SYDNEY GIRLS HIGH SCHOOL
TRIAL HIGHER SCHOOL CERTIFICATE

LABOR OMNIA VINCIT

1999

MATHEMATICS

3 UNIT (Additional)
and
3/4 UNIT (Common)

Time allowed - 2 hours
(Plus 5 minutes' reading time)

DIRECTIONS TO CANDIDATES

NAME [redacted]

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Each question attempted should be started on a new sheet. Write on one side of the paper only.

This is a trial paper ONLY. It does not necessarily reflect the format or the contents of the 1999 HSC Examination Paper in this subject.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \; x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \; a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \; a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \; a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \; a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \; a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \; a \neq 0$$

$$\int \frac{1}{\sqrt{(a^2 - x^2)}} \, dx = \sin^{-1} \frac{x}{a}, \; a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{(x^2 - a^2)}} \, dx = \ln \left\{ x + \sqrt{(x^2 - a^2)} \right\}, \; |x| > |a|$$

$$\int \frac{1}{\sqrt{(x^2 + a^2)}} \, dx = \ln \left\{ x + \sqrt{(x^2 + a^2)} \right\}$$

NOTE : $\ln x = \log_e x; \quad x > 0$

QUESTION ONE

a) If $3\cot x = 4$, find the value of

$$\frac{6\sin x - 4\cos x}{\operatorname{cosec} x + \sec x} \qquad (\ x \text{ is acute})$$ [2]

b) Evaluate $\int_0^2 x e^{x^2}\, dx$ [2]

c) Differentiate $x^3 \sin^{-1} 4x$ [2]

d) Given $\log_a b = 0.3$ and $\log_a c = 0.4$, find $\log_a\left(\frac{b}{c}\right) + \log_a ac$ [2]

e) Find the exact value of $\cos 2x$ if $\sin x = \sqrt{3} - 1$ [2]

f) A cosine curve has an amplitude of 5 and a period of 3π. It has a minimum turning point at $(0, 5)$. Find it's equation. [2]

$2\pi \div \frac{2}{3}$

$\sin^{-1} 4x$

$(\sqrt{3}-1)(\sqrt{3}-1)$

$3 - 2\sqrt{3} + 1$

$4 - 2\sqrt{3}$

$\frac{1}{\sqrt{1-x^2}}$

$(4x)^2$

$c^2 - s^2$

QUESTION TWO

a) Write down the domain of the function

$$y = \frac{1}{x^2+5x+6}$$ [1]

b) The roots of $x^3+5x^2+8x+2=0$ are $\alpha, \beta,$ and γ [4]

i) Find $(\alpha+1)+(\beta+1)+(\gamma+1)$

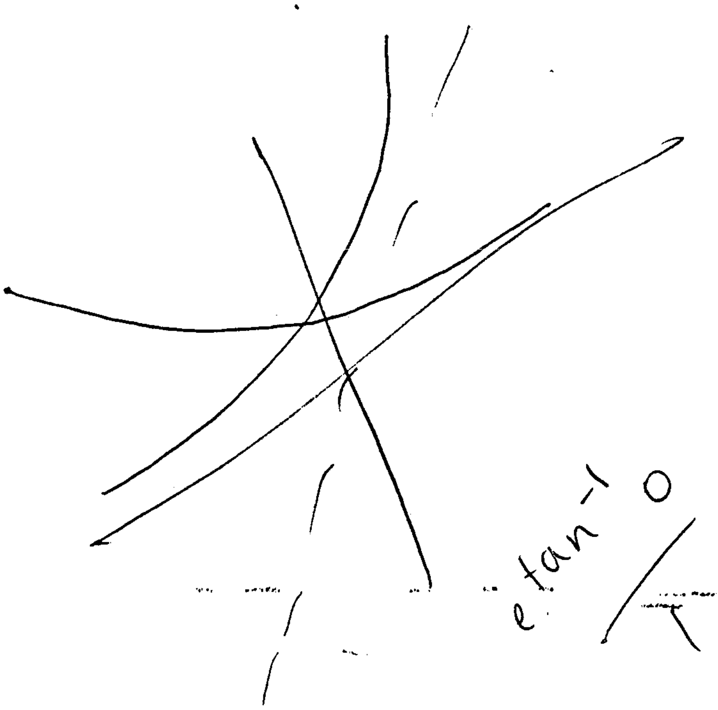
ii) Find $(\alpha+1)(\beta+1)(\gamma+1)$

