SYDNEY GIRLS HIGH SCHOOL

2012

YEAR 11

YEARLY EXAMINATION

Advanced Mathematics

Working time – 80 minutes (+ 5 minutes reading time)

General Instructions

- All necessary working should be shown in every question. Full marks may not be awarded for carelessly set out or incomplete work.
- Approved calculators may be used in all parts of the test.
- Start a new page for each question.

Total marks – 72

- There are 4 questions.
- Questions are of equal value.

Name:_____

Teacher (please circle):

Harbridge	Viswanathan
Damianos	Kalina
Ladmore	Brown
Stokes	Makar

Question 1: *(18 marks)*

a) Evaluate, correct to 2 significant figures: $\sqrt[4]{\dfrac{86.54\times 3.16}{2.6^3}}$ **2**

b) Factorise completely: $6x^2+11x-10$ **2**

c) Find all values of x for which $\tan x=-\dfrac{1}{\sqrt{3}}$, if $0°\le x\le 360°$. **2**

d) Find the values of a and b if $\dfrac{2\sqrt{3}+1}{\sqrt{3}-2}=a+b\sqrt{3}$ **2**

e) Find $\lim\limits_{x\to 2}\dfrac{x^2-4}{x-2}$. **2**

f) Find the exact area of equilateral triangle PQR, where $RQ=12$ cm. **2**

g) Simplify: $\dfrac{3^n\times 5^{2n}}{15^n}$ **2**

h) Solve for x: $|2x+5|=|3x-1|$ **2**

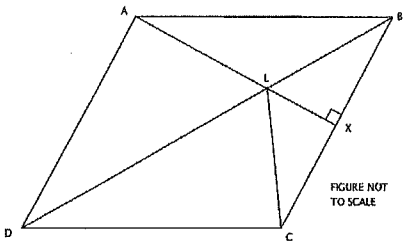
i) Show that the function $f(x)=\sqrt{1-x}$ is neither odd nor even. **2**

START QUESTION 2 ON A NEW PAGE

Question 2: *(18 marks)*

a) Find the gradient of the tangent to the curve $y = x^2 - \frac{2}{x}$ when $x = 4$. **3**

b) $ABCD$ is a rhombus. AX is perpendicular to BC and intersects BD at L.



(i) Prove $\Delta ALD \equiv \Delta CLD$. **2**

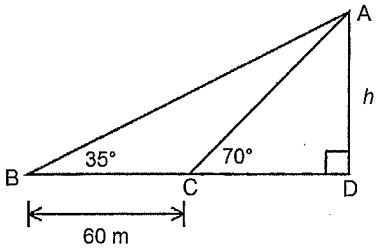
(ii) Hence, find the size of $\angle LCD$, giving reasons. **2**

c) If $\sec\alpha = -4$, and $\sin\alpha > 0$, find the exact value of $\tan\alpha$. **3**

d) Sketch $y = \cos x$ from $0° \le x \le 360°$. **2**

e) Find the perpendicular distance of $(2,-3)$ to $3x - 4y + 6 = 0$. **2**

f) Kim finds the angle of elevation of the top of a building to be 35°. After walking on horizontal ground for 60m towards the building, she now finds the angle of elevation to be 70°. Find the height, h, of the building to the nearest metre. **4**



