SYDNEY GIRLS H.S.

MATHEMATICS - EXT 1 - ASSESSMENT TASK 2 - March 2009.

QUESTION ONE (15 marks)

a) Change $\frac{2\pi}{5}$ radians into degrees. (1)

b) Write down the degree of the polynomial $P(x) = x^4(x^3+1)$ (1)

c) Change $300°$ into radians in exact form. (1)

d) Given $P(x) = x^4 + 2x^3 - 6x^2 + 12$ and $Q(x) = 7x^3 + 3x^2 + 9$, find

i) $Q(x) + P(x)$ (1)

ii) $Q(x) - P(x)$ (2)

e) Differentiate $y = 4\sin 3x$ (1)

f) Evaluate i) $\int_0^{\pi} \cos x \, dx$ (2)

ii) Find the area bounded by the curve $y = \cos x$, the x-axis and the lines $x = 0$ and $x = \pi$ (Hint: Draw a sketch) (3)

g) Evaluate $\lim_{x\to 0} \frac{\sin 3x}{2x}$ (1)

h) Differentiate $y = \tan\frac{x}{3}$ (2)

QUESTION TWO (15 marks)

a) Differentiate and express in simplest form:

i) $y = \frac{\sin x}{1+\cos x}$ (3)

ii) $y = (1+\sin x)^5$ (1)

b) Sketch the curve $y = 4\sin\frac{x}{2}$ in the domain $0 \le x \le 4\pi$ (3)

c) Find the quotient and remainder when the $P(x) = 5x^3 - 17x^2 - x + 11$ is divided by $A(x) = x - 2$. (3)

d) The gradient of a function $f(x)$ is given by $f'(x) = -6\sin 2x$.

If $f\left(\frac{\pi}{2}\right) = 4$, find the function $f(x)$. (3)

e) Find $\int \frac{dx}{\sec 2x}$ (2)

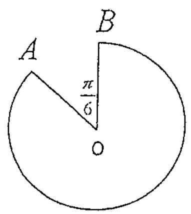
QUESTION THREE (15 marks)

a) Find $\int \frac{\cos x}{\sin x}\, dx$ (1)

b) The radius of a circle is $OA = 4cm$ and the acute angle $AOB = \frac{\pi}{6}$ radians. Find

i) The exact perimeter of the major sector OBA (2)

ii) The exact area of the major sector OBA (2)



c) Find the exact value of $\sin\frac{7\pi}{6} \times \tan\frac{11\pi}{6} \times \cos\frac{2\pi}{3}$ (3)

d) Find the size of the acute angle between the lines $y = 2x - 3$ and $3x + 2y = 6$ to the nearest minute. (2)

e) i) Differentiate $y = x\sin x$ (2)

ii) Hence find the value of $\int_0^{\pi} x\cos x\, dx$ (3)

QUESTION FOUR (15 marks)

a) Find $\int_0^{\frac{\pi}{4}} \sin^2 x\, dx$ (3)

b) i) Show that

$$[\sin(A+B)][\sin(A-B)] = (\sin A\cos B)^2 - (\cos A\sin B)^2$$ (2)

ii) Hence find the exact value of $\sin 75^\circ \sin 15^\circ$ (3)

c) Show that $\frac{2\sin^3 A - 2\cos^3 A}{\sin A - \cos A} = \sin 2A + 2$ (3)

d) Find all the values of x in the domain $0 \le x \le 2\pi$ for which $\sin 2x = \cos x$ (4)

QUESTION FIVE (15 marks)

a) If $\cos A = \frac{7}{9}$ and $\sin B = \frac{1}{3}$ when $0 \le A \le \frac{\pi}{2}$ and $0 \le B \le \frac{\pi}{2}$

i) Show that $A = 2B$ by considering $\sin A = \sin 2B$ (2)

(do not use a calculator)

ii) Find the value of $\tan(A+B)$ in simplest surd form (2)

b) i) Express $5\sin x + \sqrt{11}\cos x$ in the form $R\sin(x+\alpha)$ where $R > 0$ and α is acute. (2)

ii) Hence or otherwise solve $5\sin x + \sqrt{11}\cos x = 3$ for $0^\circ \le x \le 360^\circ$ (3)

c) i) Sketch the curve $y = 1 + \cos 2x$ for $0 \le x \le \pi$ (2)

ii) The area enclosed between the x-axis, the curve $y = 1 + \cos 2x$ and the ordinates $x = 0$ and $x = \frac{\pi}{2}$ is rotated about the x-axis. Find the exact volume of the solid of revolution formed. (4)

