SYDNEY GIRLS HIGH SCHOOL

2004 HSC Assessment Task 1

November 26, 2004

MATHEMATICS Extension 2

Year 12

Time allowed: 90 minutes

Topics: Curve Sketching, Circular Motion

DIRECTIONS TO CANDIDATES:

- Attempt all questions
- Questions are not of equal value
- There are 10 questions
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work
- Board approved calculators may be used
- Use g = 10 $m.s^{-2}$
- Write on one side of the paper only

Extension Two – Assessment 1 November 2004

1. Sketch the following curves showing any important features [8 marks]

   a) $y=-2^{-x}$

   b) $\dfrac{x^2}{4}+\dfrac{y^2}{1}=1$

   c) $\dfrac{x^2}{4}-\dfrac{y^2}{1}=1$

   d) $|2x-3y|=6$

2. a) Sketch the curve $f(x)=\dfrac{2x}{x-1}$ [10 marks]

   b) Find $f'(x)$ and hence explain why $f(x)$ has an inverse function

   c) Determine the equation of the inverse function $f^{-1}(x)$

   d) Sketch the graph of $f^{-1}(x)$ on the same set of axes as $f(x)$

   e) State any points of intersection between $f(x)$ and $f^{-1}(x)$.

3. Sketch the following curves showing the nature of any roots or asymptotes

   a) $y=(1-x)^2(3-x)^2(5-x)^3$

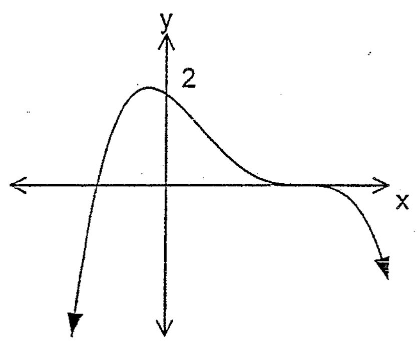
   b) $y=\dfrac{x^2}{x-1}$

   c) $y=\dfrac{x}{(x-1)^2}$

   d) $y=\dfrac{x-1}{x(x-2)^2}$

   e) $y=\dfrac{x^2}{(x-2)^2}$ [14 marks]

4. The diagram shows a function y = f(x).



On separate sets of axes, sketch

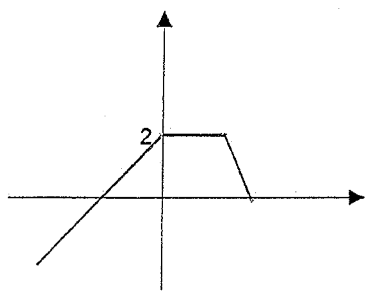
a) $y=\left(f(x)\right)^2$

b) $y=f|x|$

c) $y^2=f(x)$

d) $y=2^{f(x)}$ [12 marks]

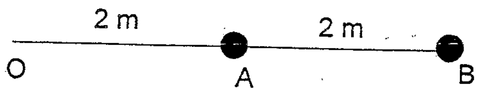
5. The diagram shows a function y = f(x). Reproduce this function and on the same set of axes sketch



a) $y=\dfrac{1}{f(x)}$

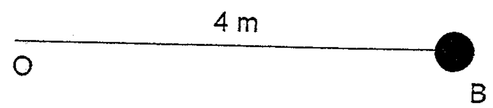
b) $y=\sqrt{f(x)}$ [6 marks]

6. A light string of length 4 metres has a 2 kilogram mass attached at A (at the centre) and a second 2 kilogram mass at B (the end). The string is rotated in a horizontal circle about O.



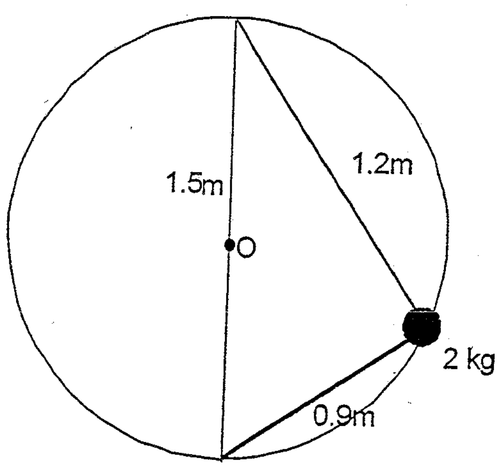
a) With the masses connected as shown in the diagram the maximum speed of rotation is 4 $rev.sec^{-1}$ before the string breaks. Find the maximum tension possible in the string.

b) If the mass at A is moved to join the mass at B (resulting in a 4 kg mass at B), what is the new maximum speed of rotation of the combined masses.



