# Sydney Girls High School

# MATHEMATICS EXTENSION 2

**2010 HSC Assessment Task 1**
**November 2009**

**Time Allowed - 90 minutes**
**(plus 5 minutes reading time)**

**Topics:** Circular Motion (Exercises 1-8) , Curve Sketching (Exercises 1-8)

**General Instructions:**

- There are TEN (10) Questions which are NOT of equal value.
- Questions do not necessarily appear in order of difficulty.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write your student number clearly at the top of each question and clearly number each question.
- Use $g = 10 \text{ ms}^{-2}$

**Total: 90 marks**

## QUESTION 1 (8 Marks)

MARKS

Sketch the following on separate number planes, showing important features :

a. $y = (x+1)^2(x-2)(x-4)^3$ — 2

b. $x = \sqrt{4-y^2}$ — 2

c. $\frac{x^2}{16} + \frac{y^2}{25} = 1$ — 2

d. $x^2 - y^2 = 4$ — 2

## QUESTION 2 (12 Marks)

MARKS

a. Given $F(x) = 1 - x^2$ , sketch :

i. $y = F(x)$ — 1

ii. $y = -F(x)$ — 1

iii. $y = |F(x)|$ — 2

iv. $y = \{F(x)\}^2$ — 2

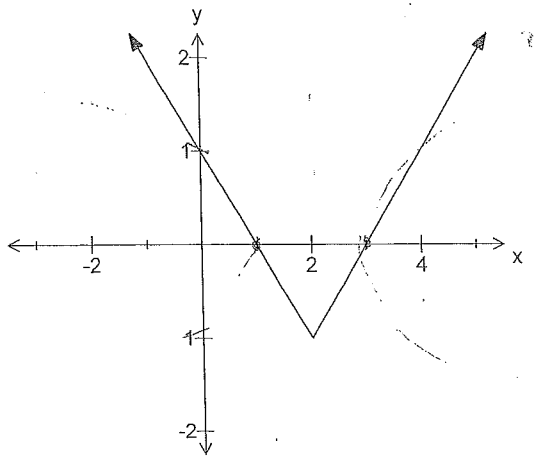
b. For the following, sketch $y = f(x)$, $y = f^{-1}(x)$ and $y = x$ on the same number plane. Show any key points, including all intersection points.

i. $f(x) = x^3$ — 3

ii. $f(x) = 2^x$ — 3

## QUESTION 3 (7 Marks)

MARKS




Copy the function $y = f(x)$ as shown above and sketch on the same number plane :

a. $y = f(|x|)$ 2

b. $y = \dfrac{1}{f(x)}$ 2

c. $y^2 = f(x)$ 2

d. $y = f(x-2)$ 1

## QUESTION 4 (10 Marks)

MARKS

Sketch the following on separate number planes, showing important features :

a. $y = \dfrac{3x+4}{x+2}$ 2

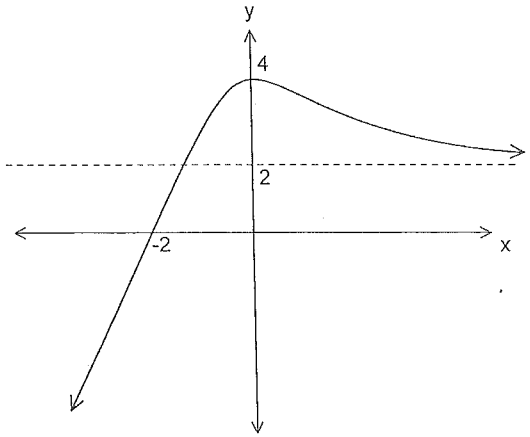
b. $25y^2 - 9x^2 = 225$ 2

c. $y = x - \dfrac{2}{x+1}$ 3

d. $y = \dfrac{3}{(x+2)(x-1)^3}$ 3

## QUESTION 5 (8 Marks)

MARKS




Copy the function $y = f(x)$ as shown above and sketch on the same number plane :

a. $y = f(-x)$ 1

b. $y = 2^{f(x)}$ 2

c. $y = -\sqrt{f(x)}$ 2

d. $y = f^{-1}(x)$ 2

e. $|y| = f(x)$ 1

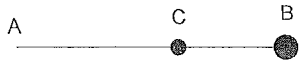
## QUESTION 6 (8 Marks)

MARKS

A 3 metre piece of string AB has a mass of 5 kg attached at point B. The string is rotated in a horizontal circle about A and breaks as soon as it exceeds a speed of rotation of 45 revolutions per minute.

a. Find the maximum possible tension in the string. 3

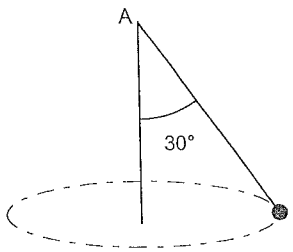
b. The mass at B is replaced by a 3 kg mass and an additional 1 kg mass is attached to the string at C, 2 metres from A (as shown below). Find the new maximum number of revolutions per minute that the string can be rotated. 5



## QUESTION 7 (8 Marks)

MARKS

A mass is rotating in circular motion as a conical pendulum as shown in the diagram.




