Sydney Girls High School

Year 9

Yearly Examination 2008

Mathematics

Time allowed - 75 minutes

Instructions

- Attempt all five questions.
- Questions are of equal value.
- All necessary working should be shown in every question.
- Marks may be deducted for careless or badly arranged work.
- Start each question on a new page. Write on one side of the paper only.
- Diagrams are not to scale.

QUESTION ONE (20 marks) Marks

a) i) Two dice are thrown once. Write all the possible outcomes. (2)

ii) What is the probability of tossing a pair of numbers which are the same? (1)

b) Simplify $\frac{30}{5a^4}$ (1)

c) Write with the positive index $\frac{1}{4p^{-5}}$ (1)

d) Expand and simplify $5(1-2x)-3(x-1)$ (2)

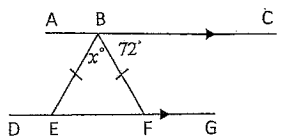
e) For the points $A(3,-1)$ and $B(-5,0)$. Find the distance of AB in surd form. (2)

f) Evaluate $3^3 \times 3^2 \div 3^7$ in exact form. (1)

g) Simplify $\frac{x}{4}-\frac{2x+3}{6}$ (2)

h) Factorise fully $12n^3-8n^2$ (1)

i) Find the value of x (3)



j) Expand and simplify $(2\sqrt{5}+\sqrt{3})(\sqrt{5}-2\sqrt{3})$ (2)

k) Rationalise the denominator $\frac{5-\sqrt{3}}{3\sqrt{5}}$ (2)

QUESTION TWO (20 Marks) Marks

a) Solve : i) $7x+6=13-3x$ (2)

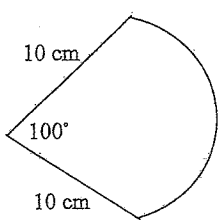
ii) $3(x-2)=7-2(3-x)$ (2)

iii) $\frac{1-2x}{5} \leq \frac{3(2-x)}{3}$ (4)

b) If $x=\frac{-b+\sqrt{b^2-4ac}}{2a}$, find x (in surd form), (2)

when $a=4, b=3$ and $c=-4$.

c) Calculate the exact perimeter of the following sector below:



(2)

d) Solve the following equation simultaneously

$2x-y+2=0$
$3x-y=0$ (3)

e) A cube has a surface area of $1350cm^2$. Find its volume? (3)

f) Find the volume of a cylinder with radius 15 *cm* and height 23 *cm* (correct to two decimal places). (2)

QUESTION THREE (20 marks) Marks

a) Use the table of tax rates to calculate Bart's tax if he earns $46852 per year and claims deductions of $72 per week. (4)

Taxable Income	Tax on this income
$1 - $6000	**Nil**
$6001 - $20000	**17 cents for each $1 over $6000**
$20001 - $50000	**$2380 + 30 cents for each $1 over $20000**
$50001 - $60000	**$11380 + 42 cents for each $1 over $50000**
$60001 and over	**$15580 + 47 cents for each $1 over $60000**

b) Merv's annual salary is $29744. Merv takes his annual holidays and is paid 4 weeks normal pay plus a holiday leave loading of 17.5% on top of his pay. Calculate Merv's total holiday pay. (3)

c) Calculate the GST included in a retail price of $250 (1)

d) A store is offering customers a choice: '10% discount or $10 off all items'. For which items would a discount of $10 be a better deal than a discount of 10%. (1)

e) A retailer purchased an electric fan for $30 and sold it for $42. Calculate the profit as a percentage of the cost price. (2)

f) For the interval joining the points $A(2, 7)$ and $B(-5, 21)$ find: (9)

i) the midpoint of AB
ii) the length of AB
iii) the gradient of AB
iv) the equation of the line which passes through the points A and B
v) the equation of the line which passes through the point $C(3, 4)$ and is perpendicular to AB expressing your answer in general form.

QUESTION FOUR (20 marks) — Marks

a) The frequency distribution table below shows the results of a quiz taken by shoppers at a supermarket.

