Sydney Girls High School

2005 Assessment Task 2

MATHEMATICS

Year 12

Time allowed - 90 minutes

Topics: Probability, Series and Sequences , The Quadratic Polynomial

Instructions

NAME ______________________

- Attempt all five questions.
- Questions are of equal value.
- All necessary working should be shown in every question.
- Marks may be deducted for careless or badly arranged work.
- Write on one side of the paper only.

Question One (18 marks)

a) Given the parabola $y = 2x^2 + 7x - 4$ [8]

- i.) Find the X intercepts
- ii.) Find the Y intercept
- iii.) Find the equation of the axis of symmetry
- iv.) Find the co ordinates of the vertex
- v.) Sketch the parabola
- vi.) Find the Max/Min value of the parabola

b) In a census of a large Girls High School 65% of the population had been to the beach on the previous weekend. Three students are selected at random. Find the probability that: [2]

- i.) None of the girls had been to the beach
- ii.) At least one of the girls had been to the beach

c) Anh throws a pair of dice. Find the probability of getting: [5]

- i.) Any Pair
- ii.) A pair of twos
- iii.) At least one six
- iv.) A total of four
- v.) A total greater than nine

d) The sum of n terms of an arithmetic sequence is given by $S_n = 11n + 2n^2$. Find: [3}

- i.) The first term
- ii.) The common difference

Question Two (18marks)

a) If α and β are the roots of the quadratic equation $2x^2 - 6x - 1 = 0$ find: [8]

i.) $\alpha + \beta$

ii.) $\alpha\beta$

iii.) $\dfrac{2}{\alpha} + \dfrac{2}{\beta}$

iv.) $\alpha^2 + \beta^2$

v.) $\alpha^3 + \beta^3$

b) In a fund raising raffle, tickets are numbered consecutively from 1 to 1000. Participants draw a ticket at random and pay an amount in dollars corresponding to the number on the ticket except for those tickets with numbers divisible by 5, which are free. How much money is raised if all 1000 tickets are sold? [4]

c) Jai decides to get fit. After little thought he decides on the following approach. On the first day he will jog for 10 minutes. On the second day he will jog for 15 minutes. On each subsequent day he will jog for five minutes longer than the previous day. [6]

i.) How long will Jai jog for on the twentieth day?

ii.) How many hours of jogging will Jai have completed by the end of the twentieth day?

iii.) During which day will Jai have completed a total forty hours of jogging?

all together

Question Three (18marks)

a) $(2m-8)$, $(2m+4)$ and $(5m-2)$ are successive terms of a geometric sequence. Find the value of m [2]

b) Find the values of p for which $3x^2 - px + 3 = 0$ has no real roots [2]

c) Solve $(x+1)^2 + 5(x+1) + 6 = 0$ [2]

d) Find the values of A, B and C if $2x^2 + 3x - 5 = A(x+1)^2 + B(x+1) + C$ [3]

e) Prove that the equation $2ax + 3bx - 2b = 3ax^2$ has rational roots if a and b are rational [3]

f) A box contains contains 8 chocolates of identical size and shape. Five of the chocolates are soft centered and three are hard centered. Two are chosen at random: [4]

i.) What is the probability that the first drawn is soft centered?
ii.) What is the probability that both are soft centered?
iii.) What is the probability that both are hard centered?
iv.) What is the probability that one is hard and one is soft?

g) Given that the line $y = mx - 6$ is a tangent to the parabola $y = x^2 - 2x + 3$, find m. [2]

Question Four (18 marks)

a) If one root of $3x^2 - kx + 12 = 0$ is 2, find the value of k [2]

b) Mama Chiu's has a new lunchtime menu to celebrate the New Year. [2]

Entrées	Mains
Spring Rolls	Crispy Chicken
Dumplings	Lemon Chicken
Chicken Wings	Roast Chicken
	Chicken Noodle Salad

i.) In how many ways can I choose an entrée and a main?
ii.) Find the probability that I will choose Dumplings as an entrée and either Crispy Chicken or Lemon Chicken as a main course

Chicken or L
$1/4 + 1/4 = 1/2$

c) Find the first two terms of a geometric sequence in which the sixth term is 160 and the seventh term is 320. [3]

d) Freida frog has spotted a fly 1.99m away, which she fancies for lunch [6]
Freida jumps 1m on her first leap, 0.5m on her second, and 0.25m on her third and so on.

i.) How long will her seventh jump be?
ii.) How far in total will she have jumped by the end of her seventh jump?
iii.) Will Freida ever reach her lunch? Justify your answer

e) Find the first term and common difference of an arithmetic sequence in which the fifth term is three times the second term, and the sum of the first six terms is 36. [3]

f) A bag contains black and white marbles. The probability of choosing a black marble is $\frac{3}{4}$. [2]

i.) What is the probability of choosing a white marble at random?
ii.) If I add another 10 white marbles to the bag, the probability of choosing a white marble changes to $\frac{1}{2}$. How many marbles are there now in the bag?