START QUESTION 3 ON A NEW PAGE

Question 3: *(18 marks)*

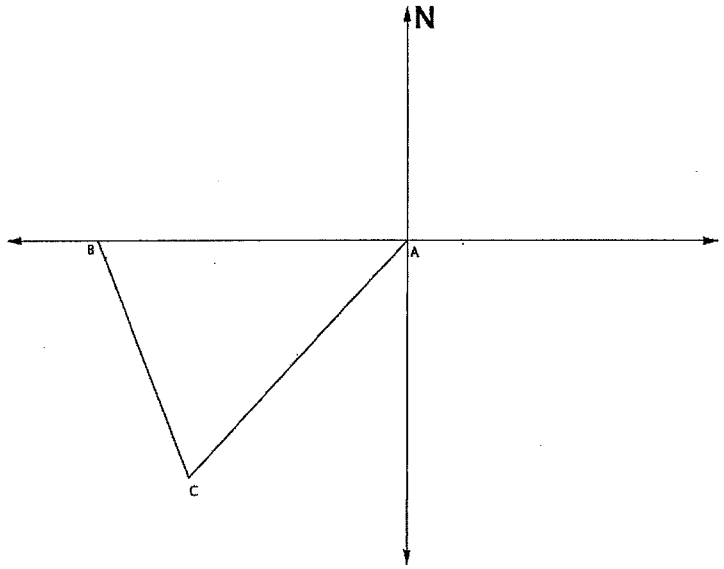
a) Given the points $A(-1,-4)$, $B(3,2)$, $C(-4,-6)$:

(i) find the gradient of AB **1**

(ii) find the equation of AB **1**

(iii) find the equation of the line which passes through C and is perpendicular to AB. **2**

b) A plane flies from an airport A 160 nautical miles due west to an airport B. It then flies on a bearing of 150° to an airport C. The bearing of C from A is 230°. Find the distance of AC, to the nearest nautical mile. **3**



c) On a number plane, sketch the region defined by the intersection of $y \le x^2 - 2x$ and $x + y - 2 > 0$ **3**

QUESTION 3 CONTINUES ON NEXT PAGE...

d) The sides of a triangle are in the ratio 5 :16 :19. Find the size of the smallest angle, to the nearest minute. **2**

e) If $f(x)=x^3+2x^2-15x+1$:

(i) find the coordinates of the turning points of $y=f(x)$ and determine their nature. **4**

(ii) sketch the graph of $y=f(x)$. **2**

START QUESTION 4 ON A NEW PAGE.

Question 4: *(18 marks)*

a) At what point on the curve $y=x\sqrt{x}$ is the tangent perpendicular to $2x+9y-9=0$? **3**

b) Prove $\dfrac{1}{\tan\theta-\sin\theta}=\dfrac{\cos\theta(1+\cos\theta)}{\sin^3\theta}$ **3**

c) Find the domain and range of $y=\sqrt{x^2-9}$. **3**

d) A coal shute is built in the shape of an upturned cone, in which the sum of the radius r and the height h is 12 metres.

(i) Show that the volume V of the shute is given by **2**

$V=4\pi r^2-\dfrac{1}{3}\pi r^3$.

(ii) Find the radius of the cone which yields the maximum volume. **4**

e) Factorise $a^2-b^2-c^2-2bc$ **3**

END OF TEST

Year 11

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2012

Q1 a) $\sqrt[4]{\frac{86.54 \times 3.16}{2.6^2}}$

$= 1.986076544$

$= 2.0$ (2 sig fig)

