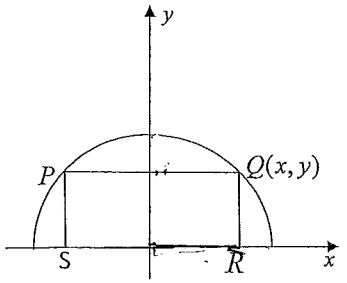
$f'(3) = 0$

$f'(x) > 0$ for $3 < x < 7$

$f'(7) = 0$

$f'(x) > 0$ for $7 < x \le 10$

d)



Rectangle PQRS is inscribed in a semicircle with radius 6 units as shown in the diagram. Q has coordinates (x, y).

i) Show that the area A of the rectangle is given by $A = 2x\sqrt{36 - x^2}$ (2)

ii) Find the dimensions of the rectangle with greatest area. (4)

THE END

Mematics 2010 Assessment task 2

1) a) i)

$ar = -18$ —> ①

$ar^4 = \frac{2}{3}$ — ②

from ①

$a = \frac{-18}{r}$

$\frac{-18}{r} \times r^4 = \frac{2}{3}$

$-18r^3 = \frac{2}{3}$

$r^3 = -\frac{1}{27}$

$\boxed{r = -\frac{1}{3}}$

ii) $a = \frac{-18}{-\frac{1}{3}}$

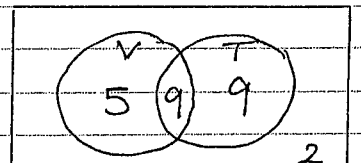
$\boxed{a = 54}$

iii) $S_6 = \frac{54\left(1 - (-\frac{1}{3})^6\right)}{1 - -\frac{1}{3}}$

$= \frac{364}{9}$

$= 40\frac{4}{9}$

b) i)



ii) $P(TT) = \frac{9}{} \times \frac{8}{} = \frac{3}{}$

c) $y' = 6x^2 + 2ax$

$y'' = 12x + 2a$

$y'' = 0$ at $x = 1$

$0 = 12 + 2a$

$12 = -2a$

$\boxed{a = -6}$

d) i)

a) $P(B) = \frac{11}{18}$

b) $P(\bar{R}H) = \frac{8}{18}$

$= \frac{4}{9}$

c) $P(G\bar{R}) = \frac{2}{18}$

$= \frac{1}{9}$

ii) a) $P(GG) = \frac{7}{18} \times \frac{6}{17}$

$= \frac{7}{51}$

b) $P(NR) = \frac{90}{306}$

$= \frac{5}{17}$

c) $P(RR) = \frac{5}{51}$

d) $P(\text{atleast 1 Red hair}) = 1 - \left(\frac{13}{\text{pair}} \frac{12}{}\right)$

2) a)

i) $T_1 = S_1 = 34 - 1$

$= 33$

ii) $S_2 = 68 - 4$

$= 64$

$T_1 + T_2 = 64$

$33 + T_2 = 64$

$T_2 = 31$

$d = 31 - 33$

$= -2$

iii) $T_n = 33 + (n-1)(-2)$

$= 33 + 2 - 2n$

$\boxed{T_n = 35 - 2n}$

iv) $35 - 2n < 0$

$-2n < -35$

$n > 17.5$

$\boxed{n = 18}$

b) $T_6 = 17$

$T_{13} = 31$

$a + 5d = 17$ — ①

$a + 12d = 31$ — ②

$7d = 14$

$\boxed{d = 2}$

$a + 10 = 17$

$\boxed{a = 7}$

ii) $S_{40} = \frac{40}{2}(14 + 39 \times 2)$

$\boxed{S_{40} = 1840}$

c) i)

$A = 500(1.06)^{50}$

$= \$9210$

ii) $A_1 = 500(1.06)^{50}$

a) $A_2 = 500(1.06)^{49}$

$A_3 = 500(1.06)^{48}$

$A_4 = 500(1.06)^{47}$

⋮

b) $A_{50} = 500(1.06)^1$

Total $= 500(1.06 + 1.06^2 + \cdots 1.06^{50})$

$= 500\left(\frac{1.06(1.06^{50} - 1)}{0.06}\right)$

$= \$153878.03$

Total $= 153878.03 + \$9210$

3) a)