THE END

2009 Ex 1 Task 2

1) a) $= \frac{2 \times 180}{5}$

$= 72^\circ$ 1

b) degree $= 7$ 1

c) $\frac{300\pi}{180}$ 1

$= \frac{5\pi}{3}$

d) i) $Q(x) + P(x) = x^4 + 9x^3 - 3x^2 + 21$ 1

ii) $7x^3 + 3x^2 + 9 - x^4 - 2x^3 + 6x^2 - 12$

$= -x^4 + 5x^3 + 9x^2 - 3$ 2

e) $y' = 12\cos 3x$ 1

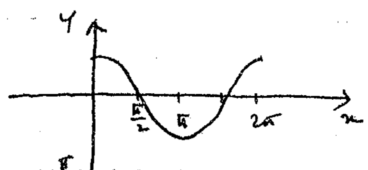
f) i) $\int_0^{\pi} \cos x \, dx = \left[\sin x\right]_0^{\pi}$

$= \sin\pi - \sin 0$

$= 0 - 0$

$= 0$ 2

ii)



$A = 2\int_0^{\frac{\pi}{2}} \cos x \, dx$ 3

$= 2\left[\sin x\right]_0^{\frac{\pi}{2}}$

$= 2\left[1 - 0\right] = 2u^2$

g) $\lim_{x \to 0} \frac{\sin 3x}{2x}$

$= \frac{3}{2} \lim_{x \to 0} \frac{\sin 3x}{3x}$

$= \frac{3}{2}$ 1

h) $y' = \frac{1}{3}\sec^2 \frac{x}{3}$ 2

2a) i) $y' = \frac{\cos x(1+\cos x) + \sin x(\sin x)}{(1+\cos x)^2}$

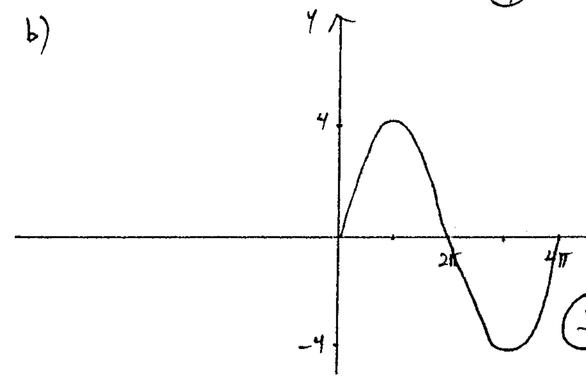
$= \frac{\cos^2 x + \cos x + \sin^2 x}{(1+\cos x)^2}$

$= \frac{\cancel{1+\cos x}}{(1+\cos x)^{\cancel{2}}}$ 3

$= \frac{1}{1+\cos x}$

ii) $y' = 5(1+\sin x)^4 \cos x$ 1

b)



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c)

$$\begin{array}{r|l} & 5x^2 - 7x - 15 \\ x-2 & 5x^3 - 17x^2 - x + 11 \\ & 5x^3 - 10x^2 \\ \hline & -7x^2 - x \\ & -7x^2 + 14x \\ \hline & -15x + 11 \\ & -15x + 30 \\ \hline & -19 \end{array}$$

3

Quotient $= 5x^2 - 7x - 15$

remainder $= -19$

d) $f(x) = \frac{6\cos 2x}{2} + c$

$= 3\cos 2x + c$

$f\left(\frac{\pi}{2}\right) = 3\cos 2\times\frac{\pi}{2} + c$

$4 = 3\cos\pi + c$

$4 = -3 + c$

$c = 7$ 3

$f(x) = 3\cos 2x + 7$

e) $\int \frac{dx}{\sec 2x}$

$\int \cos 2x \, dx$

$= \frac{\sin 2x}{2} + c$ 2

QUESTION 3.