b) $6x^2 + 11x - 10$

$= (2x+5)(3x-2)$

c) $\tan x = \frac{-1}{\sqrt{3}}$

related Angle 30°

$180^\circ - 30^\circ$, $360^\circ - 30^\circ$

$\therefore 150^\circ, 330^\circ$

d) $\frac{2\sqrt{3}+1}{\sqrt{3}-2} \times \frac{\sqrt{3}+2}{\sqrt{3}+2}$

$= \frac{(2\sqrt{3}+1)(\sqrt{3}+2)}{(\sqrt{3}-2)(\sqrt{3}+2)}$

$= \frac{2\sqrt{3}(\sqrt{3}+2) + (\sqrt{3}+2)}{3-4}$

$= \frac{6+4\sqrt{3}+\sqrt{3}+2}{-1}$

$= \frac{8+5\sqrt{3}}{-1}$

$= -8 - 5\sqrt{3} = a + b\sqrt{3}$

$\therefore a = -8, \quad b = -5$

e) $\lim_{x \to 2} \frac{x^2-4}{x-2}$

$= \lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)}$

$= \lim_{x \to 2} x+2$

$= 2+2$

$= 4$

f) $A = \frac{1}{2} a b \sin\theta$

$= \frac{1}{2} \times 12 \times 12 \times \sin 60^\circ$

$= 72 \times \frac{\sqrt{3}}{2}$

$= 36\sqrt{3}$

g) $\frac{3^n \times 5^{2n}}{15^n}$

$= \frac{3^n \times 5^{2n}}{(3\times 5)^n}$

$= \frac{3^n \times 5^{2n}}{3^n \times 5^n}$

$= 5^{2n-n}$

$= 5^n$

h) $|2x+5| = |3x-1|$

$2x+5 = 3x-1$ or $2x+5 = -(3x-1)$

$6 = x$ $\quad 2x+5 = -3x+1$

$\therefore x = 6$ $\quad 5x = -4$

$x = \frac{-4}{5}$

$f(x) = \sqrt{1-x}$

$f(-x) = \sqrt{1--x}$

$= \sqrt{1+x}$

$\therefore f(x) \neq f(-x)$

Not even

also $f(-x) \neq -f(x)$

$\therefore$ Not odd

f) $\angle ACB = 110^\circ$ (Str. $\angle$)

$\angle BAC = 180 - 110 - 35$ ($\angle$ sum $\triangle$)

$= 35^\circ$

$\therefore \angle BAC = \angle ABC$

$\therefore \triangle ABC$ is isos. ($\angle$s equal, cs)

$\therefore AC = BC$ (sides of isos. $\triangle$)

$= 60$m

In $\triangle ACD$: $\sin 70^\circ = \frac{h}{AC}$

$h = AC \sin 70^\circ$

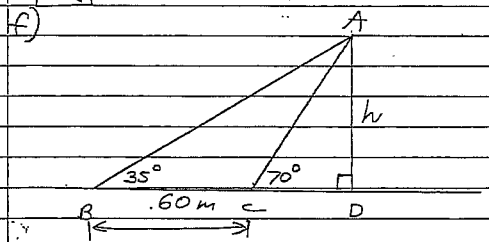
$= 60 \sin 70^\circ$

$\doteq 56$ m (to nearest m)

/4

OR

f)



In $\triangle ACD$: $\tan 70^\circ = \frac{h}{CD}$

$CD = \frac{h}{\tan 70^\circ}$

In $\triangle ABD$: $\tan 35^\circ = \frac{h}{60+CD}$

$h = (60+CD)\tan 35^\circ$

$= 60\tan 35^\circ + CD \tan 35^\circ$

$h = 60\tan 35^\circ + \frac{h}{\tan 70^\circ} \times \tan 35^\circ$

$h - \frac{h\tan 35^\circ}{\tan 70^\circ} = 60\tan 35^\circ$

$h\left(1 - \frac{\tan 35^\circ}{\tan 70^\circ}\right) = 60\tan 35^\circ$

$h = \frac{60\tan 35^\circ}{1 - \frac{\tan 35^\circ}{\tan 70^\circ}}$

$\doteq 56.38$ m

$\doteq 56$ m (to nearest m)

-1

Q2

a) $y = x^2 - \frac{2}{x}$

$= x^2 - 2x^{-1}$

$\frac{dy}{dx} = 2x + 2x^{-2}$

$= 2x + \frac{2}{x^2}$

when $x = -4$: $m = 2(-4) + \frac{2}{4^2}$

$= -8 + \frac{2}{16}$

$= -8\frac{1}{8}$

/3

$\angle LCD = \angle DAL$ (Corresp. $\angle$s of Congn. $\triangle$s)

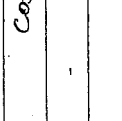
$= 90^\circ$

/2

c) $\sec x = -4$

$\frac{1}{\cos x} = -4$

$\cos x = -\frac{1}{4}$



$\cos x < 0$; 2nd + 3rd Quad

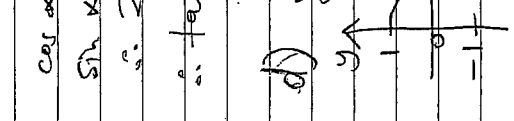
$\sin x > 0$, 1st + 2nd Quad

$\therefore$ 2nd Quad

$\therefore \tan x = -\sqrt{15}$

/3

d) $y = \cos x$



/2

b) i) In $\triangle$s ALD and CLD:

1. $AD = CD$ (sides in rhombus)

2. DL is common side

3. $\angle ADL = \angle CDL$ (diag. bisects $\angle ADC$)

$\therefore \triangle ALD \equiv \triangle CLD$ (SAS)

/2

ii) $\angle DAL = \angle AXB$

$= 90^\circ$

(Alt. $\angle$s, DA // CB)

e) $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \frac{|3(2) + -4(-3) + 6|}{\sqrt{9+16}}$

$= \frac{|6+12+6|}{\sqrt{25}}$

$= \frac{24}{5}$ units

/2

Question 3.

a) i) $m_1 = \frac{-4-2}{-1-3}$

$= \frac{-6}{-4}$ ①

$= \frac{3}{2}$

ii) $y-(-4) = \frac{3}{2}(x-(-1))$

$y+4 = \frac{3}{2}(x+1)$

$2y+8 = 3x+3$ ①

$0 = 3x-2y-5$

iii) $m_2 = -\frac{2}{3}$

$y-(-6) = -\frac{2}{3}(x-(-4))$

$y+6 = -\frac{2}{3}(x+4)$

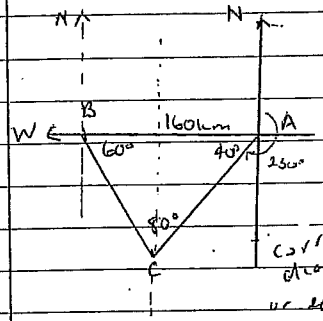
$3y+18 = -2x-8$

$2x+3y+26 = 0$ ②

1 mark for correct gradient

1 mark for correct resulting equation

b)



correct diagram or equivalent

(i) $\frac{AC}{\sin 60°} = \frac{160}{\sin 80°}$ ①

1 mark for correct ∠s in Δ

$AC = \frac{160 \sin 60°}{\sin 80°}$ (i)

1 mark for correct use of sine rule

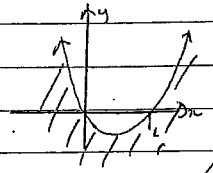
$= 140.7$ (i)

is 141 n. miles. ③

1 mark for correct evaluation

c) $y \leq x^2-2x$

$\leq x(x-2)$

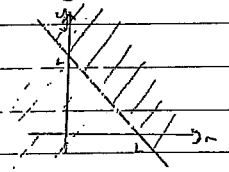


test (0,−1)

$-1 \leq 0$ ✓

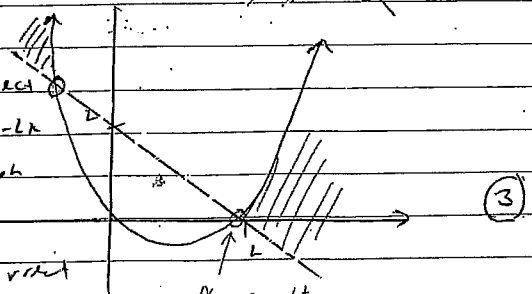
$x+y-2 \geq 0$

$x+y \geq 2$



test (0,0)

$0+0 \not\geq 2$

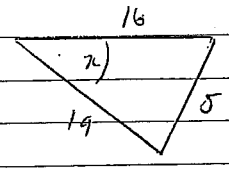


1 mark for correct graph of $y = x^2-2x$

1 mark for graph of $x+y=2$

1 mark for correct diagram showing both regions ③

d)