The mass of 5 kg is attached to A by a light string of length 2 m and the angle between the string and the vertex is $30°$.

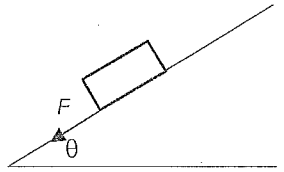
a. Find the tension in the string. 4

b. Find the speed that the mass is moving. Give your answer in m/s. 4

## QUESTION 8 (10 Marks)

MARKS

A section of a car-racing track is being constructed and is to be banked at an angle of $\theta$ as shown below. A car of mass $m$ kg is travelling around the track at $v$ m/s.




a. Copy the diagram and mark in the normal force N and the gravitational force. 1

b. Resolve the forces vertically and horizontally. 2

c. Hence, find an expression for the : 4

i. lateral force F in terms of $m, v, r, g$ and $\theta$.

ii. normal force N in terms of $m, v, r, g$ and $\theta$.

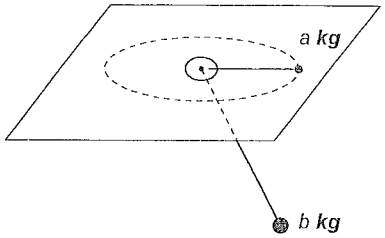
d. The road is constructed around a curve of radius 150 m to allow for an optimum speed of 120 km/h. Calculate the angle (answer to the nearest minute) at which the track should be banked. 3

## QUESTION 9 (7 Marks)

MARKS

Two masses are connected by a light inelastic string. One mass of $a$ kg is rotating in circular motion on a smooth horizontal table with angular velocity $w$ rad/s. The other mass of $b$ kg is at the other end of the string, rotating in a conical pendulum below the table with angular velocity $2w$ rad/s. 7

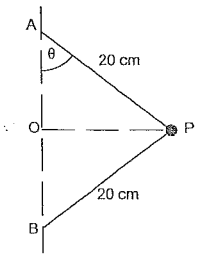
If $a:b=5:2$, find what fraction of the string lies above the table.




## QUESTION 10 (12 Marks)

MARKS

A particle, P, of mass $m$, is attached by two strings, each of length 20 cm, to two fixed points, A and B, which are 24 cm apart and lie on a vertical line as shown in the diagram.




The particle P moves with constant speed, $v$ m/s, in a horizontal circle about the midpoint of AB so that both pieces of string experience a tension.

The tension in AP is $T_1$ and the tension in BP is $T_2$. The acceleration due to gravity is $g$ ms$^{-2}$.

a. Copy the diagram and mark on it all the forces acting on P. 2

b. Find the length of the radius of motion OP. 1

c. Resolve the forces on P horizontally and vertically. 2

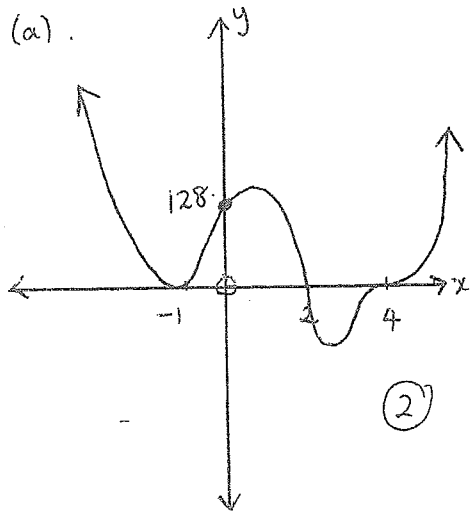
d. Find the tension in each part of the string in terms of $m, v$ and $g$. 4

e. Show that $v > \frac{4\sqrt{3g}}{15}$. 3

# 2010 (Nov '09) - Extension 2 - TASK 1 : SOLUTIONS

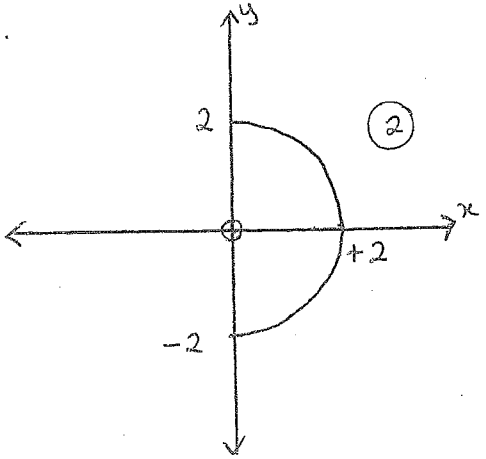
## Question 1

(a).