c) The half life of a radioactive substance is 24 hrs. How long will it take for only 15% of the substance to remain. (Assume $M = M_o e^{-kt}$ and give your answer to the nearest hour) [2]

d) Find the equation of the tangent to the curve $y = e^{\tan^{-1}x}$ at the point where it cuts the y-axis. [2]

e) The area of the region below the curve $y = e^{-x}$ and above the x-axis, between x= 0.5 and x=1.5 is rotated about the x-axis. Find the volume of the solid generated. (Answer correct to 2 decimal places)

[3]



$\tan^{-1}x = \frac{1}{1+x^2}$

e^{5x}

$e^{\tan^{-1} 0}$

m

0.7m

0.15m

QUESTION THREE

a) If $\frac{dx}{dt} = 5(x-3)$ [3]

i) Show that $x = 3 + Ae^{5t}$ is a solution ,where A is a constant.

ii) Find A if $x = 20$ when $t = 0$.

b) [5]

The points $P(2ap, ap^2)$, $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

i) If PQ passes through $(4a, 0)$ show that $pq = 2(p+q)$

ii) Hence find the locus of M , the mid point of PQ.

c) Find the size of the acute angle between the lines [2]

$y = -x$ and $\sqrt{3}y = 2x$ (Answer to the nearest minute)

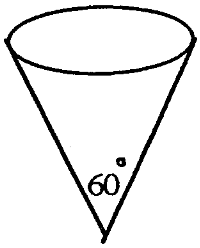
d) Differentiate $\log_e\left(\frac{3+x}{3-x}\right)$ [2]

$\frac{1}{1} - \frac{2}{\sqrt{3}}$ $\frac{\sqrt{3}-2}{\sqrt{3}}$

$\frac{-1}{1} - \frac{2}{\sqrt{3}}$ $\frac{-\sqrt{3}-2}{\sqrt{3}}$

QUESTION FOUR

a) A right circular cone of vertical angle 60° is being filled with liquid. The depth of liquid in the cone is increasiong at a rate of 4cm/ s. Find the rate of increase of the volume of the liquid in the cone when the depth is 9 cm. [3]

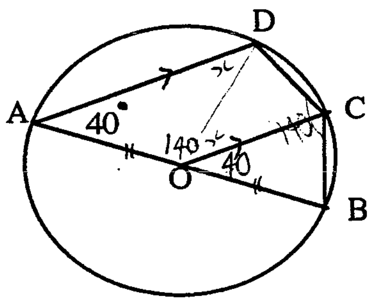


b) A projectile is fired at an angle of $\tan^{-1}(\frac{5}{12})$ to the horizontal with initial velocity 130 m/s. Using $g=10\ m/s^2$ [6]

i) Derive equations for the horizontal and vertical position of the projectile at time t.

ii) What is the horizontal range of this projectile?

c) AB is the diameter of the circle centre O. AD is parallel to OC, and angle BAD = 40°. Find the size of angle DCO, giving reasons. [3]



(figure not to scale)

QUESTION FIVE

a)

i) Find the remainder when $P(x) = x^3 - (k+1)x^2 + kx + 12$ is divided by $A(x) = x - 3$ [9]

ii) Find k if $P(x)$ is divisible by $A(x)$

iii) Find the zeros of $P(x)$, for this value of k

iv) Solve $P(x) > 0$

b) It is known that $\log_e x + \sin x = 0$ has one root close to $x = 0.5$. Use one application of Newton's method to obtain a better approximation of the root correct to 3 decimal places.