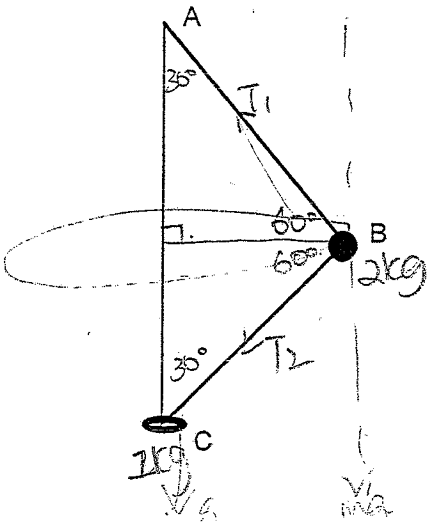
[6marks]

7. A smooth disc of diameter 1.5 m has a mass of 2kg on its edge held in place by two light strings of length 1.2m and 0.9m. If the disc and mass are rotating in a horizontal circle at 4 $rad.sec^{-1}$, find the tension in each of the strings.



[8 marks]

8. A 4 metre string is attached to a vertical rod AC at A and C. The string has a 2kg mass attached at its centre B and a 1 kg mass attached at C. The mass at C is free to move smoothly up or down the rod AC

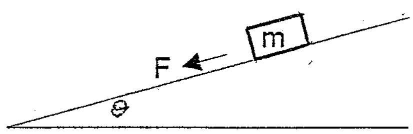


Find ω

[10 marks]

The mass at B is rotating around the vertical rod in a horizontal circular motion. If $< ABC = 120^{o}$, find the angular velocity in rad.sec^{-1} that keeps the system in equilibrium (i.e C does not move).

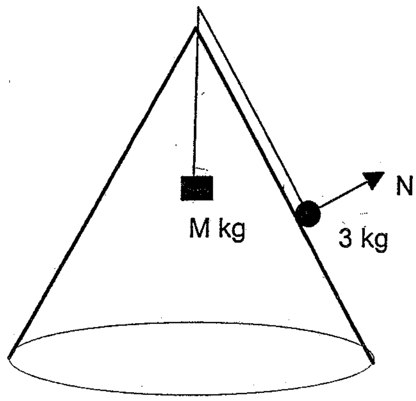
9. A track is banked at an angle of θ. A car of mass m is traveling around the track of radius r metres at v metres/second. The sideways frictional force is shown.



a) Copy down the diagram and mark in the correct position of mg, N and $\dfrac{mv^2}{r}$ on the diagram.
b) Resolve vertically and horizontally and then solve to find expressions for F and N.
c) If the radius of the track is 1kilometre and a car of mass 1.2 tonnes negotiates the corner at 72 km/hr, the sideways frictional force is zero. Find the angle of banking.
d) If the car stops on the corner find the normal force and the sideways frictional force F.
e) If the car travels at 108 km/hr around the corner find the sideways frictional force and indicate if this force is up or down the slope.

[14 marks]

10. An inverted cone has a semi –vertical angle of $30°$. Two masses are connected by a 3 metre string. On one end inside the cone is a mass of M kg hanging in a stationary position. At the other end is a mass of 3 kg rotating about the cone in a horizontal circle at 2 rad.sec^{-1}. The length of the string inside the cone is 1 metre.



