Sydney Girls' High School

2012
MATHEMATICS
YEAR 11
HALF-YEARLY EXAMINATION

Time Allowed: 60 minutes

TOPICS: Arithmetic, Algebra, Equations and Geometry.

Directions to Candidates

- There are four (4) questions.
- Attempt ALL questions.
- Questions are of equal value.
- Start each question on a new page.
- Write on one side of the paper only.
- Show all necessary working. Marks will be deducted for careless or badly arranged work.
- Diagrams are NOT drawn to scale.
- Board-approved calculators may be used.

Total: 60 marks

STUDENT NAME:.....

TEACHER NAME:.....

QUESTION 1 (15 marks) — Marks

a) Evaluate correct to three significant figures

$$\frac{(1.76)^3 - 0.65}{\sqrt{2.15}}$$ — 2

b) Express $0.6\dot{7}$ as a fraction in its simplest form — 2

c) Simplify

i) $(-3a^2b)^3$ — 2

ii) $\frac{18a^3b^5}{12ab^7}$ — 2

d) Expand and simplify

i) $(2a+3)(a-2)$ — 2

ii) $(3x-4y)^2$ — 1

e) An importer buys 800 watches for $6 800 and sells them at a profit of 66%. What is the selling price of each watch? — 2

f) Express $\frac{6}{\sqrt{7}+2}$ with a rational denominator. — 2

QUESTION 2 (15 marks)

a) Simplify the following

i) $\frac{2a-1}{3}+\frac{a+4}{4}$ — 2

ii) $\frac{75-3x^2}{x^2+3x-10} \div \frac{125-x^3}{x^2-4}$ — 3

b) Solve each of the following

i) $|2x-1|=11$ — 2

ii) $12-(x-2) \le 24$ — 2

iii) $x^2+2x=5$ (leave in simplest exact form) — 2

iv) $|3x+1| \le 7$ — 2

c) Each interior angle of a regular polygon is $165°$. How many sides are there in the polygon? — 2

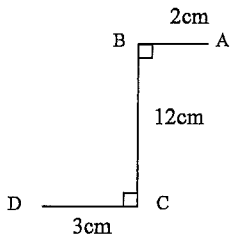
QUESTION 3 (15 marks)

Marks

a) Solve the following pair of simultaneous equations. 3

$$x^2 - y = 0$$
$$x + y - 6 = 0$$

b) Find the distance AD in the diagram below 2



c) If $\frac{6^{\frac{2}{3}} \times 10^{\frac{1}{3}}}{30} = 3^a.5^b$, find the values of a and b. 3

d)

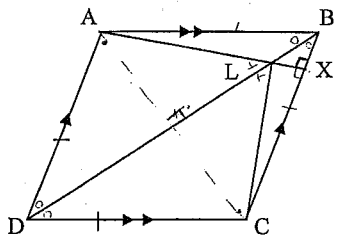


FIGURE NOT TO SCALE

ABCD is a rhombus. AX is perpendicular to BC. AX intersects BD at L.

i) Copy the diagram onto your examination paper and state why $\angle ADB = \angle CDB$. 1

ii) Prove $\Delta ALD \equiv \Delta CLD$ 3

iii) Find the size of $\angle DAL$, giving reasons. 2

iv) Hence or otherwise find the size of $\angle LCD$ 1

QUESTION 4 (15 marks)

Marks

a) The formula $r = \sqrt[3]{\frac{3V}{4\pi}}$ can be used to find the radius (r) of a sphere given its volume (V).

Find the radius of a sphere with volume $2144 \cdot 7cm^2$ (answer to nearest cm). 1

b) Factorise fully $(2x+1)^2 - (x+2)^2$ 2

c) Simplify $\frac{a^2 - a^{-1}}{a^{-1} - 1}$ 3

d) Find the values of a and b such that $\sqrt{8 - 4\sqrt{3}} = \sqrt{a} - \sqrt{b}$ 2

e)

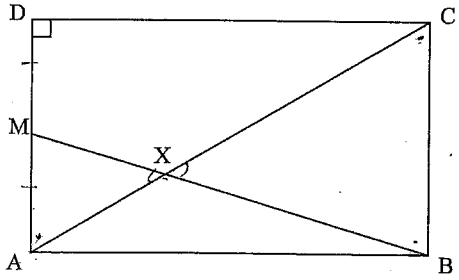


FIGURE NOT TO SCALE

In the diagram, ABCD is a rectangle and AB=2AD.
The point M is the midpoint of AD. The line BM meets AC at X.

i) Prove ΔAXM and ΔBXC are similar. 3

ii) Show that $3CX = 2AC$ 2

iii) Show that $9(CX)^2 = 5(AB)^2$ 2

THE END

Year 11 half yearly
Mathematics
2012

Q1 a) $\frac{(1.76)^3 - 0.65}{\sqrt{2.15}}$

$= 3.274784051$

$= 3.27$ (2)

b) $x = 0.6\dot{7}$

$10x = 6.\dot{7}$ ..... ①

$100x = 67.\dot{7}$ ..... ②

eq. ② − eq. ①

$90x = 61$

$x = \frac{61}{90}$ (2)

c) I) $\left(-a^2 b^3\right)^3$

$= -a^6 b^3$ (2)

