SYDNEY GIRLS HIGH SCHOOL

YEAR 11 Mathematics
Extension 1

Half Yearly Examination

May 2012

Time allowed: 60 minutes
Plus 3 minutes reading time

Instructions:

- There are Four (4) questions. Questions **are** of equal value.

- Attempt all questions.

- Show all necessary working. Marks may be deducted for badly arranged work.

- Start each question on a new page. Write on one side of the paper only.

Student Name:- _____

Teacher:------------_____

QUESTION ONE (15 marks)

a) Fully Factorise $x^3 - 4x$ (1)

b) If $\tan\theta = \dfrac{2}{3}$, and θ is acute, find the exact value of $\sin\theta$.   (2)

c) The angle of elevation of a tower is $39°44'$ when measured at a point 100 m from its base.

   Find the height of the tower, to 1 decimal place.   (2)

d) Write down the exact value of $\sec(-30°)$.   (2)

e) Solve $\dfrac{5x}{|x+3|} = 2$ (3)

f) Solve $\cos x = -0.6$ for $0° \le x \le 360°$ (answer correct to the nearest degree)   (2)

g) Find the range of $x^2 + y^2 - 10y + 21 = 0$ (2)

h) Find the domain of $y = \dfrac{1}{\sqrt{x-4}}$ (1)

QUESTION TWO (15 marks)

a) Sketch the graph $y = \operatorname{cosec} x$ for $0° \leq x \leq 360°$. (2)

b) Draw a neat sketch of the following showing their important features:

i) $y = -x^2 - 5x$ (2)

ii) $y = \dfrac{2}{x+1}$ (2)

iii) $y = 3 - |x+2|$ (3)

c) Determine whether the function $f(x) = 3x^4 - 5x$ is odd, even or neither. (2)

d) Given that $f(x) = 2x^2 + 5x - 3$, for what values of x, is $f(x) > 0$ (2)

e) Find the value of x given, $\sin(2x+40)° = \cos 30°$ (2)

QUESTION THREE (15 marks)

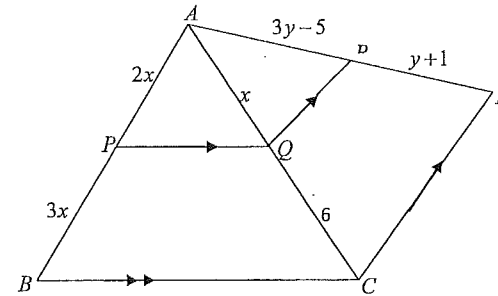a) Given $PQ \parallel BC$ and $QR \parallel CD$, find the value of x and y, giving reasons. (3)

Figure not to scale

b) Solve $\dfrac{x}{x+2} \geq 3$ (3)

c) i) On the same axis sketch $y = \sqrt{4 - x^2}$ and $y = |x|$. (3)

ii) Shade the region defined by $y \leq \sqrt{4-x^2}$ and $y \geq |x|$. (2)

iii) Find the area of the shaded region. (1)

d) Given $(\sqrt{3} - 1)^4 = a + b\sqrt{3}$, find the value of a and b. (3)

QUESTION FOUR (15 marks)

a) Find the exact value of $(0.2\dot{6})^3$. Show all your working. (2)

b) A function is defined by :

$$f(x) = \begin{cases} x+3 & \text{if } x < -3 \\ -1 & \text{if } -3 \leq x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$$
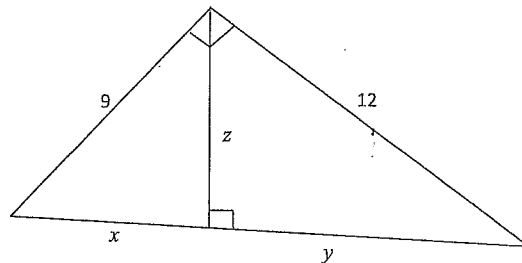
    i) Sketch this function for $-4 \leq x \leq 2$ (3)

    ii) Evaluate $f(-3) + f(-2) + f(2)$ (2)

    iii) $f(a^2)$ (1)

c) Simplify $\sin(270° - \theta)$. (2)

d) In the following diagram find the value of x, y and z. (2)

e)   Edward , Bella and Reneesme are hiding in Forks forest from The Vulturi Family . Bella is 1km due east of Edward, and Reneesme is 1 km due east of Bella. They all sight the Vulturi family at the same time being due North of Reneesme and on a bearing of 032°T from Edward. Find the bearing of Bella from TheVulturi family correct to the nearest degree. (Draw the information on a diagram) (3)
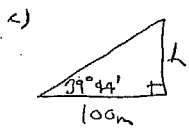
END OF THE PAPER

1 a) $x(x^2 - 4)$

$= x(x-2)(x+2)$ ✓

b)