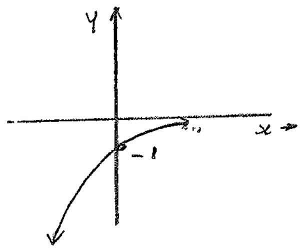
Find the exact value of

a) The normal force exerted by the cone on the rotating mass.
b) The tension in the string
c) The size of M, the stationary mass.
d) The speed in rad.sec^{-1} the outside mass would need to travel at for the system to remain in equilibrium if the lengths of the strings are reversed (so that 2 metres is inside the cone and 1 metre outside)

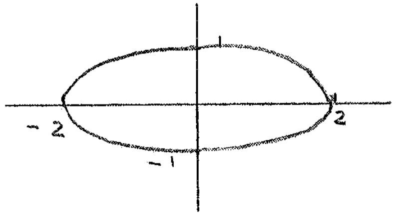
[12 marks]

4U - Nov 2004

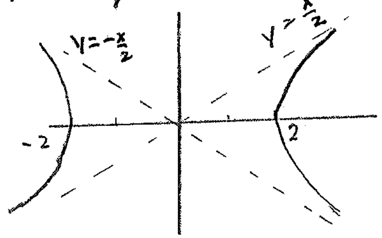
1a) $y = -2^{-x}$



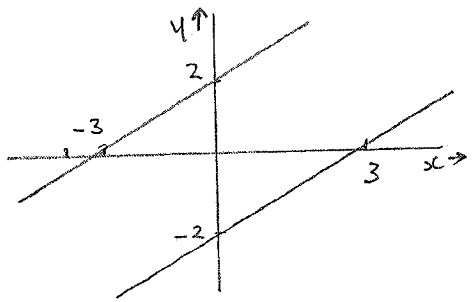
b) $\frac{x^2}{4} + \frac{y^2}{1} = 1$



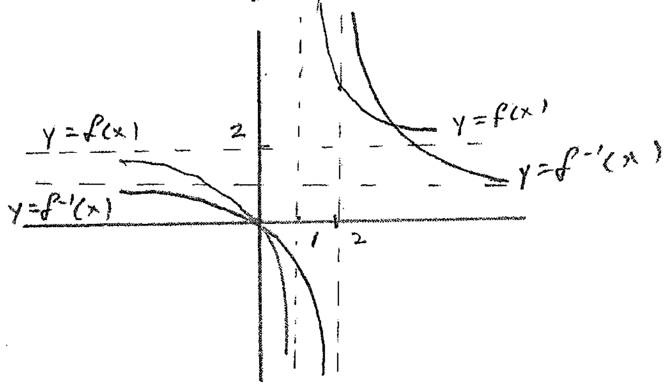
c) $\frac{x^2}{4} - \frac{y^2}{1} = 1$



d) $|2x - 3y| = 6 \Rightarrow 2x - 3y = 6$
$2x - 3y = -6$



2a) $f(x) = \frac{2x}{x-1}$



b) $f'(x) = \frac{(x-1)2 - 2x(1)}{(x-1)^2}$