II) $\frac{18 a^3 b^5}{12 a b^7}$

$= \frac{3 a^2}{2 b^2}$ (2)

d) i) $(2a+3)(a-2)$

$= 2a(a-2) + 3(a-2)$

$= 2a^2 - 4a + 3a - 6$

$= 2a^2 - a - 6$ (2)

ii) $(3x-4y)^2$

$= (3x)^2 - 2(3x)(4y) + (4y)^2$

$= 9x^2 - 24xy + 16y^2$ (1)

e) Selling price = \$6800 × 1.66

= \$11288

Price of each = \$11288 ÷ 800

= \$14.11 (2)

f) $\frac{6}{\sqrt{7}+2} \times \frac{\sqrt{7}-2}{\sqrt{7}-2}$

$= \frac{6\sqrt{7} - 12}{7 - 4}$

$= \frac{6\sqrt{7} - 12}{3}$

$= \frac{3(2\sqrt{7} - 4)}{3}$

$= 2\sqrt{7} - 4$ (2)

Yr 11 Ext 1

Question 2

a) i) $\frac{2a-1}{3} + \frac{a+4}{4}$

$= \frac{4(2a-1)+3(a+4)}{12}$

$= \frac{8a-4+3a+12}{12}$

$= \frac{11a+8}{12}$ /2

ii) $\frac{75-3x^2}{x^2+3x-10} \div \frac{125-x^3}{x^2-4}$

$= \frac{3(25-x^2)}{(x+5)(x-2)} \times \frac{x^2-4}{5^3-x^3}$

$= \frac{3(5-x)(5+x)}{(x+5)(x-2)} \times \frac{(x-2)(x+2)}{(5-x)(25+5x+x^2)}$

$= \frac{3(x+2)}{25+5x+x^2}$ /3

b) i) $|2x-1| = 11$

$2x-1 = 11$ or $2x-1 = -11$

$2x = 12$ $\quad 2x = -10$

$x = 6$ $\quad x = -5$

$x = 6$ or -5 /2

ii) $12-(x-2) \leq 24$

$12-x+2 \leq 24$

$14-x \leq 24$

$-x \leq 10$

$x \geqslant -10$

/2

iii) $x^2+2x-5 = 0$

$x = \frac{-6 \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-2 \pm \sqrt{4-4(-5)}}{2}$

$= \frac{-2 \pm \sqrt{24}}{2}$

$= \frac{-2 \pm 2\sqrt{6}}{2}$

$= -1 \pm \sqrt{6}$ /2

iv) $|3x+1| \leq 7$

$-7 \leq 3x+1 \leq 7$

$-8 \leq 3x \leq 6$

$\frac{-8}{3} \leq x \leq 2$

/2

c) Int. $\angle = \frac{(n-2)\times 180}{n}$

$165n = 180n - 360$

$360 = 15n$

$n = 24$

$\therefore$ 24 sides

/2

Question Three

a) $x^2 - y = 0$ ①, $x + y - 6 = 0 \Rightarrow y = 6 - x$ sub in ①

$x^2 - (6 - x) = 0$

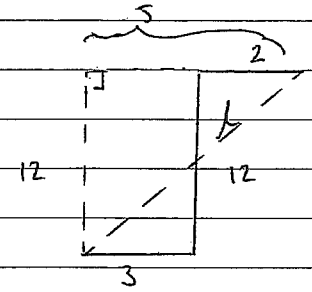
$x^2 + x - 6 = 0$ ✓

$(x + 3)(x - 2) = 0$

$x = -3, \; x = 2$

$y = 9, \; y = 4$ ✓ ③

b)



$l^2 = 5^2 + 12^2$ ✓

$l = \sqrt{169}$

$= 13$ ✓ ②

c) $\dfrac{6^{\frac{2}{3}} \times 10^{\frac{1}{3}}}{30} = \dfrac{(2\times3)^{\frac{2}{3}} \times (2\times5)^{\frac{1}{3}}}{2\times3\times5}$

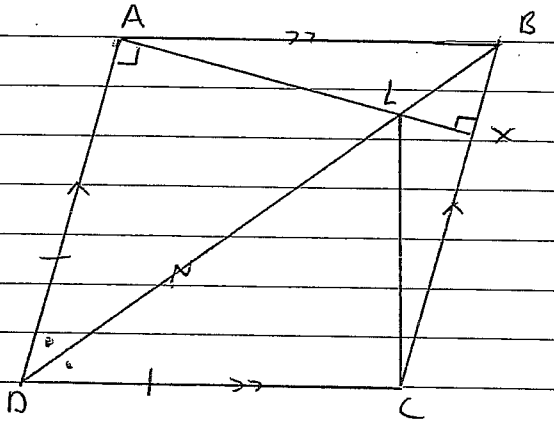
$= \dfrac{2^{\frac{2}{3}} \times 3^{\frac{2}{3}} \times 2^{\frac{1}{3}} \times 5^{\frac{1}{3}}}{2\times3\times5}$

