Sydney Girls' High School

LABOR OMNIA VINCIT

2010

MATHEMATICS

YEAR 11

YEARLY EXAMINATION

Time Allowed: 90 minutes

TOPICS: Ch1 to ch11 Jones and Couchman

<u>Directions to Candidates</u>

- There are four (4) questions.
- Attempt ALL questions.
- Questions are of equal value(20 marks).
- Start each question on a new page.
- Write on one side of the paper only.
- Show all necessary working. Marks will be deducted for careless or badly arranged work.
- Diagrams are **NOT** drawn to scale.
- Board-approved calculators may be used.

Total: 80 marks

Student Name

Class

Question 1

a) Find the value of $\frac{3.6+4.25}{\sqrt{5.1^2+2.3^2}}$ correct to 2 sig.figs.. 2

b) Simplify $\sqrt{12}-3\sqrt{3}$ 2

c) Factorise x^3-27y^3 2

d) Solve $\frac{x}{3}+\frac{x}{4}=\frac{x+1}{6}$ 3

e) Find the exact value of $\cos 150°$ 2

f) Solve $3-x>9$ 2

g) Solve $|2x+3|=13$ 3

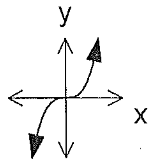
h) Expand and simplify $(t-1)^2-2t$ 2

i) If $\tan\theta=1$ and $0°\le\theta\le 360°$, find θ 2

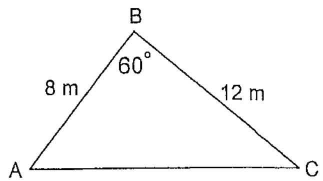
Question 2

a) Rationalise the denominator and simplify $\frac{\sqrt{2}}{\sqrt{5}-1}$ 2

b) Is the Function below odd, even or neither? Give reasons 2



c) Find the exact area of ΔABC 2



d) Differentiate the following, simplify where possible

i) $5x^2 + 4x$ 2

ii) $(x + 2)(x^2 + 1)$ 3

e) Evaluate the limit 2

$$\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$$

f) i) Simplify $\sin\theta \cos(90° - \theta) + \cos\theta \sin(90° - \theta)$ 2

ii) Prove that $(cosec^2\theta - 1)tan^2\theta = 1$ 2

g) Differentiate $f(x) = x^2 + 2x$ from first principles using 3

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Question 3

a) For the points $A(-2,-5)$, $B(6,7)$ and $C(0,5)$.

i) Draw a number plane and plot the points A, B and C. 1

ii) Show that the gradient of AB is $\frac{3}{2}$. 1

iii) Find the equation of AB(in general form). 2

iv) Find the perpendicular distance of the point C from the line AB. 2

v) Find the midpoint of AB. 1

vi) Find the distance AB. 1

vii) Find the exact area of ΔABC . 2

b) Find the coordinates of the point on the curve $y = 5x^2 - 4x + 1$ where the tangent to the curve is horizontal. 2

c) i) Find the points of intersection of the circle $x^2 + y^2 = 1$ and the parabola $y = x^2 - 1$ 3

ii) Sketch the region represented by $x^2 + y^2 \leq 1$ and $y < x^2 - 1$ 2

d) i) Show that $\frac{x}{x+1} = 1 - \frac{1}{x+1}$ 1

ii) State the domain and range . 2

Question 4

a) Sketch $y = \sin x$ for $0° \leq x \leq 360°$ 1

b) If $\sin\theta = \frac{1}{2}$ and $0° < \theta < 90°$, Find the exact value of $\tan(180° - \theta)$ 2

c) $ABCD$ is a parallelogram. X lies on AB and W lies on DC such that $AX = CW$. Prove $XD = BW$ 3

d) For the function $y = x^2(3 - x)$

i) Find the coordinates of the point where the graph meets the y-axis 1

ii) Find the intercept of the graph on the x-axis 2

iii) Find the coordinates of the stationary points and determine their nature. 3

iv) Sketch the graph showing all important features 2

e) If $y = Ax^n$, Show that $x\frac{dy}{dx} = ny$ 3

f) If the interior angle of a regular polygon is k times as large as each exterior angle . Find the number of sides the polygon has in terms of k. 3