$f'(x) = -6x - 12$

$-6x - 12 < 0$

$-6x < 12$

$x > -2$

b) $\frac{1}{4} \times \frac{7}{7} \times 4$

$= \frac{1}{7}$

c) i)

$x+1 - (x-2) = 3x - 3 - (x+1)$

$x+1 - x + 2 = 3x - 3 - x - 1$

$3 = 2x - 4$

$2x = 7$

$x = 3\frac{1}{2}$

ii) $\frac{x+1}{x-2} = \frac{3x-3}{x+1}$

$x^2 + 2x + 1 = 3x^2 - 6x - 3x + 6$

$2x^2 - 11x + 5 = 0$

$2x^2 - 10x - x + 5 = 0$

$2x(x-5) - (x-5) = 0$

$(2x-1)(x-5) = 0$

$x = \frac{1}{2}$ or 5

d) i)

$0.37 = \frac{37}{100} + \frac{37}{10000} + \frac{37}{1000000} + \ldots$

ii) $S = \frac{\frac{37}{100}}{1 - \frac{1}{100}}$

$= \frac{37}{99}$

e) $y' = 6x^2 + 6x - 12$

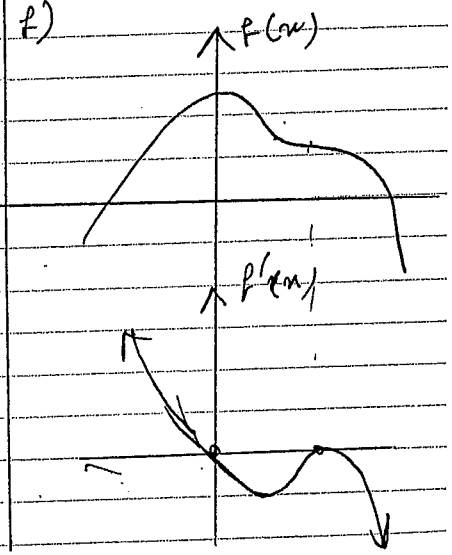
$y'' = 12x + 6$

$y'' > 0$

$12x > -6$

$x > -\frac{1}{2}$

f)



4)

$y' = -3x^2 + 24x - 36$

$y'' = -6x + 24$

$y' = 0$

$-3x^2 + 24x - 36 = 0$

$x^2 - 8x + 12 = 0$

$(x-2)(x-6) = 0$

$x = 2 \quad x = 6$

$x = 2 \rightarrow y = 9$

$x = 6 \rightarrow y = 41$

at $x = 2$

$y'' = 12$

$y'' > 0 \therefore (2, 9)$ min t.p

at $x = 6$

$y'' = -12$

< 0

$\therefore (6, 41)$ max t.p

ii) $y'' = 0$

$-6x + 24 = 0$

$-6x = -24$

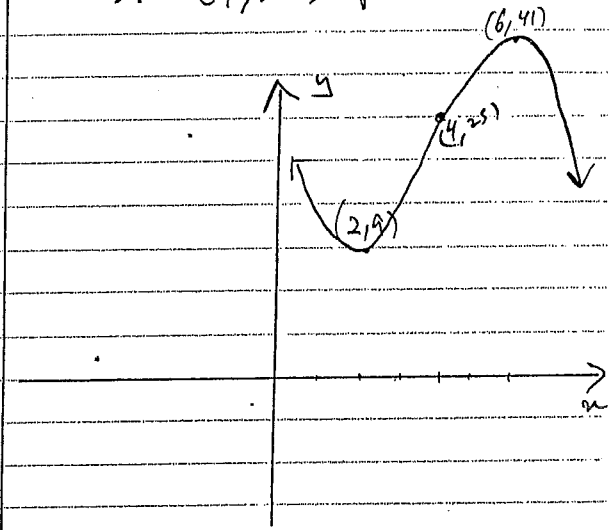
$x = 4$ at $x = 4$

$y = 25$

x	3	4	5
y'	+	0	−

concavity changes

$\therefore (4, 25)$ p. of inflex.