$= \frac{2x - 2 - 2x}{(x-1)^2}$

$= \frac{-2}{(x-1)^2}$

Since $f'(x) \neq 0$, there are no turning points & an inverse function exists

c) $x = \frac{2y}{y-1}$

$xy - x = 2y$

$xy - 2y = x$

$y(x-2) = x \quad \therefore y = \frac{x}{x-2}$

i.e. $f^{-1}(x) = \frac{x}{x-2}$

e) Let $f(x) = x$

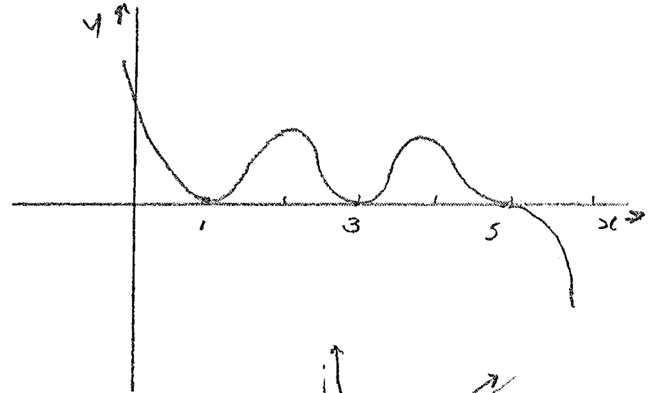
$\therefore \frac{2x}{x-1} = x$

$2x = x^2 - x$

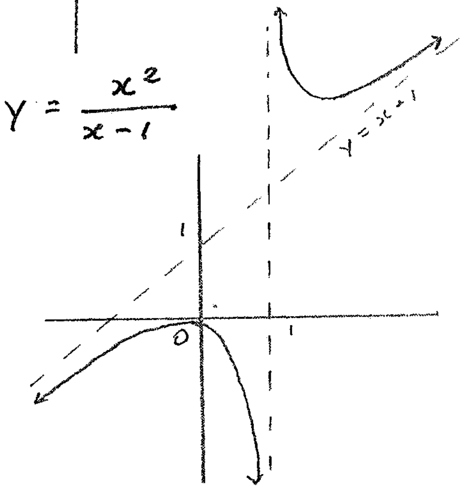
$\therefore 0 = x^2 - 3x$

$\therefore x = 0, 3 \quad \therefore$ pts are $(0,0)$ $(3,3)$

3a) $y = (1-x)^2(3-x)^2(5-x)^3$

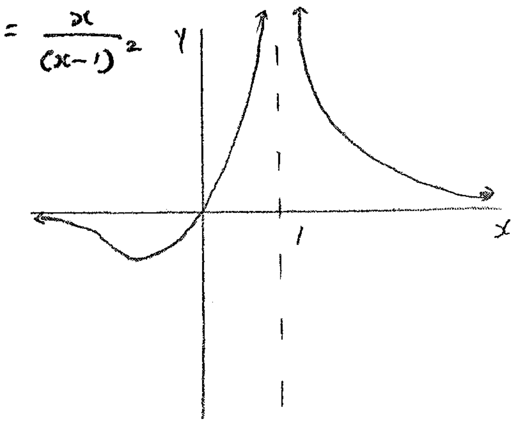


b) $y = \frac{x^2}{x-1}$

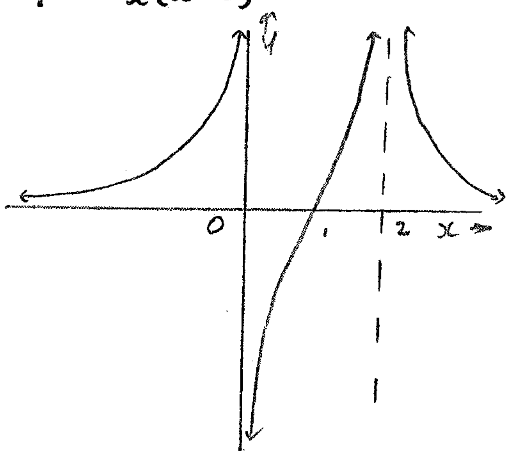


$$\begin{array}{r} x+1 \\ x-1\,\overline{)\,x^2+0x+0} \\ x^2 - x \\ x + 0 \\ x - 1 \\ \hline 1 \end{array} \quad = x + 1 + \frac{1}{x-1}$$