End of Exam

2U Mathematics 2010

Q1

a) 1.4

b) $2\sqrt{3} - 3\sqrt{3}$

$= -\sqrt{3}$

c) $(x-3y)(x^2+3xy+9y^2)$

d) $4x+3x = 2x+2$

$7x = 2x+2$

$5x = 2$

$x = \frac{2}{5}$

e) $-\frac{\sqrt{3}}{2} = -\cos 30^\circ$

f) $-x > 6$

$x < -6$

g) $2x+3 = 13$ or $2x+3 = -13$

$2x = 10$ or $2x = -16$

$x = 5$ or $x = -8$

h) $t^2 - 2t + 1 - 2t$

$t^2 + 1 - 4t$

i) $\theta = 45^\circ$ or 225°

Q2

a) $\frac{\sqrt{2}}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$

$= \frac{\sqrt{10}+\sqrt{2}}{5-1}$

$= \frac{\sqrt{10}+\sqrt{2}}{4}$

b) Odd, the function has a point of symmetry around the origin

c) $A = \frac{1}{2} \times 8 \times 12 \times \sin 60$

$A = \frac{1}{2} \times 8 \times 12 \times \frac{\sqrt{3}}{2}$

$A = 24\sqrt{3}$

d) i) $y = 5x^2 + 4x$

$y' = 10x + 4$

ii) $y = (x+2)(x^2+1)$

$y = x^3 + 2x^2 + x + 2$

$y' = 3x^2 + 4x + 1$

e)

$\lim_{x\to 3} \frac{x^2 - x - 6}{x - 3}$

$= \lim_{x\to 3} \frac{(x-3)(x+2)}{(x-3)}$

$= \lim_{x\to 3} x + 2$

$= 5$

f) i) $\sin\theta\cos(90-\theta) + \cos\theta\sin(90-\theta)$

$= \sin\theta \times \sin\theta + \cos\theta \times \cos\theta$

$= \sin^2\theta + \cos^2\theta$

$= 1$

ii) $(\mathit{cosec}^2\theta - 1)\tan^2\theta$

$= \cot^2\theta \times \tan^2\theta$

$= \frac{1}{\tan^2\theta} \times \tan^2\theta$

$= 1$

g)

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 + 2(x+h) - (x^2+2x)}{h}$

$f'(x) = \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h}$

$f'(x) = \lim_{h\to 0} \frac{2xh + h^2 + 2h}{h}$

$f'(x) = \lim_{h\to 0} \frac{h(2x+h+2)}{h}$

$f'(x) = \lim_{h\to 0}(2x+h+2)$

$f'(x) = 2x + 2$

d) $y = x^2(3-x)$

i) On y axis $\left.\begin{array}{l} x=0 \\ y=0 \end{array}\right\}$ ✓ (1)

ii) On x axis $y=0$

$x=0$ (above) or $x=3$ ✓✓ (2)

iii) $y = 3x^2 - x^3$

$\frac{dy}{dx} = 6x - 3x^2$

for a stry pt $\frac{dy}{dx} = 0$

$6x - 3x^2 = 0$

$3x(2-x) = 0$

$\left.\begin{array}{l} x=0 \\ y=0 \end{array}\right\}$ ✓ $\quad$ $\left.\begin{array}{l} x=2 \\ y=12-8 \\ \quad =4 \end{array}\right\}$ ✓