Question Five (18 marks)

a) Find $\sum_{n=1}^{12}(2n-1)$ [2]

b) Find the quadratic equation with roots $\left(3+\sqrt{7}\right)$ and $\left(3-\sqrt{7}\right)$ [2]

c) In a class of 24 students, 10 study Chemistry, 11 study Physics and 6 study neither. If a student is chosen at random, find the probability that she studies: [4]

i.) Both Chemistry and Physics

ii.) Physics only

d) Jessie decides to borrow \$80 000 to buy a new BMW. She is to repay it in equal monthly instalments over five years. These repayments are due at the end of each month. The interest is calculated on the balance owing at the start of each month and is charged at the rate of 12%pa. Let \$M be the monthly repayment and $\$A_n$ be the amount owing after n months. [7]

i.) How much interest is charged for the first month?

ii.) Show that $A_n = 80\ 000 \times 1.01^n - \left(\frac{M\left(1.01^n - 1\right)}{0.01}\right)$

iii.) Find \$M to the nearest dollar.

iv.) How much does Jessie owe at the end of four years?

e) Solve $3x^2 + 2x + \sqrt{3x^2 + 2x - 1} = 1$ [3]

Question One 2005

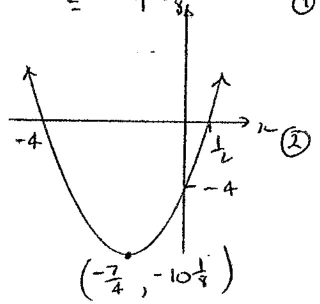
a) i) $y = 2x^2 + 7x - 4$
$= (2x-1)(x+4)$
$x = \frac{1}{2},\ x = -4$ ②

ii) $y = -4$ ①

iii) $x = -\frac{b}{2a}$
$= \frac{-7}{4}$ ①

iv) When $x = \frac{-7}{4}$
$y = 2(-\frac{7}{4})^2 + 7(-\frac{7}{4}) - 4$
$= -9\frac{5}{8}$ ①

v)



②

vi) $Y_{min} = -10\frac{1}{8}$ ①

b) i) $P = \frac{35}{100} \times \frac{35}{100} \times \frac{35}{100}$
$= \frac{42875}{1000000}$
$= \frac{1715}{40000}$
$= \frac{343}{8000}$ ①

ii) $P = 1 - \frac{343}{8000}$
$= \frac{7657}{8000}$ ①

c)

	1	2	3	4	5	6
1			x			o
2		x				o
3	x					o
4						o
5					o	o
6	o	o	o	o	o	o

i) $\frac{6}{36} = \frac{1}{6}$ ①
ii) $\frac{1}{36} = \frac{1}{36}$ ①
iii) $\frac{11}{36} = \frac{11}{36}$ ①
iv) $\frac{3}{36} = \frac{1}{12}$ ①
v) $\frac{6}{36} = \frac{1}{6}$ ①

Q1 d) $S_1 = T_1$
$= 11(1) + 2(1)^2$
ie $a = 13$ ①

$S_2 = 11(2) + 2(2)^2$
$= 30$ ①
$\therefore T_2 = 30 - 13$
$= 17$, $d = 4$ ①

Question Two

a) $2x^2 - 6x - 1 = 0$

i) $\alpha + \beta = \frac{-b}{a}$
$= 3$ ①

ii) $\alpha\beta = \frac{c}{a}$
$= -\frac{1}{2}$ ①

iii) $\frac{2}{\alpha} + \frac{2}{\beta} = \frac{2(\alpha+\beta)}{\alpha\beta}$
$= 6 \div -\frac{1}{2}$
$= -12$ ②

iv) $(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$
$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
$= (3)^2 - 2(-\frac{1}{2})$
$= 10$ ②

v) $(\alpha+\beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$
$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$
$= (3)^3 - 3(-\frac{1}{2})(3)$
$= 31\frac{1}{2}$ ②

b) Required Sum $= (1+2+3+\dots+1000) - (5+10+15+\dots+1000)$
$S_n = \frac{n}{2}(2a + (n-1)d)$ ①