[3]

QUESTION SIX

a) Show that $7^n + 2$ is divisible by 3, for all positive integral n. [3]

b) Find the general solution of $\cos 2x = \sin x$

[3]

c) Find the area bounded by the curve $y = \dfrac{1}{\sqrt{25 - x^2}}$, the x axis and the ordinates at $x = -2$ and $x = 2$. (Answer correct to 2 decimal places)

[2]

d) Differentiate $\log_e(\sec x + \tan x)$ and hence find $\int_0^{\frac{\pi}{4}} \sec x \, dx$, in simplest exact form.

[4]

-4

tan

3

$)(x+1)$

$\log_e sx$

-4

cos 2x

$3\pi/2$

270

180

$(1-s^2)$

7+3+2

QUESTION SEVEN

a) [5]

A Particle moving on a horizontal line has a velocity of v m / s given by $v^2 = 64 - 4x^2 + 24x$

i) Prove that the motion is simple harmonic

ii) Find the centre of the motion

iii) Write down the period and amplitude of the motion

iv) Initially the particle is at the centre of the motion and moving to the left. Write down an expression for the displacement as a funtion of time.

b) [4]

i)Write the expression for $\sqrt{2}\cos\theta + \sin\theta$ in terms of t. (where $t = \tan\frac{\theta}{2}$)

ii) Hence or otherwise solve $\sqrt{2}\cos\theta + \sin\theta = 1$ for $0^\circ < \theta < 360^\circ$

c) Find $\int \frac{x\,dx}{(25+x^2)^{\frac{3}{2}}}$ using the substitution $x = 5\tan\theta$ [3]

$(1-\sqrt{2})(\sqrt{2}-1)$

$\frac{2(1-\sqrt{2})}{2(\sqrt{2}+1)} \times \frac{\sqrt{2}-1}{(\sqrt{2}-1)}$ $\sqrt{2}$

$2-1$

$4(\sqrt{2}-2+1-\sqrt{2})$

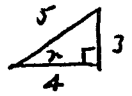
$\sqrt{2}-1-2+\sqrt{2}$

$-4(\sqrt{2}+1)(1-\sqrt{2})$

$\sqrt{2}$

Q1. a) $3\cot x = 4$

$\cot x = \frac{4}{3}$



$\dfrac{6\sin x - 4\cos x}{\operatorname{cosec} x + \sec x}$

$= \left[6\left(\frac{3}{5}\right) - 4\left(\frac{4}{5}\right)\right] \div \left[\frac{5}{3} + \frac{5}{4}\right]$

$= \dfrac{24}{175}$

b) $\dfrac{d}{dx} e^{x^2} = 2x e^{x^2}$

$\therefore \int 2x e^{x^2} dx = e^{x^2} + c,$

$\therefore \int_0^2 x e^{x^2} dx = \frac{1}{2}\left[e^{x^2}\right]_0^2$

$= \frac{1}{2}(e^4 - 1)$

c) $y = x^3 \sin^{-1} 4x$

put $u = x^3$, $\dfrac{du}{dx} = 3x^2$

$v = \sin^{-1} 4x$, $\dfrac{dv}{dx} = \dfrac{4}{\sqrt{1-16x^2}}$

$\dfrac{dy}{dx} = 3x^2 \sin^{-1} 4x + \dfrac{4x^3}{\sqrt{1-16x^2}}$

or $\dfrac{x^3}{\sqrt{\frac{1}{16} - x^2}} + 3x^2 \sin^{-1} 4x$

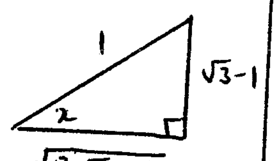
d) $\log_a\left(\frac{b}{c}\right) + \log_a ac$

$= (\log_a b - \log_a c) + (\log_a a + \log_a c)$

$= (0.3 - 0.4) + (1 + 0.4)$

$= 1.3$

e)



$\sin x = \sqrt{3} - 1$

$= \dfrac{\sqrt{3}-1}{1}$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 2\sqrt{3} - 3 - (\sqrt{3}-1)^2$