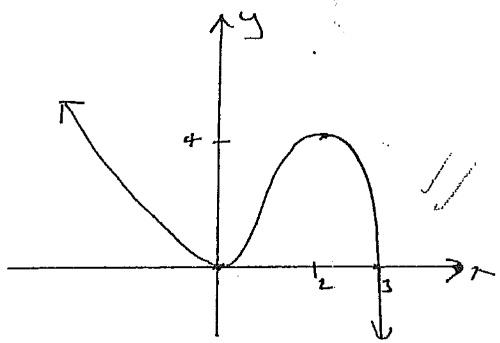
x	0^-	0	0^+
$\frac{dy}{dx}$	$-$	0	$+$

$\therefore$ min at $(0,0)$ ✓

x	2^-	2	2^+
$\frac{dy}{dx}$	$+$	0	$-$

max at $(2,4)$ (3)

iv)



✓✓ (2)

* e) $y = Ax^n$

$\frac{dy}{dx} = Anx^{n-1}$ ✓

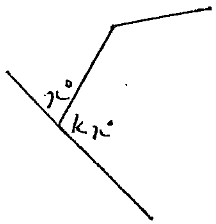
R.T.P. $x\frac{dy}{dx} = ny$ (3)

LHS $= x \times A_n x^{n-1}$
$= Anx^n$ ✓

RHS $= n \times Ax^n$
$= Anx^n$
$=$ LHS ✓

* Proof must be logical and flow

* f)



let the exterior $\angle = x°$

then the interior $\angle = kx°$

Now $x + kx = 180°$

$x(k+1) = 180°$

$\therefore x = \frac{180°}{k+1}$ ✓

Ext $\angle = \frac{360°}{n}$ ✓ (n – number of sides)

$\therefore n = \frac{360°}{x}$

$= 360° \times \frac{k+1}{180°}$

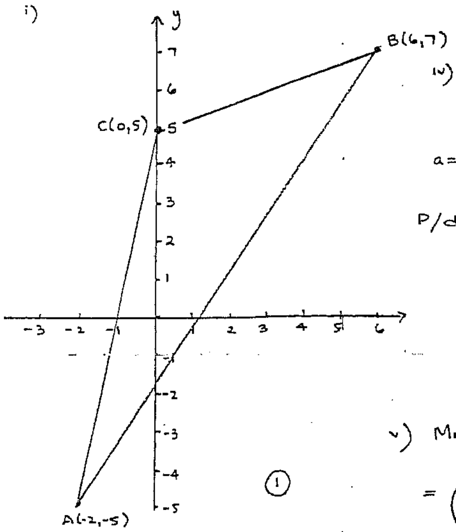
$= 2(k+1)$ ✓ (must have correct area for) *no small (3)

* no small error allowance allowance

Question 3 (20 marks.)

a) $A(-2,-5)$ $B(6,7)$ and $C(0,5)$ ($x_1\ y_1$, $x_2\ y_2$)

i)



①

ii) Gradient $AB = \dfrac{y_2-y_1}{x_2-x_1}$

$= \dfrac{7+5}{6+2}$

$= \dfrac{12}{8}$

$m = \dfrac{3}{2}$ ①

iii) Equation of AB

$y - y_1 = m(x - x_1)$

$y + 5 = \frac{3}{2}(x+2)$

$2y + 10 = 3x + 6$

iii) $3x - 2y - 4 = 0$ ②

iv) Perpendicular distance $= \left|\dfrac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

$a = 3,\ b = -2,\ c = -4$

$C(0,5)$

P/distance $= \dfrac{|3(0)+(-2)5-4|}{\sqrt{9+(-2)^2}}$

$= \dfrac{|-14|}{\sqrt{13}}$

$= \dfrac{14}{\sqrt{13}}$ units ②

v) Midpoint $AB\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

$= \left(\dfrac{-2+6}{2}, \dfrac{-5+7}{2}\right)$

$= \left(\frac{4}{2}, \frac{2}{2}\right)$ ①

$= (2,1)$

vi) distance $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(6+2)^2+(7+5)^2}$