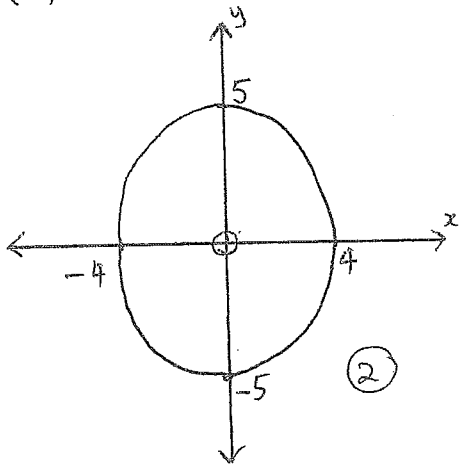

(2)

(b).



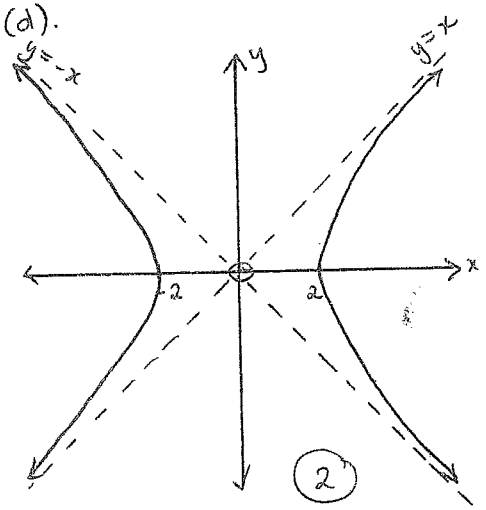

(2)

(c).




(2)

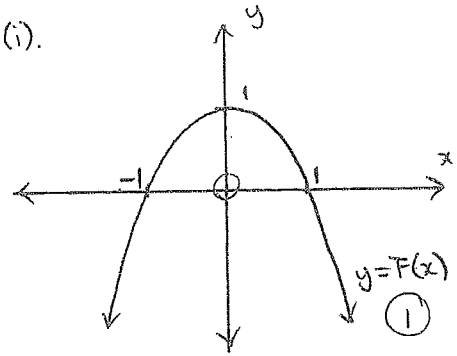
(d).




(2)

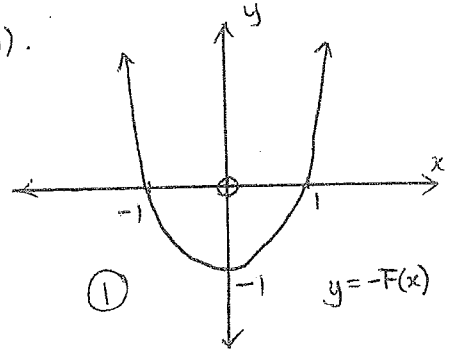
## Question 2

(a). (i).



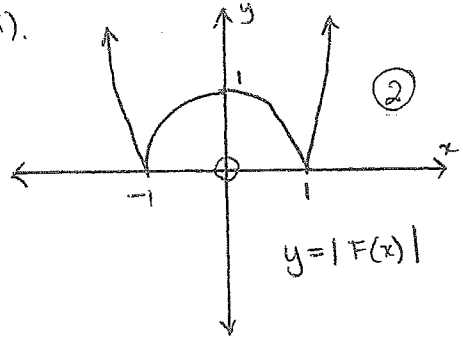

(1)

(ii).



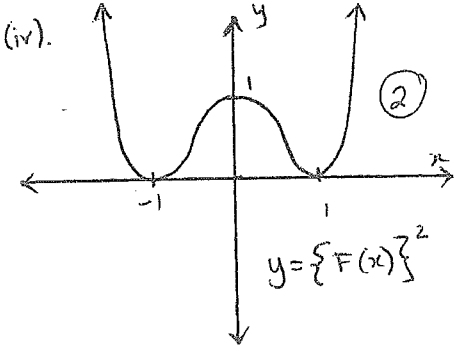

(1)

(iii).