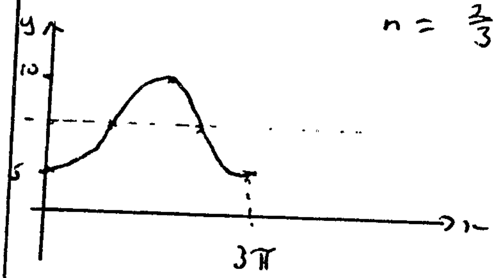
$= 2\sqrt{3} - 3 - (4 - 2\sqrt{3})$

$= 2\sqrt{3} - 3 - 4 + 2\sqrt{3}$

$4\sqrt{3} - 7$

f) $a = 5$, $\dfrac{2\pi}{n} = 3\pi$

$n = \frac{2}{3}$



$y = 10 - 5\cos\dfrac{2x}{3}$

or $y = 10 + 5\cos\left(\dfrac{2x}{3} - \pi\right)$

or $y = 5\sin\left(\dfrac{2x}{3} - \dfrac{\pi}{2}\right) + 10$

Q2 a) all real except $x^2 + 5x + 6 =$

ie all real except $x = -2,$

b)

$P(x) = x^3 + 5x^2 + 8x + 2$

$\alpha + \beta + \gamma = \dfrac{-b}{a}$

$= -5$

$\alpha\beta + \alpha\gamma + \beta\gamma = \dfrac{c}{a}$

$= 8$

$\alpha\beta\gamma = \dfrac{-d}{a}$

$= -2$

i) $(\alpha+1) + (\beta+1) + (\gamma+1)$

$= \alpha + \beta + \gamma + 3$

$= -5 + 3$

$= -2$

ii) $(\alpha+1)(\beta+1)(\gamma+1)$

$= (\alpha+1)(\beta\gamma + \beta + \gamma + 1)$

$= (\alpha\beta\gamma) + (\alpha\beta + \alpha\gamma + \gamma\beta) + (\alpha + \beta + \gamma) +$

$= -2 + 8 - 5 + 1$

$= 2$

c) $M = M_0 e^{-kt}$

when $t = 24$, $M = \dfrac{M_0}{2}$

ie $\dfrac{M_0}{2} = M_0 e^{-24k}$

$0.5 = e^{-24k}$

$-24k = \log_e 0.5$

$k = \dfrac{\log_e 0.5}{-24}$ $[\doteqdot 0.029$

If 15% remains

$m = 0.15\,M_0$

ie $0.15 = e^{-kt}$

$\log_e(0.15) = \log_e e^{-kt}$

$-kt = \log(0.15)$

$t = \dfrac{\log(0.15)}{-k}$

$= 65\text{ hrs } 41'\ 14''$

d) $y = e^{\tan^{-1}x}$

On the y axis $x=0$

$y = e^{\tan^{-1}(0)}$

$= e^0$

$= 1$ ie $(0,1)$

$\dfrac{dy}{dx} = \dfrac{1}{1+x^2} \times e^{\tan^{-1}x}$

$= \dfrac{e^{\tan^{-1}x}}{1+x^2}$

when $x = 0$

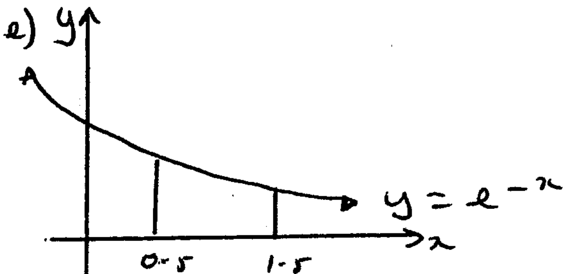
$m = \dfrac{e^{\tan^{-1}(0)}}{1+0}$

$= 1$

$y - y_1 = m(x - x_1)$

$y - 1 = 1(x - 0)$

$y = x + 1$

e) 

$V = \pi \int_{0.5}^{1.5} (e^{-x})^2\, dx$

$= \pi \int_{0.5}^{1.5} e^{-2x}\, dx$

$\pi\left[-\frac{1}{2} e^{-2x}\right]_{0.5}^{1.5}$

$-\frac{\pi}{2}\left[e^{-3} - e^{-1}\right]$

$= \frac{\pi}{2}\left[\frac{1}{e} - \frac{1}{e^3}\right]$ units^3

$= 0.50$ (2d.p.)