$= \dfrac{2 \times 3^{\frac{2}{3}} \times 5^{\frac{1}{3}}}{2\times3\times5}$ ✓

$= 3^{-\frac{1}{3}} \times 5^{-\frac{2}{3}}$ ✓ ✓

$= 3^a \times 5^b \;\therefore\; a = -\frac{1}{3}, \; b = -\frac{2}{3}$

d)



i) Diagonals of a rhombus bisect the angles through which the pass ✓ ①

ii) In $\triangle$'s ALD, CLD

AD = DC (sides of a rhombus) ✓

$\angle ADB = \angle BDC$ (see i))

DL common ✓

$\therefore \triangle ALD \equiv \triangle CLD$ (SAS) ✓ ③

iii) $\angle DAL = 90°$ ✓ (alt $\angle$'s AD || BC) ✓ ②

iv) $\angle LCD = \angle DAL$ (corresp $\angle$'s) ✓ ①

$= 90°$ (cong $\triangle$'s)

cont

Solutions – Question 4 (15 marks)

(a) 1 mark

$$r=\sqrt[3]{\frac{3V}{4\pi}}=\sqrt[3]{\frac{3\times 2144.7}{4\pi}}$$

$\therefore r\approx 8$ cm

(b) 2 marks

$$\begin{aligned}(2x+1)^2-(x+2)^2&=\big(2x+1+(x+2)\big)\big(2x+1-(x+2)\big)\\&=(3x+3)(x-1)\\&=3(x+1)(x-1)\end{aligned}$$

(c) 3 marks

$$\begin{aligned}\frac{a^2-a^{-1}}{a^{-1}-1}&=\frac{a^2-\frac{1}{a}}{\frac{1}{a}-1}\\&=\frac{a^3-1}{1-a}\\&=\frac{(a-1)(a^2+a+1)}{-(a-1)}\\&=-(a^2+a+1)\quad\text{or}\quad -a^2-a-1\end{aligned}$$

(d) 2 marks

$\sqrt{8-4\sqrt{3}}=\sqrt{a}-\sqrt{b}$

$8-4\sqrt{3}=a-2\sqrt{ab}+b$

$a+b=8 \qquad ab=12$

$a+\frac{12}{a}=8 \qquad \Rightarrow a^2-8a+12=0$

$(a-6)(a-2)=0$

$\therefore a=6,b=2$ or $a=2,b=6$

$\sqrt{a}-\sqrt{b}>0$

$\therefore a>b\Rightarrow \qquad a=6,b=2$ only

(e)(i) 3 marks

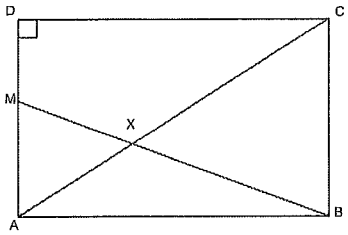
$AD\parallel BC$ (opp. sides of parallelogram)

In ΔAXM and ΔCXB :

(i) $\angle MXA=\angle BXC$ (vert. opp. $\angle$s)

(ii) $\angle XMA=\angle XBC$ (alt. $\angle$s, $AD\parallel BC$)

$\therefore \Delta AXM \equiv \Delta CXB$ (equiangular)



(e)(ii) 2 marks

$BC=AD$ (opp. sides of rectangle)

$MA=\frac{AD}{2}$ (M is the midpoint of AD)

$\frac{AX}{CX}=\frac{MA}{BC}$ (corr. sides in same ratio in similar Δs)

$\frac{AC-CX}{CX}=\frac{\frac{AD}{2}}{AD}$

$\frac{AC-CX}{CX}=\frac{1}{2}$

$2AC-2CX=CX$

$\therefore 3CX=2AC$

(e)(iii) 2 marks

$AB=2AD$ (given)

$AD=BC$ (opp. sides of rectangle)

$\therefore AB=2BC$ i.e. $BC=\frac{AB}{2}$

$3CX=2AC$

$$\begin{aligned}\therefore\quad 9(CX)^2&=4(AC)^2\\&=4\left[(AB)^2+(BC)^2\right]\quad\text{(Pythag. Thm.)}\\&=4\left[(AB)^2+\left(\frac{AB}{2}\right)^2\right]\\&=4\left[(AB)^2+\frac{(AB)^2}{4}\right]\\&=4(AB)^2+(AB)^2\end{aligned}$$

$$\therefore\quad 9(CX)^2=5(AB)^2$$