Smallest ∠ opp smallest side

$\cos x° = \frac{16^2+19^2-5^2}{2\times 16\times 19}$ ②

$x = 13°10'$

1 mark for correct expression

1 mark for correct evaluation

e) i) $f(x) = x^3+2x^2-15x+1$

$f'(x) = 3x^2+4x-15$ $f''(x) = 6x+4$

for a stn. pt $\frac{dy}{dx} = 0$

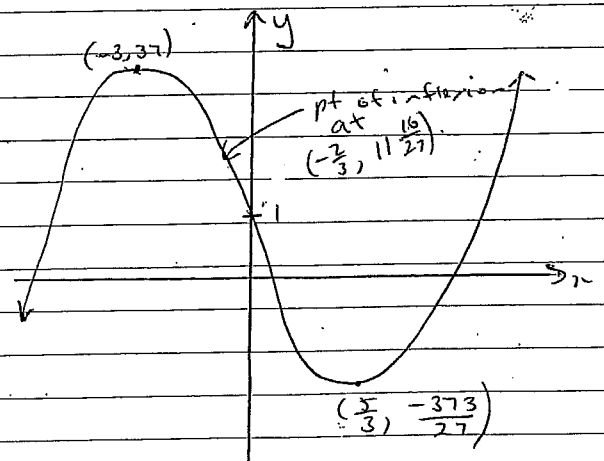
$3x^2+4x-15 = 0$

$(3x-5)(x+3) = 0$

$x = -3$, $y = 37$, $\frac{dy}{dx} < 0$, ∴ max } 2 marks

or $x = \frac{5}{3}$, $y = -\frac{373}{27}$, $\frac{dy}{dx} > 0$, ∴ min } 2 marks. (−13·81)

ii)



1 mark for correct shape of graph

1 mark for y int. or pt of inflexion ✓

QUESTION 4 (18 MARKS)

(a) $y = x^{\frac{3}{2}}$

$y' = \frac{3}{2}x^{\frac{1}{2}}$ (1) ← OR

$\therefore \frac{3\sqrt{x}}{2} = \frac{9}{2}$ (1)

$\sqrt{x} = 3 \qquad x = 9$

$\therefore (9, 27)$ (1)

$9y = -2x + 9$

$y = -\frac{2}{9}x + 1$

$m_1 = -\frac{2}{9}$

$m_2 = \frac{9}{2}$

3

(b) $LHS = \dfrac{1}{\tan\theta - \sin\theta}$

$= \dfrac{1}{\frac{\sin\theta}{\cos\theta} - \sin\theta}$ (1)

$= \dfrac{\cos\theta}{\sin\theta - \sin\theta\cos\theta}$ (1)

$= \dfrac{\cos\theta\,(1+\cos\theta)}{\sin\theta(1-\cos\theta)(1+\cos\theta)}$

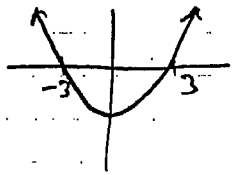
$= \dfrac{\cos\theta\,(1+\cos\theta)}{\sin\theta\,(1-\cos^2\theta)}$ (1)

$= \dfrac{\cos\theta(1+\cos\theta)}{\sin^3\theta} = RHS$

3

(c) D: $x^2 - 9 \geq 0$ (1)

$\therefore x \geq 3$ or $x \leq -3$ (1)



R: $y \geq 0$ (1) 3

QUESTION 4 (continued)

(d) $r + h = 12$

(i) $V = \frac{1}{3}\pi r^2 h$ OR $h = 12 - r$ (1)

$= \frac{1}{3}\pi r^2(12 - r)$ (1)

$\therefore V = 4\pi r^2 - \frac{\pi r^3}{3}$

2

(ii) $r = ?$ for max. V (1) $V' = 8\pi r - \pi r^2$

$V'' = 8\pi - 2\pi r$

Stat. pt. when $V' = 0$

$8\pi r - \pi r^2 = 0$ (1)

$\pi r(8 - r) = 0 \quad \therefore r = 0$ or $r = 8$

$r > 0 \quad \therefore r = 8$ (1)

when $r = 8 \quad V'' = 8\pi - 16\pi$

$V'' = -8\pi < 0$ (1)

$\therefore$ maximum volume when $r = 8$

4

(e) $a^2 - b^2 - c^2 - 2bc$

$= a^2 - (b^2 + 2bc + c^2)$ (1)

$= a^2 - (b + c)^2$ (1)

$= (a + (b+c))(a - (b+c))$ (1)

$= (a + b + c)(a - b - c)$

3