$\therefore$ Required Sum $= \{\frac{1000}{2}(2 + (999)1)\} - \{\frac{200}{2}(10 + (199)5)\}$ ① ①
$= 500\,500 - 100\,500$
$= \$400\,000$ ①

c) $10 + 15 + 20 + \dots$
$a = 10,\ d = 5$

i) $T_n = a + (n-1)d$
$n = 20$
$T_{20} = 10 + (19)5$
$= 105$ mins ②

ii) $S_n = \frac{n}{2}(a + l)$
$S_{20} = \frac{20}{2}(10 + 105)$
$= 1150$ mins
$= 19$ hrs 10 min ②

3/8

ii) 40 hrs = 2400 mins

$2400 = \frac{n}{2}(20 + (n-1)5)$

$4800 = n(20 + 5n - 5)$

$4800 = n(15 + 5n)$

$4800 = 15n + 5n^2$

$5n^2 + 15n - 4800 = 0 \Rightarrow n^2 + 3n - 960 = 0$

$n = \frac{-3 \pm \sqrt{9 + 4 \times 960}}{2}$

$= \frac{-3 \pm \sqrt{3849}}{2}$

$= 29.52$

ie 30th day (2)

Question Three

a) $\frac{2m-8}{2m+4} = \frac{2m+4}{5m-2}$

$(2m-8)(5m-2) = (2m+4)^2$

$10m^2 - 44m + 16 = 4m^2 + 16m + 16$

$6m^2 - 60m = 0$

$6m(m-10) = 0$

$m = 0$ or $m = 10$ (2)

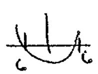
b) No real roots $\Delta < 0$

$b^2 - 4ac < 0$

$(-p)^2 - 4(3)(3) < 0$

$p^2 - 36 < 0$

$-6 < p < 6$



(2)

4/8

c) $(x+1)^2 + 5(x+1) + 6 = 0$

put $m = (x+1)$

$m^2 + 5m + 6 = 0$

$(m+2)(m+3) = 0$

$m = -2$, $m = -3$

$x + 1 = -2$, $x + 1 = -3$

$x = -3$, $x = -4$ (2)

d) $2x^2 + 3x - 5 = A(x+1)^2 + B(x+1) + C$

put $x = -1$, $-6 = C$ (1)

Coeff't x^2 $\quad 2 = A$ (1)

Coeff't x $\quad 3 = 2A + B$

$3 = 4 + B$

$B = -1$ (1)

e) $2ax + 3bx - 2b = 3ax^2$

$3ax^2 + 2ax + 3bx + 2b = 0$

$3a(x^2) + (2a+3b)x + 2b = 0$

$\Delta = (2a+3b)^2 - 4(3a)(+2b)$

$= 4a^2 + 12ab + 9b^2 - 24ab$

$= 4a^2 - 12ab + 9b^2$ (2)

$= (2a - 3b)^2$ a perfect square (1)

which is rational if a, b rational

f) i) $P = \frac{5}{8}$ (1)

ii) $P = \frac{5}{8} \times \frac{4}{7}$

$= \frac{20}{56}$

$= \frac{5}{14}$ (1)

iii) $P = \frac{3}{8} \times \frac{2}{7}$

$= \frac{6}{56}$

$= \frac{3}{28}$ (1)

iv) $P =$ SH or HS

$= \frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{4}{7}$

$= \frac{30}{56}$

$= \frac{15}{28}$ (1)

g) Solving. $mx - 6 = x^2 - 2x + 3$

$0 = x^2 - 2x - mx + 9$

$0 = x^2 - (2+m)x + 9$

If a tangent $\Delta = 0$

$b^2 - 4ac = 0$

$(2+m)^2 - 4(1)(9) = 0$ (1)

$4 + 4m + m^2 - 36 = 0$

$m^2 + 4m - 32 = 0$

$(m-4)(m+8) = 0$

$m = 4 \; , \; m = -8$ (1)

Question Four

a) $3x^2 - kx + 12 = 0 \qquad x = 2$

$12 - 2k + 12 = 0$ (1)

$-2k = -24$

$k = 12$ (1)

b) i) $3 \times 4 = 12$ ways (1) ii) $p = \frac{1}{3} \times \frac{2}{4}$

$= \frac{1}{6}$ (1)

c) $ar^5 = 160$ (A) $\quad ar^6 = 320$ (B) (1)

(B) ÷ (A) $\quad r = 2 \quad$ subst in (A) (1)