a) $\int \frac{\cos x}{\sin x}\,dx = \ln(\sin x) + C$ 1

b) i) $P = 4 \times {}^{11\pi}/_{6} + 8$

$= \left({}^{22\pi}/_{3} + 8\right)\text{cm}$ 2

ii) $A = \tfrac{1}{2} r^2 \theta$

$= \tfrac{1}{2} \times 4^2 \times {}^{11\pi}/_{6}$

$= {}^{88\pi}/_{6}$ 2

$= {}^{44\pi}/_{3}\ \text{cm}^2$.

c) $\sin {}^{7\pi}/_{6} \times \tan {}^{11\pi}/_{6} \times \cos {}^{2\pi}/_{3} = -\sin {}^{\pi}/_{6} \times -\tan {}^{\pi}/_{6} \times -\cos {}^{\pi}/_{3}$

$= -\tfrac{1}{2} \times \frac{-1}{\sqrt{3}} \times -\tfrac{1}{2}$

$= \frac{-1}{4\sqrt{3}}$ 3

d) $y = 2x - 3$ $\qquad$ $y = 6 - {}^{3}/_{2}x$

$m_1 = 2$ $\qquad$ $m_2 = -{}^{3}/_{2}$

$\tan\theta = \left| \frac{2 + {}^{3}/_{2}}{1 + 2 \times -{}^{3}/_{2}} \right|$ 2

$= \left| \frac{3.5}{-2} \right|$

$\theta = 60^\circ 15'$

e) i) $y = x \sin x$

$y' = u'v + uv'$ 2

$= \sin x + x\cos x$

ii) $\int_0^{\pi} \sin x + x\cos x \, dx = x \sin x \Big]_0^{\pi}$

$\int_0^{\pi} x\cos x \, dx = x\sin x \Big]_0^{\pi} - \int_0^{\pi} \sin x \, dx$

$= x\sin x + \cos x \Big]_0^{\pi}$

$= -1 - 1$

$= -2$ 3

<u>Question Four</u>

a) $\int_0^{\frac{\pi}{4}} \sin^2 x \, dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} (1-\cos 2x)\, dx$ ✓

$= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$ ✓

$= \frac{1}{2}\left[\left(\frac{\pi}{4} - \frac{1}{2}(1)\right] - [0-0]\right.$ ✓ $\quad /3$

$= \frac{\pi}{8} - \frac{1}{4}$ or $\frac{\pi - 2}{8}$

b) i) LHS $= (\sin(A+B))(\sin(A-B))$

$= (\sin A \cos B + \cos A \sin B)(\sin A \cos B - \cos A \sin B)$ ✓

$= (\sin A \cos B)^2 - (\cos A \sin B)^2$ ✓ $\quad /2$

ii) $\sin 75^\circ \sin 15^\circ = \sin(45^\circ + 30^\circ)\sin(45^\circ - 30^\circ)$

$= (\sin 45^\circ \cos 30^\circ)^2 - (\cos 45^\circ \sin 30^\circ)^2$ ✓

$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right)^2$ ✓

$= \left(\frac{\sqrt{3}}{2\sqrt{2}}\right)^2 - \left(\frac{1}{2\sqrt{2}}\right)^2$

$= \frac{3-1}{8}$ $\quad /3$

$= \frac{1}{4}$ ✓ correct evaluation must follow from above

c) LHS $= \dfrac{2\sin^3 A - 2\cos^3 A}{\sin A - \cos A}$

$= \dfrac{2(\sin A - \cos A)(\sin^2 A + \sin A \cos A + \cos^2 A)}{(\sin A - \cos A)}$

$= 2(\sin^2 A + \cos^2 A + \sin A \cos A)$

$= 2 + 2\sin A \cos A$

$= 2 + \sin 2A$ $\quad /3$

d) $\sin 2x = \cos x \qquad (0 \le x \le 2\pi)$

$2\sin x \cos x = \cos x$

$2\sin x \cos x - \cos x = 0$ ✓ lose 1 or 2 here if ÷ by $\cos x$

$\cos x(2\sin x - 1) = 0$ ✓

$\cos x = 0 \quad$ or $\sin x = \frac{1}{2}$ ✓ $\quad /4$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = \frac{\pi}{6}, \frac{5\pi}{6}$ ✓✓ [illegible]