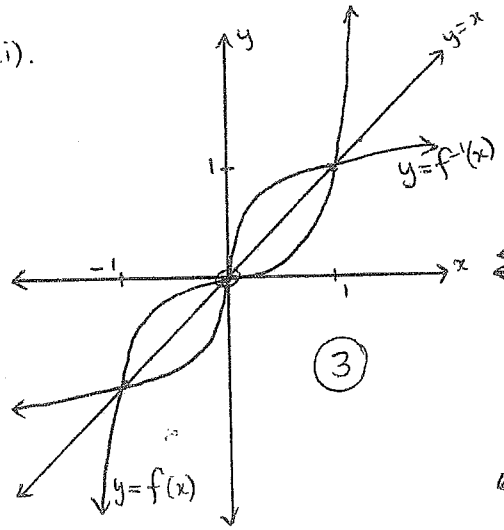

(2)

(iv).



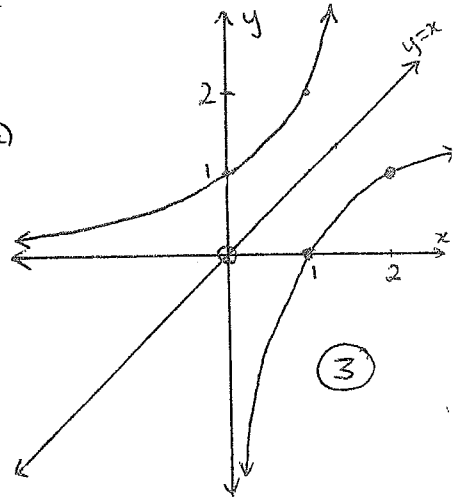

(2)

(b). (i).




(3)

$f(x) = x^3$

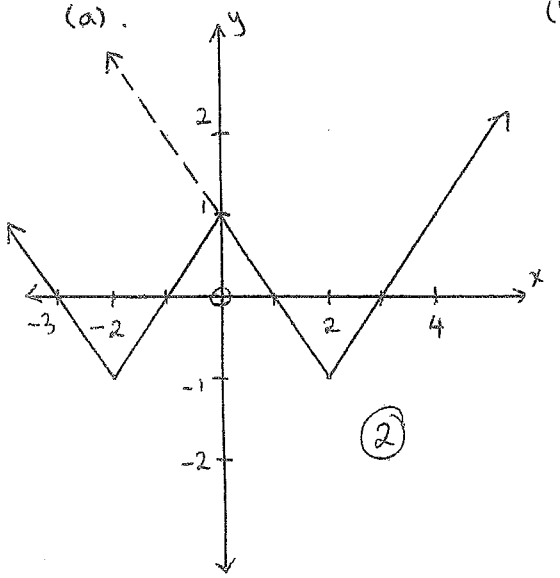



(3)

$f(x) = 2^x$

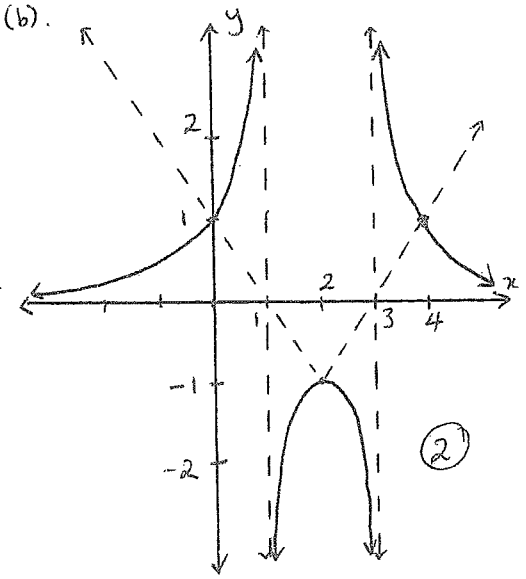
## QUESTION 3

(a).



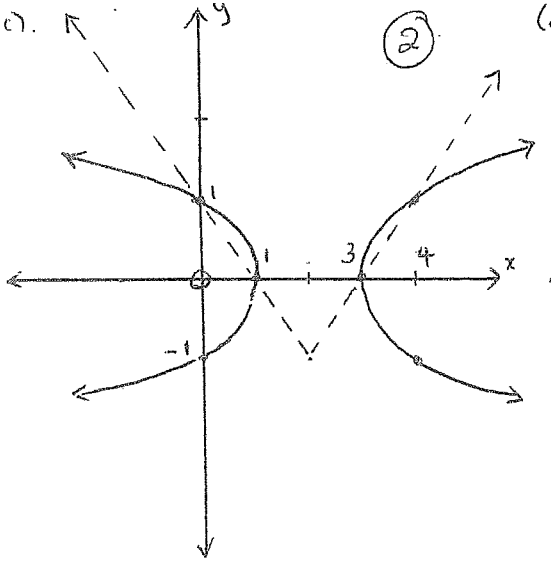

(2)

(b).