2 ... $\sqrt{2^2 + 3^2} = \sqrt{13}$

θ

3

$\sin\theta = \dfrac{2}{\sqrt{13}}$ ✓

c)

h

$39°44'$

$100m$
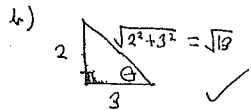
$\tan 39°44' = \dfrac{h}{100}$ ✓

$h = 100\tan 39°44'$

$= 83.1992413$

$= 83.1m$ ✓

d) $\dfrac{1}{\cos 30°} = \dfrac{1}{\frac{\sqrt{3}}{2}}$ ✓

$= \dfrac{2}{\sqrt{3}}$

e) $5x = 2|x+3|$

$|x+3| = \dfrac{5x}{2}$

$x+3 = \dfrac{5x}{2}$ on $x+3 = -\dfrac{5x}{2}$

$2x+6 = 5x$ .... $2x+6 = -5x$

$-3x = -6$ .... $7x = -6$

$x = 2$ .... $x = -\dfrac{6}{7}$

$\dfrac{5 \times 2}{2}$ .... $\dfrac{5 \times -\frac{6}{7}}{2}$

$= 2$ .... $= -\dfrac{30}{14}$

$\therefore x = 2$ is ✓ .... $\therefore$ no solution
the solution ✓

f) $x = 180° - 53°$ on $x = 180° + 53°$

$= 127°$ on $233°$ ✓

g) $y^2 - 10y + 5^2 + x^2 = -21 + 25$

$(y-5)^2 + x^2 = 4$

centre $(0,5)$ .... $r^2 = 4$

.... $r = 2$ ✓

range: $3 \leq y \leq 7$

h) domain: $\sqrt{x-4} \geq 0$

$x - 4 \geq 0$

$x \geq 4$ ✓

2) a)

$y = \dfrac{1}{\sin x}$

$90 \quad 180 \quad 270 \quad 360$

b)

i) $-x^2 - 5x = 0$

$-x(x+5) = 0$

$x = 0 \quad x = -5$

at $x = -2.5$

$y = 6.25$

ii) $y = \dfrac{2}{x+1}$

$(-1, -2)$

iii)

$-3 \quad -2$ 3
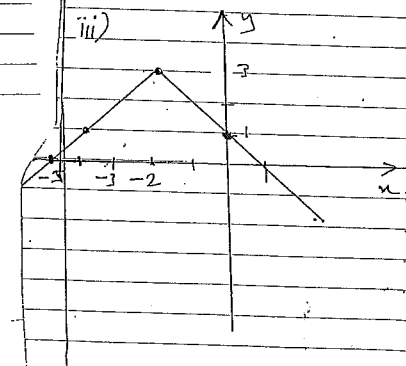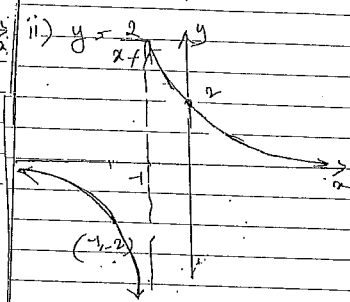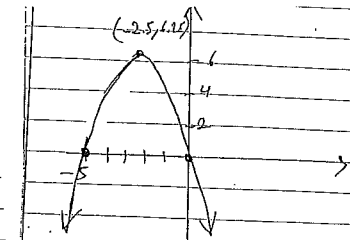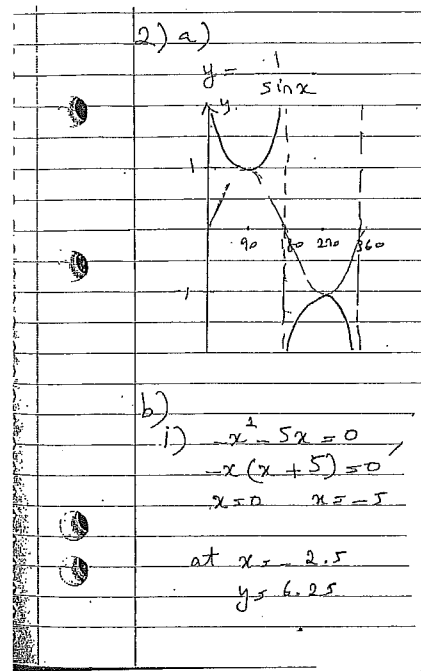
c) $f(x) = 3x^4 - 5x$

$f(-x) = 3(-x)^4 - 5(-x)$

$= 3x^4 + 5x$

$-f(x) = -3x^4 + 5x$

$f(x) \neq f(-x)$ not even

$-f(x) \neq f(-x)$ not odd

$\therefore$ neither

d) $2x^2 + 5x - 3 > 0$

$2x^2 + 6x - x - 3 > 0$

$2x(x+3) - (x+3) > 0$

$(x+3)(2x-1) > 0$

$x > \dfrac{1}{2}, \quad x < -3$
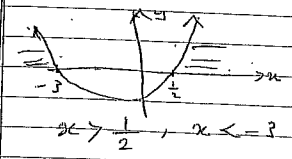
e) $\sin(2x+40)° = \sin 60$

$2x + 40 = 60$

$2x = 20$

$x = 10$

QUESTION 3

3a) $\frac{x}{6} = \frac{2x}{3x}$ (ratio of intercepts on parallel lines)