Q3

a) i) $x = 3 + Ae^{5t}$

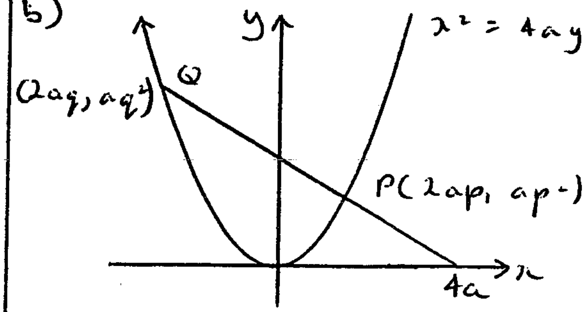
$\dfrac{dx}{dt} = 5Ae^{5t}$ ②

$= 5(x-3)$ [since $Ae^{5t} = x -$

ii) $x = 20$ when $t = 0$

$20 = 3 + A$

$A = 17$

b) 

i) Gradient $PQ = \dfrac{ap^2 - aq^2}{2ap - 2aq}$

$= \dfrac{p+q}{2}$

Eqn PQ

$y - y_1 = m(x - x_1)$

$y - ap^2 = \dfrac{p+q}{2}(x - 2ap)$

$2y - 2ap^2 = (p+q)(x - 2ap)$

subst $x = 4a$, $y = 0$

$-2ap^2 = (p+q)(4a - 2ap)$

$-\cancel{2ap^2} = 4ap - \cancel{2ap^2} + 4aq - 2$

$2apq = 4ap + 4aq$

$pq = 2p + 2q$

$= 2(p+q)$ ②

3

ii) <u>Co-ords of M</u>

$x = \dfrac{2ap + 2aq}{2}$, $y = \dfrac{ap^2 + aq^2}{2}$

$x = a(p+q)$ $\qquad y = \frac{a}{2}(p^2+q^2)$ B

$\therefore p+q = \frac{x}{a}$ A

Now $y = \frac{a}{2}[p^2+q^2]$

$= \frac{a}{2}[(p+q)^2 - 2pq]$

$= \frac{a}{2}[(\frac{x}{a})^2 - 2pq]$ from A

$= \frac{a}{2}[(\frac{x}{a})^2 - (2)(2)(p+q)]$ from part i)

$= \frac{a}{2}[(\frac{x}{a})^2 - 4(p+q)]$

$= \frac{a}{2}[\frac{x^2}{a^2} - 4\frac{x}{a}]$ from A

or $2ay = x^2 - 4ax$ 3

or $(x-2a)^2 = 2a(y+2a)$

c) $y = -x$, $y = \frac{2}{\sqrt{3}}x$

$\therefore m_1 = -1$, $m_2 = \frac{2}{\sqrt{3}}$

$\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$

$= (-1 - \frac{2}{\sqrt{3}}) \div (1 - \frac{2}{\sqrt{3}})$

$\theta = 85° 54'$ 2

d) $y = \log_e\left(\frac{3+x}{3-x}\right)$

$= \log_e(3+x) - \log_e(3-x)$

$\frac{dy}{dx} = \frac{1}{3+x} - \frac{-1}{3-x}$

$= \frac{1}{3+x} + \frac{1}{3-x}$

$= \frac{6}{(3-x)(3+x)}$

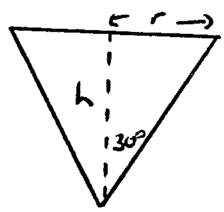
Q4 let depth be h

then $\frac{dh}{dt} = 4\ cm\,s^{-1}$

Find $\frac{dV}{dt}$ when $h = 9$

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dh} = \frac{1}{3}\pi r^2$



$\frac{r}{h} = \tan 30°$

$r = \frac{h}{\sqrt{3}}$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(\frac{h}{\sqrt{3}})^2 h$