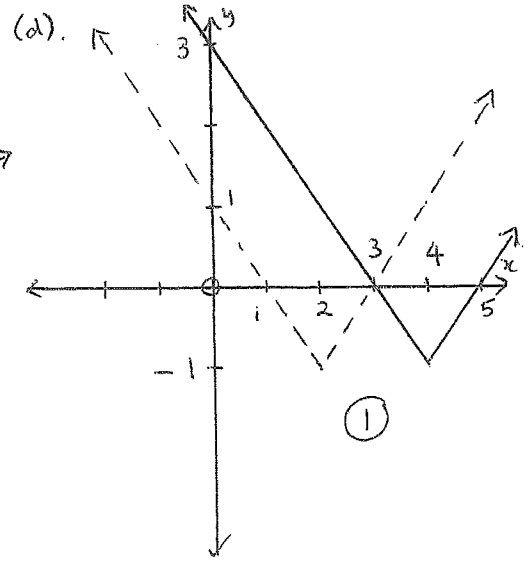

(2)

(c).




(2)

(d).

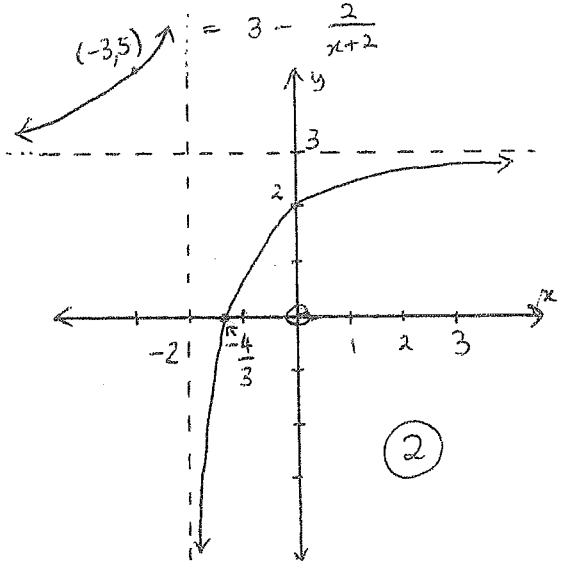



(1)

## QUESTION 4

(a). $y = \frac{3x+6}{x+2} - \frac{2}{x+2}$

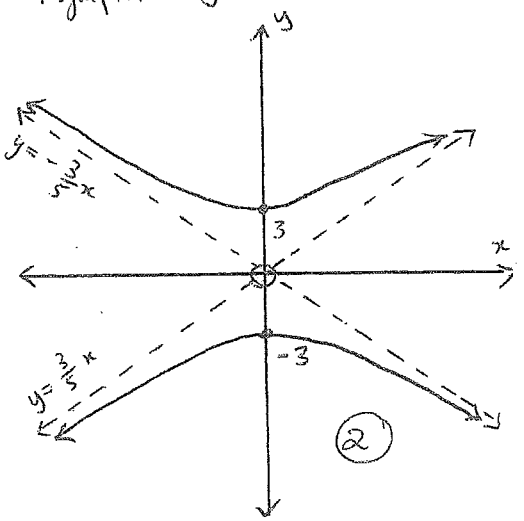
$= 3 - \frac{2}{x+2}$




(2)

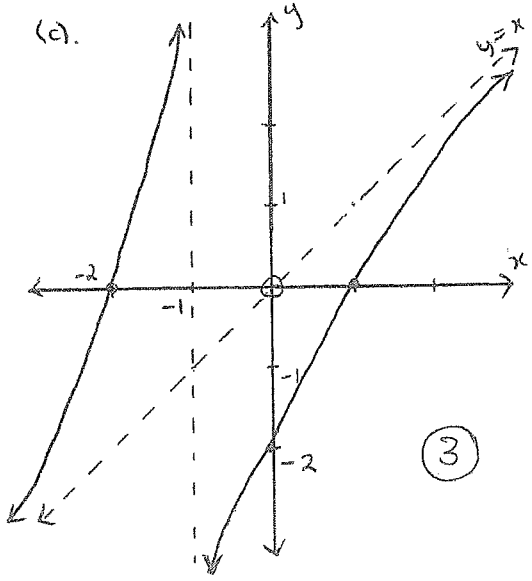
(b). $(5y+3x)(5y-3x) = 225$

Asymptotes $y = \pm \frac{3}{5}x$




(2)

(c).