Score	Frequency	Cumulative Frequency
6	7	
7	4	
8	3	
9	1	
10	2	

i) Copy the table onto your answer sheet and complete the cumulative frequency column. (1)

ii) Calculate the number of shoppers who participated. (1)

iii) Calculate the mean score. Give your answer correct to 2 decimal places. (2)

iv) Calculate the median score. (1)

v) Calculate the range. (1)

vi) Construct a cumulative frequency histogram and polygon. (3)

b) A cricketer played 12 innings at an average of 38.5 runs. She then scored 10 and 70 runs in the next two innings. Find the cricketer's new average. Give your answer correct to 2 decimal places. (3)

c) Factorise :

i) $3ax+3ay+2bx+2by$ (2)

ii) $8x^2+14x-15$ (2)

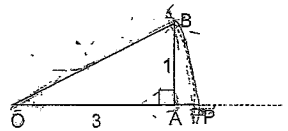
d) Express in simplest form :

$$\frac{9-x^2}{x^2-4x-21} \div \frac{1}{49x-x^3}$$ (4)

QUESTION FIVE (20 Marks) — Marks

a) Make a the subject of $a^{-1}+b^{-1}=c$ (3)

b) In the diagram below OA = 3 units, AB = 1 unit and angle BAO = 90°. An arc centre O is drawn for B to intersect OA extended, at the point P (2)



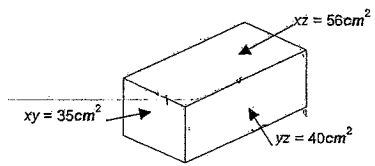
i) Find the irrational number represented by the length OP.

ii) Write down the exact length of AP

c) Graph the region on a number plane where the following hold simultaneously (4)

$$\left.\begin{array}{l} x>y \\ 2x+3y-5\le 0 \end{array}\right\}$$

d) A rectangular prism has three edges x, y and z. (2)
If the area of the three given faces are as follows:
$xy=35cm^2$, $xz=56cm^2$, $yz=40cm^2$

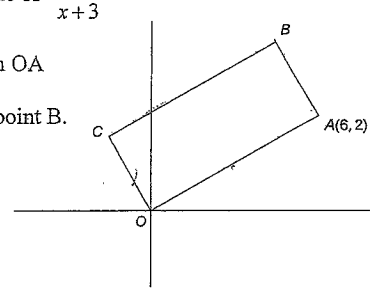


Find the volume of the rectangular prism

e) A 20 cm length of wire is cut into two pieces, the smaller piece having a length of x cm. The longer piece is bent into the shape of a rectangle with a width of 7 cm. Find an expression for the length of the rectangle. (4)

f) If $\frac{1}{x+2}=3$, find the value of $\frac{1}{x+3}$ (3)

g) OABC is a rectangle with OA twice the length of OC. Find the co-ordinates of point B. (2)



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1) a)

i)

11	21	31	41	51	61
12	22	32	42	52	62
13	23	33	43	53	63
14	24	34	44	54	64
15	25	35	45	55	65
16	26	36	46	56	66

(2)

ii) $P(\text{same}) = \frac{6}{36}$

$= \frac{1}{6}$ (1)

b) $\frac{6}{a^4}$ (1)

c) $\frac{p^5}{4}$ (1)

d) $5 - 10x - 3x + 3$

$= 8 - 13x$ (2)

e) $AB = \sqrt{(-5-3)^2 + (0+1)^2}$

$= \sqrt{64+1}$

$= \sqrt{65}$ (2)

f) $3^5 \div 3^7$

$= 3^{-2}$

$= \frac{1}{9}$ (1)

g) $\frac{6x - 4(2x+3)}{24}$

$\frac{6x - 8x - 12}{24}$

$= \frac{2x - 12}{24}$

$= \frac{-x - 6}{12}$ (2)

h) $4n^2(3n-2)$ (1)

i) $2y + x = 180$

$y + x + 72 = 180$

$2y + x = y + x + 72$

$y = 72$

$x = 36$ (3)

j) $(2\sqrt{5} + \sqrt{3})(\sqrt{5} - 2\sqrt{3})$

$10 - 4\sqrt{15} + \sqrt{15} - 6$

$4 - 3\sqrt{15}$ (2)

k) $\frac{5 - \sqrt{3}}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{5\sqrt{5} - \sqrt{15}}{15}$ (2)