$= \frac{\pi h^3}{9}$

$\frac{dV}{dh} = \frac{\pi h^2}{3}$

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

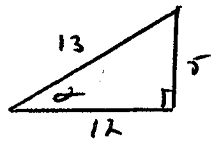
$$= \underline{\pi h^2} \times 4$$

when $h = 9$

$$\frac{dV}{dt} = \frac{\pi(81)(4)}{3}$$

$$= 108\pi \text{ cm}^3 \text{ s}^{-1}$$

b) i)

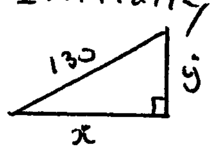


$\tan\alpha = \frac{5}{12}$

$\sin\alpha = \frac{5}{13}$

$\cos\alpha = \frac{12}{13}$

Initially



$t = 0, \ x = 0, y = 0$

$\dot{x} = 130 \cos\alpha$

$= 130 \times \frac{12}{13}$

$= 120$

$\dot{y} = 130 \sin\alpha$

$= 130 \times \frac{5}{13}$

$= 50$

Horizontal Motion

$\ddot{x} = 0$

$\dot{x} = c_1$

when $t = 0 \quad \dot{x} = 120$

$\dot{x} = 120$

$x = 120t + c_2$

when $t = 0, \ x = 0$

$x = 120t$

Vertical Motion

$\ddot{y} = -10$

$\dot{y} = -10t + c_1$

when $t = 0, \ \dot{y} = 50$

$\dot{y} = -10t + 50$

$y = -5t^2 + 50t + c$

when $t = 0, \ y = 0$

$y = -5t^2 + 50t$

ii) at max range $y = 0$

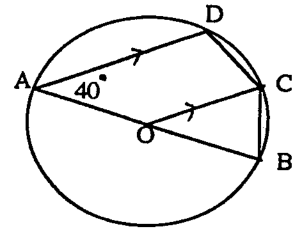
ie $-5t^2 + 50t = 0$

$-5t(t - 10), \ t = 10$

$x_{max} = 120(10)$

$= 1200$ metres

c)



$\angle AOC + 40° = 180°$ (co-int $\angle$'s AD

$\angle AOC = 140°$

Major $\angle AOC = 360° - 140°$

$= 220°$ ($\angle$'s at a p

$\angle ADC = 220 \div 2$ ($\angle$ at cent twice $\angle$ a circ)

$= 110°$

$\angle DCO = 180° - 110°$ (co-int $\angle$'s AD $\parallel$ OC)

$= 70°$

Q5

a) i) $R = P(3)$

$= 3^3 - (k+1)9 + 3k + 12$

$= 30 - 6k$

ii) If divisible $P(3) = 0$

$0 = 30 - 6k$

$k = 5$

iii) $P(x) = x^3 - 6x^2 + 5x + 12$

$$\begin{array}{r l}
 & x^2 - 3x - 4 \\
x - 3 & \overline{)\,x^3 - 6x^2 + 5x + 12} \\
 & \underline{x^3 - 3x^2} \\
 & \quad -3x^2 + 5x \\
 & \quad \underline{-3x^2 + 9x} \\
 & \qquad -4x + 12 \\
 & \qquad \underline{-4x + 12} \\
 & \qquad\qquad 0
\end{array}$$

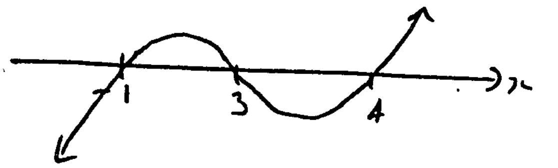
$\therefore P(x) = (x-3)(x^2 - 3x - 4)$

$= (x-3)(x-4)(x+1)$

zero's at $x = 3, \ x = 4$

$x = -1$

a)