b) $3 + 7 + 11 + \ldots$

i) $S_{20} = \frac{20}{2}(6 + 19 \times 4)$

$= 820$

ii) $T_n = 3 + (n-1)4$

$T_n > 30$

$3 + 4n - 4 > 30$

$4n > 31$

$n > 7\frac{3}{4}$

$\therefore \boxed{n = 8}$

c) $\frac{2x^4}{4} - \frac{2x^{-1}}{-1} + C$

$= \frac{x^4}{2} + \frac{2}{x} + C$

d) $f(x) = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2x + C$

$= \frac{2}{3}x^{\frac{3}{2}} + 2x + C$

at $x = 0 \quad f(x) = 4$

$4 = 0 + 0 + C$

$\therefore C = 4$

$f(x) = \frac{2}{3}\sqrt{x^3} + 2x + 4$

5a) $S_\infty = \frac{256}{1 - \frac{1}{2}}$

$= 512$

$S_8 = \frac{256\left(1 - \left(\frac{1}{2}\right)^8\right)}{1 - \frac{1}{2}}$

$= 512\left(1 - \frac{1}{256}\right)$

$= 510$

$D = 512 - 510$

$= 2$

b) i)

$A_1 =$

$20000(1.1) - 2500$

$A_2 = (20000(1.1) - 2500)1.1 - 2500$

$= 20000(1.1)^2 - 1.1(2500) - 2500$

$= 20000(1.1)^2 - 2500(1.1 + 1)$

$A_3 = 20000(1.1)^3 - 2500(1 + 1.1 + 1.1^2)$

ii)

$A_n = 20000(1.1)^n - 2500(1 + 1.1 \ldots 1.1^{n-1})$

$20000(1.1)^n - 2500\left(\frac{1(1.1^n - 1)}{0.1}\right) < 25000$

$20000(1.1)^n - 25000(1.1^n - 1) < 25000$

$20000(1.1)^n - 25000(1.1^n) + 25000 < 25000$

$-5000(1.1)^n < -22500$

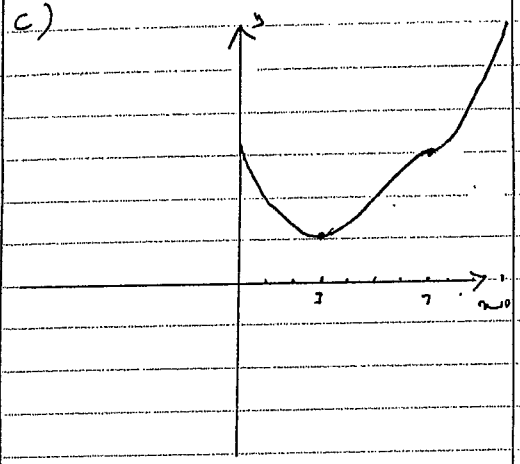
$(1.1)^n > 4.5$

$n\log(1.1) > \log 4.5$

$n > 15.78$

$n \leq 16$

c)



d) i) $PQ = 2x$

$6^2 = y^2 + x^2$

$y^2 = 36 - x^2$

$y = \sqrt{36 - x^2}$

$A = 2x \times y$

$= 2x\sqrt{36 - x^2}$

ii) $A = 2x(36 - x^2)^{\frac{1}{2}}$

$A' = 2(36 - x^2)^{\frac{1}{2}} + \frac{1}{2}(36 - x^2)^{-\frac{1}{2}} \times -2x \times 2x$

$0 = 2\sqrt{36 - x^2} - \frac{2x^2}{\sqrt{36 - x^2}}$

$2(36 - x^2) - 2x^2 = 0$

$-2x^2 + 72 - 2x^2 = 0$

$-4x^2 = -72$

$x^2 = 18$

$x = \sqrt{18}$

$x = 3\sqrt{2}$

x	4	$3\sqrt{2}$	5
y'	+	0	−

max at $x = 3\sqrt{2}$

$y = \sqrt{36 - 18}$

$= \sqrt{18}$

$x = \sqrt{18} \quad y = \sqrt{18}$

$PQ = 2\sqrt{18}$

$y = \sqrt{18}$