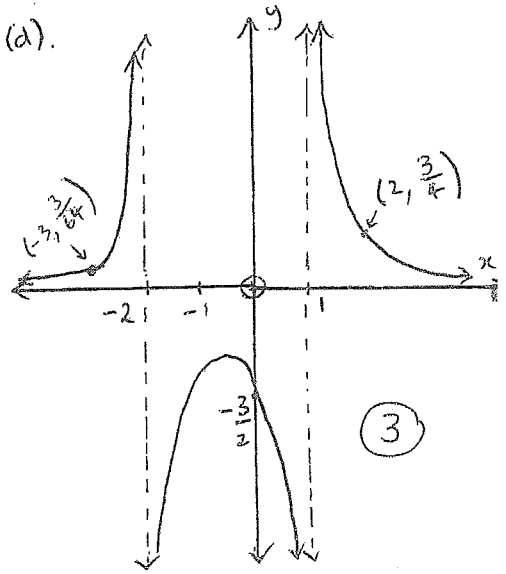



(3)

$y = x - \frac{2}{x+1}$

$= \frac{x^2+x-2}{x+1} = \frac{(x+2)(x-1)}{x+1}$

(d).

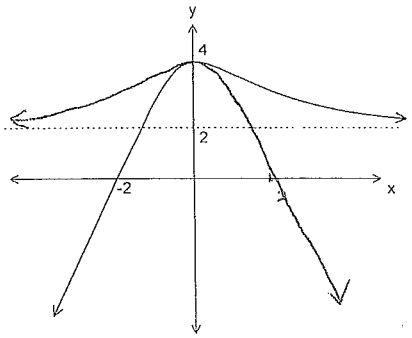



(3)

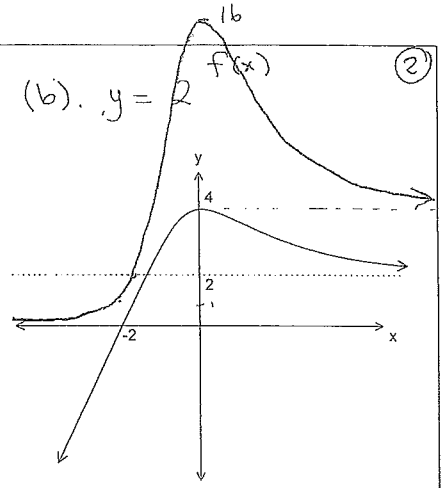
$y = \frac{3}{(x+2)(x-1)^3}$

## QUESTION 5

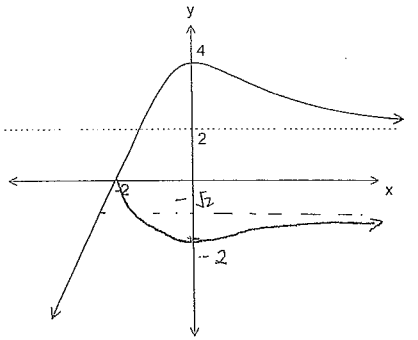
(a). $y = f(-x)$ (1)



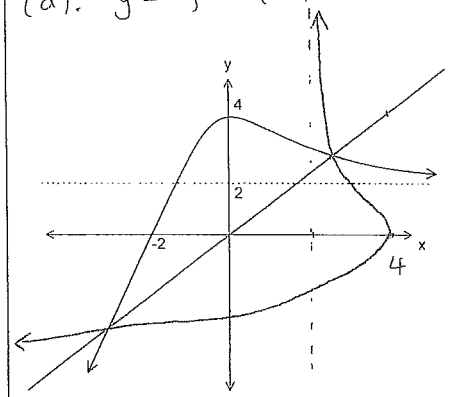

(b). $y = \frac{2}{f(x)}$ (2)



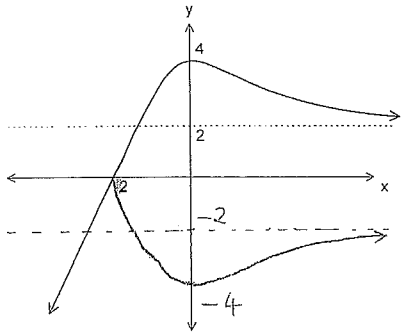

(c). $y = -\sqrt{f(x)}$ (2)




(d). $y = f^{-1}(x)$ (2)

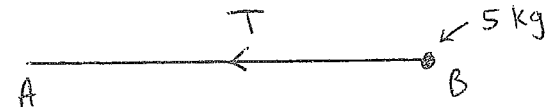



(e). $|y| = f(x)$ (1)




## QUESTION 6

(a).