$P(x) > 0$ for
$-1 < x < 3$ and $x > 4$

b) $y = \ln x + \sin x$

$$a_2 = a_1 - \frac{f(a_1)}{f'(a_1)}$$

$$y' = \frac{1}{x} + \cos x$$

$$a_2 = 0.5 - \frac{(\ln 0.5 + \sin 0.5)}{\left(\frac{1}{0.5} + \cos 0.5\right)}$$

$= 0.574$ (to 3 dp)

Question 6.

a) Step 1. Verify for $n=1$
ie $7^1 + 2 = 9$ which is divisible by 3

Step 2. a) Assume true for $n=k$
ie $7^k + 2 = 3P$ (P integer)

b) Prove true for $n = k+1$

$$7^{k+1} + 2 = 7^k.7 + 2$$
$$= 7(3P-2) + 2 \quad \text{(from assp)}$$
$$= 21P - 14 + 2$$
$$= 3(7P - 4)$$

since P is an integer, $(7P-4)$ is an integer and $7^{k+1} + 2$ is divisible by 3 if the assumption is ~~true~~.

ie true for $n = k+1$ if true for $n = k$.

Step 3 Since statement is true for $n=1$, it is true for $n=2$. Since true for $n=2$, then true for $n=3$, and so on for all positive integers (3)

b) $\cos 2x = \sin x$

$$1 - 2\sin^2 x = \sin x$$
$$\therefore 2\sin^2 x + \sin x - 1 = 0$$
$$(2\sin x - 1)(\sin x + 1) = 0$$
$$\sin x = \tfrac{1}{2} \qquad \sin x = -1$$

$\therefore x = n\pi + (-1)^n \sin^{-1}\frac{1}{2}$ or
$n\pi + (-1)^n \sin^{-1}(-1)$

ie $x = n\pi + (-1)^n\left(\frac{\pi}{6}\right)$ or
$n\pi + (-1)^n\left(-\frac{\pi}{2}\right)$ (3)

c) $y > 0$ for all x
(ie does not cut x axis)

$$\therefore A = \int_{-2}^{2} \frac{dx}{\sqrt{25 - x^2}}$$

$$= 2\int_0^2 \frac{dx}{\sqrt{25-x^2}} \quad \text{since fn. is even}$$

$$= 2\left[\sin^{-1}\frac{x}{5}\right]_0^2 = 2\left(\sin^{-1}\frac{2}{5} - \right.$$

Area $\doteqdot 0.82\ u^2$ (2)

d) $y = \log(\sec x + \tan x)$

let $u = \sec x + \tan x$
$= (\cos x)^{-1} + \tan x$

$$\frac{du}{dx} = -(\cos x)^{-2}. -\sin x + \sec^2 x$$
$$= \frac{\sin x}{\cos^2 x} + \sec^2 x$$
$$= \tan x . \sec x + \sec^2 x$$

(see bottom of next page)

(a) (i) $v^2 = 64 - 4x^2 + 24x$.

For SHM $\ddot{x} = -n^2 x$ or $\ddot{X} = -n^2 X$

Now $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = a = \ddot{x}$

$\therefore \frac{1}{2}v^2 = 32 - 2x^2 + 12x$.

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -4x + 12$

i.e $\ddot{x} = -4(x-3)$ — is of for $\ddot{X} = -n^2 X$

$\therefore$ motion is SHM.

(ii) Centre of motion $X = 0$ i.e. $x - 3 = 0$

$x = 3$.

or when $v = 0$

$4x^2 - 24x - 64 = 0$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$\therefore x = -2$ to $x = 8$

Centre then $x =$

(iii) Period $T = \frac{2\pi}{n}$ where $n = 2$

$\therefore T = \frac{2\pi}{2} = \pi$ Period $= \pi$ secs

Amplitude: is from centre to end

i.e from 3 to 8

$\therefore$ amplitude $= 5$ m.

or complete $v^2 = n^2(a^2 - x^2)$

$v^2 = n^2(a^2 - x^2)$

$v^2 = 4(16 + 6x - x^2$

$v = 4(25 - (9 - 6x$

$v = 4(25 - (x-3)^2$

$\therefore a = 5$

(iv)

$x = -5\sin 2t + 3$

$x = 5\sin(-2t) + 3$

$x = 5\cos\left(2t - \frac{3\pi}{2}\right) + 3$

Q6 (cont'd)

$\frac{dy}{dx} = \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$

$= \sec x$.