$a(2)^5 = 160$

$a = 5$

ie $5, 10$ (1)

d) i) $1 + 0.5 + 0.25 \ldots$

$a = 1 \; , \; r = \frac{1}{2} \; , \; n = 7$

$T_n = ar^{n-1}$

$T_7 = 1\left(\frac{1}{2}\right)^6$

$= \frac{1}{64}$ m (2)

ii) $S_n = \frac{a(1-r^n)}{(1-r)}$

$S_7 = \frac{1\left(1-\left(\frac{1}{2}\right)^7\right)}{\frac{1}{2}}$

$= 1\frac{63}{64}$ (2)

iii) $S_\infty = \frac{a}{r-1}$

$= \frac{1}{\frac{1}{2}}$

$= 2$m Yes since the sum to infinity is 2m. (2)

e) $T_5 = 3T_2$ and $S_6 = 36$

$a + 4d = 3a + 3d \qquad = 3[2a + 5d]$

$2a - d = 0$ (1) $\qquad 12 = 2a + 5d$ (2)

from (1) $d = 2a$ subs in (2) $\qquad 12 = 2a + 10a$

$a = 1$

$\therefore d = 2$

f) i) $\frac{1}{4}$ (1)

ii) Possibilities for (B, W) are

(3, 1) , (6, 2) , (9, 3) , (12, 4) , (15, 5)

ie $15 + 5 + 10 = 30$ marbles (1)

Question Five

a) $\sum_{n}^{12} (2n-1) = 1 + 3 + \ldots + 23$ (1)

$S_n = \frac{n}{2}(a + l)$

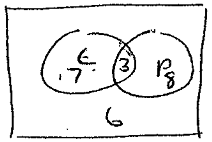
$= \frac{12}{2}(1 + 23)$

$= 144$ (1)

b) $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - (3 + \sqrt{7} + 3 - \sqrt{7}) + (3 + \sqrt{7})(3 - \sqrt{7}) = 0$ (1)

c)



$24 - 6 = 18$ in Chem & Physics

$\therefore 10 + 11 - 18 = 3$ do both

i) $P = \frac{3}{24}$

$= \frac{1}{8}$ (2)

ii) $P = \frac{8}{24}$

$= \frac{1}{3}$ (2)

d) i) 1% of \$80 000 = \$800 (1)

ii) $A_1 = \$80000 \times 1.01 - M$

$A_2 = A_1 \times 1.01 - m$

$= (\$80\,000 \times 1.01 - m) \times 1.01 - m$

$= \$80000 \times 1.01^2 - 1.01m - m$

$= \$8000 \times 1.01^2 - m(1 + 1.01)$ (2)

$A_n = \$80\,000 \times 1.01^n - m\underbrace{(1 + 1.01 + \dots 1.01^{n-1})}_{\text{geometric } a=1,\ r=1.01,\ n=n}$

$\therefore A_n = \$80\,000 \times 1.01^n - m\left(\frac{1.01^n - 1}{0.01}\right)$

iii) After 60 payment $A_n = 0$

$0 = \$80\,000 \times 1.01^{60} - m\left(\frac{1.01^{60} - 1}{0.01}\right)$ (2)

$m = \frac{\$80000 \times 1.01^{60} \times 0.01}{1.01^{60} - 1}$

$= \$1779.55$

$= \$1780$ to nearest dollar

iv) $A_{48} = 80\,000 \times 1.01^{48} - \left[\frac{m[(1.01^{48} - 1)]}{0.01}\right]$

$= 80\,000 \times 1.01^{48} - \left[\frac{1780[(1.01)^{48} - 1]}{0.01}\right]$

$= \$20\,001.00$ (2)

e)

$3x^2 + 2x + \sqrt{3x^2 + 2x - 1} = 1$

$3x^2 + 2x - 1 + \sqrt{3x^2 + 2x - 1} = 0$

put $m = \sqrt{3x^2 + 2x - 1}$

$m^2 + m = 0$

$m(m+1) = 0$

$m = 0$ or $m = -1$

$\sqrt{3x^2 + 2x - 1} = 0$ $\qquad \sqrt{3x^2 + 2x - 1} = -1$ (1)

$3x^2 + 2x - 1 = 0$ $\qquad$ has no soln

$(3x - 1)(x + 1) = 0$

$x = \frac{1}{3}$ or $x = -1$ (2)

OR. $3x^2 + 2x - 1 + \sqrt{3x^2 + 2x - 1} = 0$

Equate Rational, Irrational

$3x^2 + 2x - 1 = 0$ or $\sqrt{3x^2 + 2x - 1} = 0$ (1)

$(3x - 1)(x + 1) = 0$ $\qquad$ same soln.

$x = \frac{1}{3}$, $x = -1$ (2)