$\omega = 45$ revs/min $\qquad r = 3$ m

$= \frac{45 \times 2\pi}{60}$ rad/s

$= \frac{3\pi}{2}$ rad/s

$T = mr\omega^2$ (3)

$= 5 \times 3 \times \left(\frac{3\pi}{2}\right)^2$

$\doteq \frac{135\pi^2}{4}$ N

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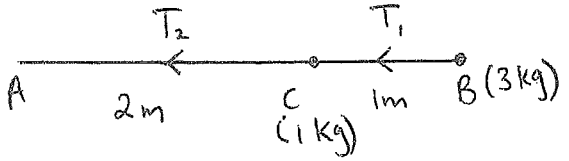
(b).




$T_1 = mr\omega^2$ $\qquad T_2 = 2 \times 1 \times \omega^2 + T_1$

$= 9\omega^2$ $\qquad = 11\omega^2$

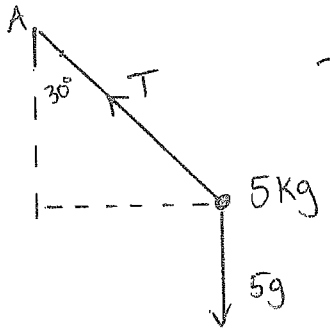
$11\omega^2 = \frac{135\pi^2}{4}$ (5)

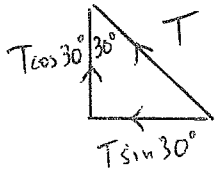
$\therefore \omega = \sqrt{\frac{135\pi^2}{44}}$ rad/s

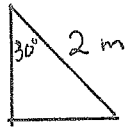
$= \sqrt{\frac{135\pi^2}{44}} \times \frac{60}{2\pi}$ rev/min.

$\omega \doteq 52.5$ rev/min.

## QUESTION 7




(a). Vert. $T\cos 30 = 5g$ (4)

$$T = \frac{5g}{\cos 30} = \frac{100}{\sqrt{3}}$$

$$T \doteq 57.7\ N$$

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(b). $T\sin 30 = \frac{mv^2}{r}$ $\quad \sin 30 = \frac{r}{2}$

$\therefore r = 1$

$$v^2 = \frac{rT\sin 30}{m}$$

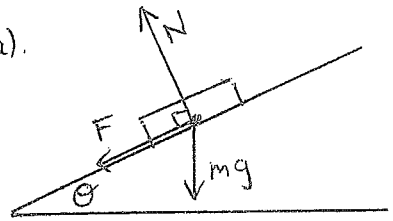
$$= \frac{100}{\sqrt{3}} \times \frac{1}{2} \times \frac{1}{5}$$

$$= \frac{10}{\sqrt{3}} \quad \therefore v = \sqrt{\frac{10}{\sqrt{3}}} \doteq 2.40\ m/s$$

(4)

## QUESTION 8

(a).




✓ ①

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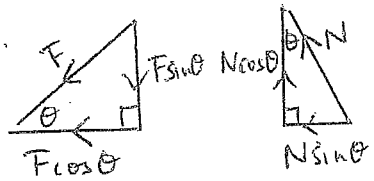
(b).




Hor. $N\sin\theta + F\cos\theta = \frac{mv^2}{r}$ ✓

Vert. (2)

$N\cos\theta - F\sin\theta - mg = 0$

i.e. $N\cos\theta - F\sin\theta = mg$ ✓

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(c). $N\sin\theta\cos\theta + F\cos^2\theta = \frac{mv^2}{r}\cos\theta$ (1)

$N\sin\theta\cos\theta - F\sin^2\theta = mg\sin\theta$ (2) ✓ ②

(1) − (2) $\quad \boxed{F = \frac{mv^2}{r}\cos\theta - mg\sin\theta}$ ✓

$N\sin^2\theta + F\sin\theta\cos\theta = \frac{mv^2}{r}\sin\theta$ (3)