$\therefore \int_0^{\pi/4} \sec x \, dx = \left[\ln(\sec x + \tan x)\right]$

$= \ln\left(\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right) - \ln(\sec 0 + \text{ta}$

$= \ln(\sqrt{2} + 1) - \ln(1 + 0)$

$= \ln(\sqrt{2} + 1)$

4

$$\left(\frac{1-t^2}{1+t^2}\right)+\frac{2t}{1+t^2}$$

$$\frac{\sqrt{2}(1-t^2)+2t}{1+t^2}$$

$$\sin\theta = \frac{2t}{1+t^2}$$

(ii) Now $\sqrt{2}\cos\theta + \sin\theta = 1$.

$$\therefore \frac{\sqrt{2}(1-t^2)+2t}{1+t^2} = 1$$

$$\sqrt{2}(1-t^2)+2t = 1+t^2$$

$$\sqrt{2}-\sqrt{2}t^2+2t = 1+t^2$$

$$t^2(1+\sqrt{2}) - 2t + (1-\sqrt{2}) = 0.$$

$$\therefore t = \frac{2 \pm \sqrt{4-4(1+\sqrt{2})(1-\sqrt{2})}}{2(1+\sqrt{2})}$$

$$t = \frac{2 \pm \sqrt{4-4(1-2)}}{2(1+\sqrt{2})}$$

$$\therefore t = \frac{2(1+\sqrt{2})}{2(1+\sqrt{2})} \quad \text{or} \quad t = \frac{1-\sqrt{2}}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$$

$$t = 1 \qquad\qquad t = \frac{2\sqrt{2}-3}{1}$$

$0° < \theta < 360°$

when $t = 1$ then $\tan\frac{\theta}{2} = 1$ i.e. $\frac{\theta}{2} = \frac{\pi}{4}$

$* \ \theta = \frac{\pi}{2}$

when $t = \frac{2\sqrt{2}-3}{1}$

$\tan\frac{\theta}{2} \doteqdot -0.1715$

$\theta = -19°28'$

But $0 < \theta < 360°$.

$\therefore \theta = 360 - 19°28'$

$$\begin{cases} \theta = 340°32' \\ \theta = \frac{\pi}{2} \end{cases}$$

(c)

$$\int \frac{x.dx}{(25+x^2)^{\frac{3}{2}}}$$

$x = 5\tan\theta$.

$dx = 5\sec^2\theta.d\theta$.

$\therefore x.dx = 25\tan\theta.\sec^2\theta.d\theta$.

$25+x^2 = 25+25\tan^2\theta$

$= 25\sec^2\theta$.

$(25+x^2)^{\frac{3}{2}} = 125\sec^3\theta$.

$$I = \int \frac{25\tan\theta.\sec^2\theta.d\theta.}{125\sec^3\theta.}$$

$$I = \frac{1}{5}\int \frac{\tan\theta}{\sec\theta}.d\theta$$

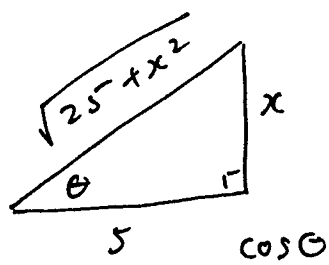
$$I = \frac{1}{5}\int \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}.d\theta.$$

$$I = \frac{1}{5}\int \sin\theta.d\theta$$

$$I = -\frac{1}{5}\cos\theta + c$$

$$\therefore I = \frac{-1}{\sqrt{25+x^2}} + c$$

But



$$\cos\theta = \frac{5}{\sqrt{25+x^2}}$$