$3x^2 = 12x$

$3x^2 - 12x = 0$

$3x(x-4) = 0$

$x = 0$ or 4

but $x \neq 0 \therefore x = 4$

Now $\frac{x}{6} = \frac{3y-5}{y+1}$ (Ratio of intercepts on parallel lines)

$x = 4, \quad \frac{4}{6} = \frac{3y-5}{y+1}$

$\frac{2}{3} = \frac{3y-5}{y+1}$

$2y + 2 = 9y - 15$

$0 = 7y - 17$

$7y = 17$

$\therefore y = \frac{17}{7} = 2\frac{4}{7}$

b) $\frac{x}{x+2} \geqslant 3 \qquad x+2 \neq 0$

$\frac{x}{(x+2)} \times (x+2)^2 \geqslant 3 \times (x+2)^2 \qquad x \neq -2$
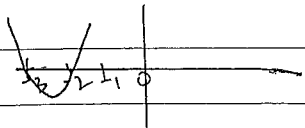
$x(x+2) \geqslant 3(x+2)^2$

$x^2 + 2x \geqslant 3(x^2 + 4x + 4)$

$x^2 + 2x \geqslant 3x^2 + 12x + 12$
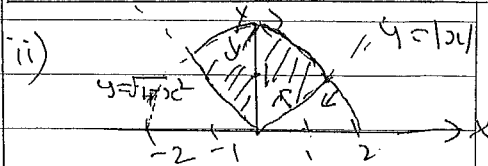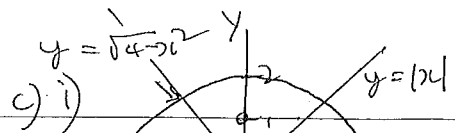
$0 \geqslant 2x^2 + 10x + 12$

$x^2 + 5x + 6 \leqslant 0$

Draw $y = (x+3)(x+2)$

$-3 \leqslant x \leqslant -2$

but $x \neq -2$

$\therefore -3 \leqslant x < -2$

c) i)

$y = \sqrt{4-x^2}$ $y = |x|$

ii)

$y = \sqrt{4-x^2}$ $y = |x|$

$y \leqslant \sqrt{4-x^2}$

Test $(0,1)$: $1 \leqslant \sqrt{4-0}$

$1 \leqslant 2$ True

$y \geqslant |x|$ Test $(0,1)$

$1 \geqslant |0|$ True

iii) Area shaded region

$= \frac{1}{4} \pi r^2 \quad (r=2)$

$= \frac{1}{4} \times \pi \times 2^2$

$= \pi$ sq units

d) $(\sqrt{3}-1)^4 = a + b\sqrt{3}$

$[(\sqrt{3}-1)^2]^2 = a + b\sqrt{3}$

$[3 - 2\sqrt{3} + 1]^2 = a + b\sqrt{3}$

$[4 - 2\sqrt{3}]^2 = a + b\sqrt{3}$

$16 - 16\sqrt{3} + 12 = a + b\sqrt{3}$

$28 - 16\sqrt{3} = a + b\sqrt{3}$

$\therefore a = 28, \ b = -16.$
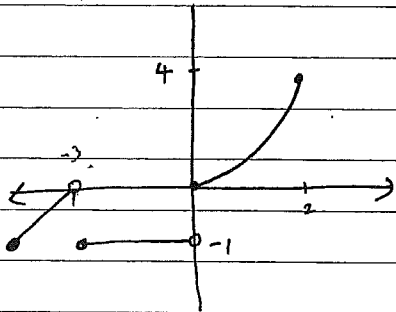
4.

a) $x = 0.2666....$

$10x = 2.666...$

$100x = 26.666...$

$90x = 24$

$x = \dfrac{4}{15}$ ②

$x^3 = \dfrac{64}{3375}$

b) i)

1 for each section
-1 if no endpoints ③

ii) $f(-3) + f(-2) + f(2) = -1 + -1 + 4$ ②
$= 2$

iii) $f(a^2) = a^4$ ①

c) $\sin(270 - \theta) = \sin(180 + 90 - \theta)$
$= -\sin(90 - \theta)$ ①
$= -\cos\theta$ ①

d) $(x + y)^2 = 9^2 + 12^2$
$x + y = 15$ ①

$x^2 + z^2 = 81$ ②

$y^2 + z^2 = 144$ ③

$x^2 - y^2 = -63$ ② − ③ → ④

$x^2 - (15 - x)^2 = -63$ sub ① into ④
$x^2 - (225 - 30x + x^2) = -63$ ①
$-225 + 30x = -63$
$30x = 162$
$x = 5.4$
$y = 9.6$ ①

$5.4^2 + z^2 = 81$
$z = 7.2$

e)
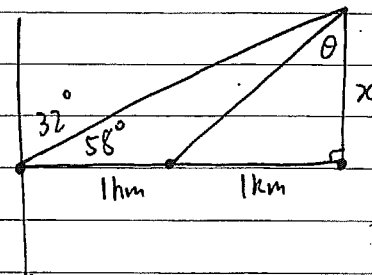
$\tan 58 = \dfrac{x}{2}$
$x = 3.2$ ①

$\tan\theta = \dfrac{1}{3.2}$

$\theta = 17°$ ①

Bearing $= 180 + 17$ ①