Question two

a) i) $7x + 6 = 13 - 3x$

$10x = 7$

$x = \frac{7}{10}$ (2)

ii) $3x - 6 = 7 - 6 + 2x$

$x = 7$ (2)

iii) $3(1 - 2x) \leq 15(2 - x)$

$3 - 6x \leq 30 - 15x$

$9x \leq 27$

$x \leq 3$ (4)

b) $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-3 + \sqrt{9 - 4(4)(-4)}}{8}$ (2)

$= \frac{-3 + \sqrt{73}}{8}$

c) Length of arc $= 2\pi r \times \frac{\theta}{360}$

$= 2\pi \times 10 \times \frac{100}{360}$

$= 20\pi \times \frac{5}{18}$

$= 5\frac{5}{9}\pi$ or $\frac{50}{9}\pi$ (2)

$\therefore$ Perimeter $= 20 + \frac{50}{9}\pi$ cm

d) $2x - y - 2 = 0$
$3x - y = 0$] subtract

$x - 2 = 0$

$x = 2$

$\therefore$ Point of intersection $(2, 6)$

$3(2) - y = 0$

$6 - y = 0$

$\therefore y = 6$ (3)

e) $SA = 6x^2$

$6x^2 = 1350$

$x^2 = 225$

$\therefore x = \sqrt{225}$

$x = 15$ cm (3)

$\therefore$ Volume of cube $= x^3$

$= 15^3$

$= 3375\ cm^3$

f) $V = \pi r^2 h$

$= \pi (15)^2 \times 23$

$= 5175\pi\ cm^3$

$\doteq 16257.74\ cm^3$ (2)

Three.

a) Taxable income = $46852 - 72 × 52.
= $46852 - $3744
= $43,108.

Tax payable = $2380 + (43 108 - 20 000) × 0·3
= $ 9312·40. (4)

b) Holiday pay = 29744 × 4/52 × 1·175
= $2688·40. (3)

c) $250 ÷ 11 = 22·73. (1)

d) Items worth less than $100. (1)

e) profit = $12
cost price = $30

% profit = 12/30 × 100
= 40% (2)

f) A(2, 7) B(-5, 21)

i) midpoint : $\left(\frac{2+-5}{2}, \frac{7+21}{2}\right)$ (1)

: $\left(\frac{-3}{2}, 14\right)$

ii)

$AB = \sqrt{(2--5)^2 + (7-21)^2}$

$= \sqrt{49 + 196}$

$= \sqrt{245}$

= 15·65 (2)

iii) $m_{AB} = \frac{21-7}{-5-2}$

$= \frac{14}{-7}$

= -2 (1)

iv) $y - y_1 = m(x - x_1)$

$y - 7 = -2(x - 2)$

$y - 7 = -2x + 4$

$y = -2x + 11$ (2)

1

v) $m_2 \times m_{AB} = -1$

$m_2 \times -2 = -1$

$m_2 = 1/2$ (3)

$y - 4 = 1/2(x - 3)$

$y - 4 = 1/2 x - 3/2$

$y = 1/2 x + 2·5$

$x - 2y + 5 = 0$

Question Four

(a) (i).

Score	Frequency	Cumulative Frequency
6	7	7
7	4	11
8	3	14
9	1	15
10	2	17

(1)

(ii) 17 shoppers (1)

(iii) $\bar{x} = \frac{\sum fx}{\sum f}$

$= \frac{123}{17}$

$= 7.24$ (2)

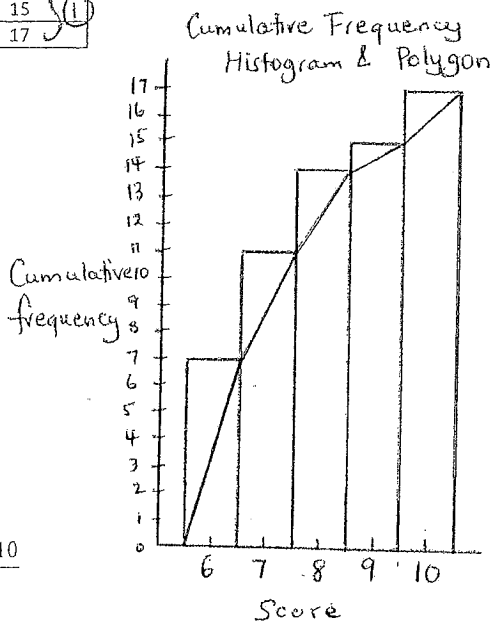
(iv) Median = 9th score

$= 7$ (1)

(v). Range $= 10 - 6$

$= 4$ (1)

(vi). See diagram at the right.