$N\cos^2\theta - F\sin\theta\cos\theta = mg\cos\theta$ (4) ✓ ②

(3) + (4) $\quad \boxed{N = \frac{mv^2}{r}\sin\theta + mg\cos\theta}$ ✓

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(d). $r = 150$ m $\quad F = 0$ $\quad v = 120$ km/h $= 33\frac{1}{3}$ m/s ✓ $\quad \theta = ?$

$$F = \frac{mv^2}{r}\cos\theta - mg\sin\theta$$

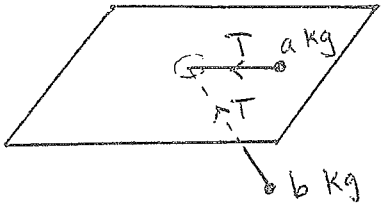
$$0 = m\left(\frac{(33\frac{1}{3})^2}{150}\cos\theta - 10\sin\theta\right) \checkmark$$

$$\therefore \tan\theta = \frac{(33\frac{1}{3})^2}{150 \times 10} \quad \therefore \theta \doteq 36°32' \checkmark$$ (3)

No Half marks

Total 10

## QUESTION 9




a kg → $\omega$ rad/s

b kg → $2\omega$ rad/s

$a : b = 5 : 2$

$\therefore 2a = 5b$

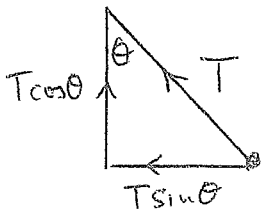
or $b = \frac{2a}{5}$ ✓

(a) kg

$T = mr\omega^2$

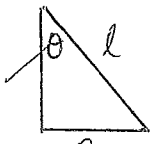
$= ar\omega^2$ ✓

(b) kg




$T\sin\theta = mr\omega^2$ ✓

$T \times \frac{R}{\ell} = \frac{2a}{5} \times R \times (2\omega)^2$ ✓




$\frac{ar\omega^2}{\ell} = \frac{8a\omega^2}{5}$

$\therefore \frac{r}{\ell} = \frac{8}{5}$ ✓

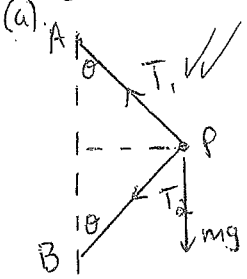
$\therefore \frac{8}{13}$ of string lies above the table ✓

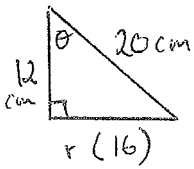
(7 marks)

Notes
- No half marks
- Methods of soln varied
  - eg - put $a=5$, $b=2$
  - Using $T_{below} = m\ell\omega^2$

## QUESTION 10

(a).



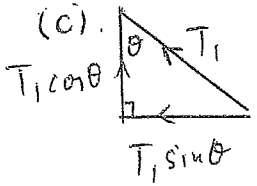



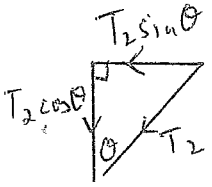

(b). $r = \sqrt{20^2 - 12^2}$ ✓

$\therefore r = 16$ cm or $0.16$ m (1)

(2)

(c).




Hor.

$T_1\sin\theta + T_2\sin\theta = \frac{mv^2}{r}$ ✓

Vert.

$T_1\cos\theta - T_2\cos\theta - mg = 0$ ✓

i.e. $T_1\cos\theta - T_2\cos\theta = mg$ (2)

---

(d). $\sin\theta = \frac{16}{20}$ or $\frac{4}{5}$ $\cos\theta = \frac{12}{20}$ or $\frac{3}{5}$ ✓

$(T_1 + T_2)\sin\theta = \frac{mv^2}{0.16}$ | $(T_1 - T_2)\cos\theta = mg$

$T_1 + T_2 = \frac{125mv^2}{16}$ ✓ (1) | $T_1 - T_2 = \frac{5}{3}$ (2)

(1) + (2) $2T_1 = \frac{125mv^2}{16} + \frac{5mg}{3}$ ✓ $\therefore T_1 = \frac{125mv^2}{32} + \frac{5mg}{6}$

(1) − (2) $2T_2 = \frac{125mv^2}{16} - \frac{5mg}{3}$ $\therefore T_2 = \frac{125mv^2}{32} - \frac{5mg}{6}$ ✓

(4)

---

(e). $T_2 > 0$ ✓ $\frac{125mv^2}{32} - \frac{5mg}{6} > 0$ ✓ No cfpa here

$\frac{125v^2}{32} > \frac{5g}{6}$

$v^2 > \frac{16g}{75}$

$v > \sqrt{\frac{g}{75}}$

$v > \frac{4\sqrt{g}}{5\sqrt{3}}$ (3)

i.e. $v > \frac{4\sqrt{3g}}{15}$ ✓

- No half marks. Total 12 marks