$= \sqrt{64+144}$

$= \sqrt{208} = 4\sqrt{13}$ ①

vii) Area of $\Delta ABC = \frac{1}{2}bh$

$= \frac{1}{2} \times 4\sqrt{13} \times \frac{14}{\sqrt{13}}$

$= 28$ units^2 ②

10

b) $y = 5x^2 - 4x + 1$

$\dfrac{dy}{dx} = 10x - 4$

tangent is horizontal at $\dfrac{dy}{dx} = 0$

$10x - 4 = 0$

$10x = 4$

$x = \frac{2}{5}$

$y = 5(\frac{2}{5})^2 - 4(\frac{2}{5}) + 1$

$y = 5 \times \frac{4}{25} - \frac{8}{5} + 1$

$y = \frac{1}{5}$

$\therefore P(\frac{2}{5}, \frac{1}{5})$ ②

c) i) $x^2 + y^2 = 1$, $y = x^2 - 1$

Point of intersection

$x^2 + (x^2-1)^2 = 1$

$x^2 + x^4 - 2x^2 + 1 = 1$

$x^4 - x^2 = 0$

$x^2(x^2-1) = 0$

$x^2(x-1)(x+1) = 0$

$x = 0,\ x = 1$ or $x = -1$

$y = -1,\ y = 0,\ y = 0$

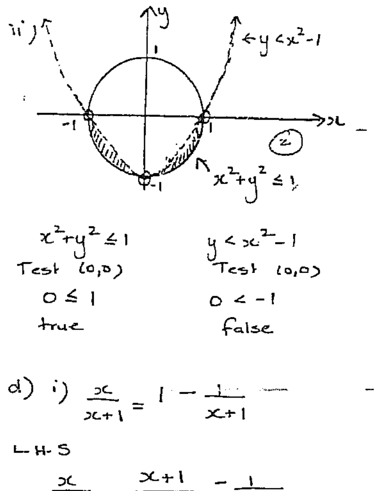
$\therefore$ Points of intersection

$P_1(0,-1)$

$P_2(1,0)$

$P_2(-1,0)$ ③

ii)



②

$x^2 + y^2 \le 1$ Test $(0,0)$ $0 \le 1$ true

$y < x^2 - 1$ Test $(0,0)$ $0 < -1$ false

d) i) $\dfrac{x}{x+1} = 1 - \dfrac{1}{x+1}$

L.H.S

$\dfrac{x}{x+1} = \dfrac{x+1}{x+1} - \dfrac{1}{x+1}$

$= 1 - \dfrac{1}{x+1}$ ①

$=$ R.H.S.

ii) Domain: all real x except $x = -1$ ①

Range: all real y except $y = 1$. ①

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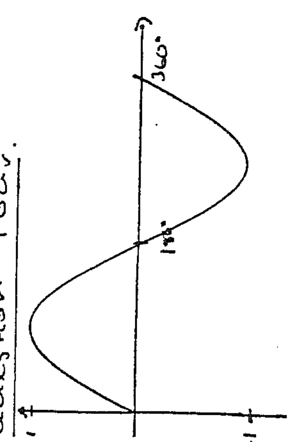
Question Four.

a)



①

must be correct

b) $\sin\theta = \frac{1}{2}$

$\therefore \theta = 30°$ $(0° \le \theta \le 90°)$

$\tan(180° - \theta) = \tan 150°$

$= -\tan 30°$

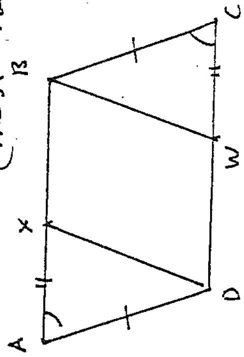
$= -\dfrac{1}{\sqrt{3}}$

(must have correct sign)

must have correct sign

②

c)



In Δ's AXD, BWC

$AX = WC$ (given)

$\angle XAD = \angle BCW$ (opp $\angle$'s p gram)

$AD = BC$ (opp sides p gram)

$\therefore \Delta AXD \equiv AWB$ (SAS)

$\therefore XD = BW$ (corresp sides cong Δ's)

③