(3)

(b). New average $= \frac{(38.5 \times 12) + 70 + 10}{12 + 2}$

$= \frac{462 + 80}{14}$

$= 38.71$ (2 d p) (3)

(c) (i). $3ax + 3ay + 2bx + 2by = 3a(x + y) + 2b(x + y)$

$= (x + y)(3a + 2b)$ (2)

(ii). $8x^2 + 14x - 15 = 8x^2 + 20x - 6x - 15$

$= 4x(2x + 5) - 3(2x + 5)$

$= (2x + 5)(4x - 3)$ (2)

(d). $\frac{9 - x^2}{x^2 - 4x - 21} \div \frac{1}{49x - x^3} = \frac{(3 - x)(3 + x)}{(x - 7)(x + 3)} \times \frac{49x - x^3}{1}$

$= \frac{3 - x}{x - 7} \times \frac{-x(x^2 - 49)}{1}$

$= \frac{3 - x}{x - 7} \times \frac{-x(x + 7)(x - 7)}{1}$

$= -x(3 - x)(x + 7)$ OR $x(x - 3)(x + 7)$ (4)

20

Q5

$a^{-1} + b^{-1} = c$

$\frac{1}{a} + \frac{1}{b} = c$

$\frac{a + b}{ab} = c$ (3)

$a + b = abc$

$abc - a = b$

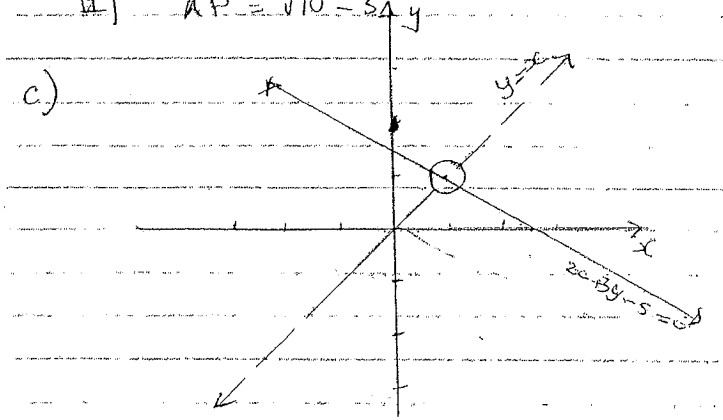
$a(bc - 1) = b$

$\therefore a = \frac{b}{bc - 1}$

b) I) $OP = \sqrt{10}$ (2)

II) $AP = \sqrt{10} - 3$

c)



(4)

d) Any way will give $V = 280\ cm^3$ (2)

e) $20 - x$

$2(7 + y) = 20 - x$

$14 + 2y = 20 - x$

$y = 6 - \frac{x}{2}$ $(x = 4, y = 4)$ (4)

F) $\frac{1}{x+2} = 3$

$1 = 3x + 6$

$3x = -5$

$x = \frac{-5}{3}$

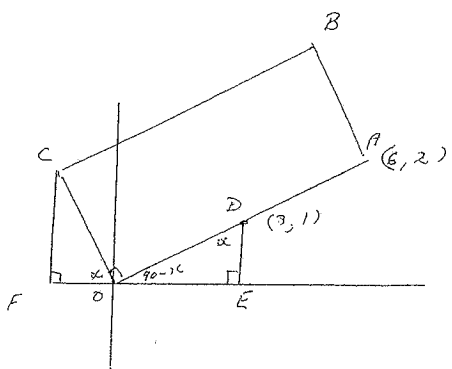
$\therefore \frac{1}{x+3}$ (3)

$= \frac{1}{\frac{-5}{3} + 3}$

$= \frac{1}{\frac{4}{3}}$

$= \frac{3}{4}$

g)



$OA = 2OC$

$\therefore \frac{1}{2}$ of $OA = OC$, $\therefore OD = OC$

$\triangle ODF \equiv \triangle CDF$

$\therefore OC = CF = 3$

$DE = OF = 6$

$\therefore C$ is $(-1, 3)$

$OABC$ is a rectangle

O to A move 6 to the right & 2 up

$\therefore C$ to B is the same

$\therefore B$ is $(-1+6, 3+2) = (5, 5)$ (2)