Question Five

a) $\cos A = \frac{7}{9}$ $\sin B = \frac{1}{3}$

$\sqrt{32} = 4\sqrt{2}$ $\sqrt{8} = 2\sqrt{2}$

i) $\sin A = \frac{4\sqrt{2}}{9}$ ✓ , $\sin 2B = 2\sin B\cos B$
$= 2 \times \frac{1}{3} \times \frac{2\sqrt{2}}{3}$
$= \frac{4\sqrt{2}}{9}$ ✓ /2

since $\sin A = \sin 2B$ (A, B acute)
then $A = B$

ii) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
($\tan A = \frac{4\sqrt{2}}{7}$, $\tan B = \frac{1}{2\sqrt{2}}$)
$= \left(\frac{4\sqrt{2}}{7} + \frac{1}{2\sqrt{2}}\right) \div \left(1 - \frac{4\sqrt{2}}{7} \times \frac{1}{2\sqrt{2}}\right)$ ✓
$= \frac{16+7}{14\sqrt{2}} \div \left(1 - \frac{2}{7}\right)$
$= \frac{23}{14\sqrt{2}} \times \frac{7}{5}$
$= \frac{23}{10\sqrt{2}}$ ✓
$= \frac{23\sqrt{2}}{20}$ /2

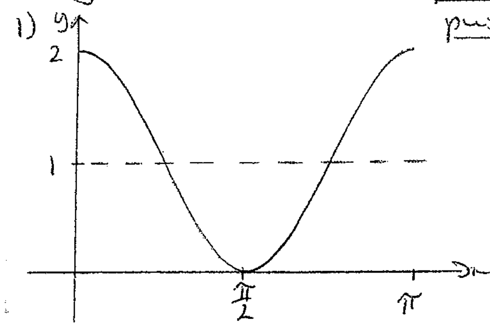
b) i) $5\sin x + \sqrt{11}\cos x = R\sin(x+\alpha)$ $R = \sqrt{25+11} = 6$
Divide by $R = 6$ ✓
$\frac{5}{6}\sin x + \frac{\sqrt{11}}{6}\cos x = \sin(x+\alpha)$
$\frac{5}{6}\sin x + \frac{\sqrt{11}}{6}\cos x = \sin x\cos\alpha + \cos x\sin\alpha$
$\therefore$ $\cos\alpha = \frac{5}{6}$, $\sin\alpha = \frac{\sqrt{11}}{6}$
$\tan\alpha = \frac{\sqrt{11}}{6} \times \frac{6}{5}$ [ie $6\sin(x + 33°33')$] 2/
$= \frac{\sqrt{11}}{5}$ ← (accept $\tan^{-1}\frac{\sqrt{11}}{5}$) but can lose in ii)
$\alpha = 33°33'$ ✓ (nearest minute)
0.586 radian
0.58

ii) $5\sin x + \sqrt{11}\cos x = 3$
ie $6\sin(x + 33°33') = 3$
$\sin(x + 33°33') = \frac{1}{2}$ ✓
$x + 33°33' = 30°$ or $150°$
$x = -3°33'$ ✓ $x = 116°27'$ ✓ /3
$= 356°27'$

Question Five Continued

c) $y = 1 + \cos 2x$ period $\frac{2\pi}{2} = \pi$, amp = 1
push up 1 unit, y incpt 2

i)



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ii) $V = \pi\int_0^{\frac{\pi}{2}} (1+\cos 2x)^2\,dx$
$= \pi\int_0^{\frac{\pi}{2}} (1 + 2\cos 2x + \cos^2 2x)\,dx$ ✓

Now $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
$\therefore \cos^2 2x = \frac{1}{2}(1 + \cos 4x)$
$\therefore V = \pi\int_0^{\frac{\pi}{2}} \left(1 + 2\cos 2x + \frac{1}{2}(1 + \cos 4x)\right)dx$ ✓
$= \pi\int_0^{\frac{\pi}{2}} \left(\frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x\right)dx$
$= \pi\left[\frac{3x}{2} + \sin 2x + \frac{1}{8}\sin 4x\right]_0^{\frac{\pi}{2}}$ ✓
$= \pi\left[\left(\frac{3\pi}{4} + 0 + 0\right) - (0+0+0)\right]$ ✓ correct evaluation before simplify
$= \frac{3\pi^2}{4}$ units^3 /4

1 2 3 4 5 6 7 8