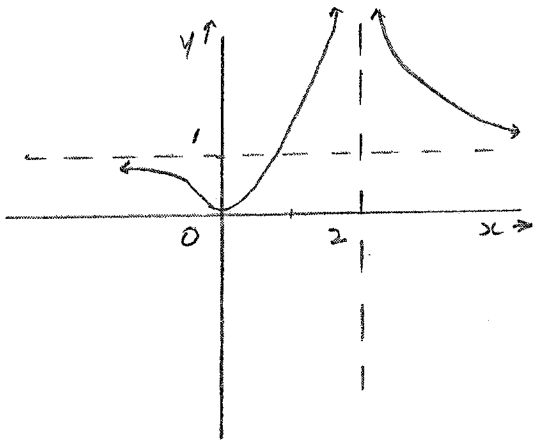
c) $y = \frac{x}{(x-1)^2}$



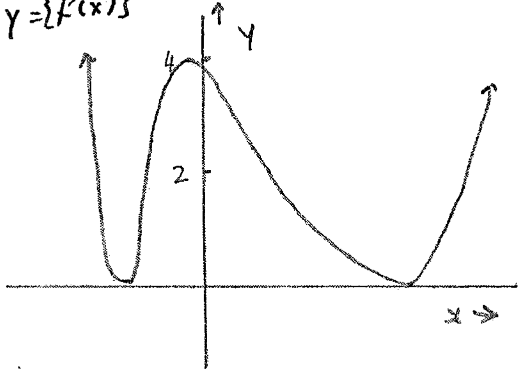
) $y = \dfrac{x-1}{x(x-2)^2}$



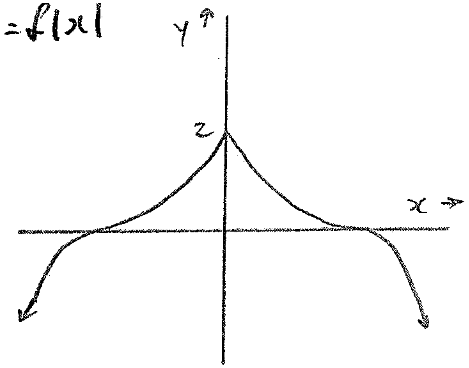
$y = \dfrac{x^2}{(x-2)^2}$



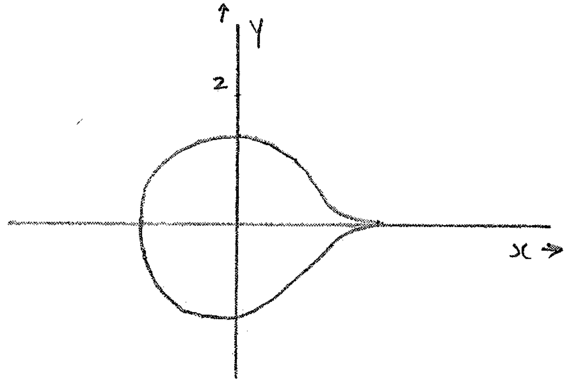
a) $y = \{f(x)\}^2$



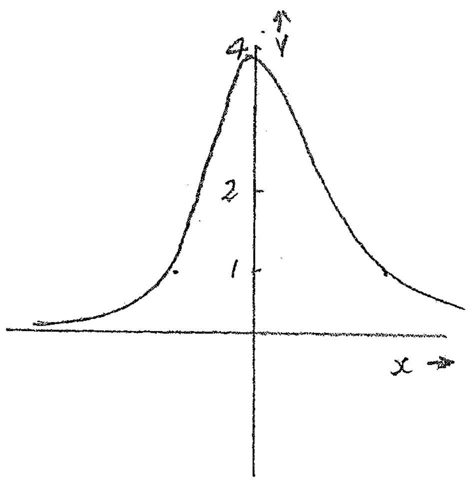
b) $y = f|x|$



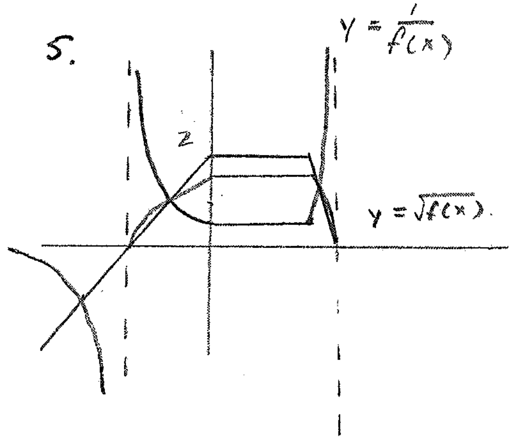
c) $y^2 = f(x)$



d) $y = 2^{f(x)}$



5.



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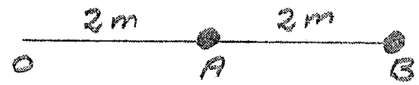


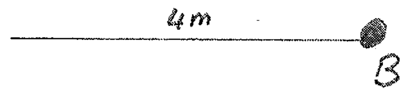
4 rev/sec = 8π rad/sec.

$T = T_A + T_B$

$T_A = m r_1 w^2 = 2 \times 2 \times 64\pi^2 = 256\pi^2$

$T_B = m r_2 w^2 = 2 \times 4 \times 64\pi^2 = 512\pi^2$

$\therefore T = 192\pi^2$ Newtons



$T = m r w^2$

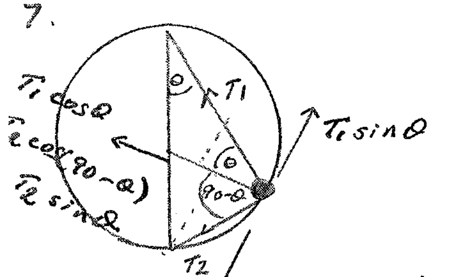
$768\pi^2 = 4 \times 4 \times w^2$

$w^2 = 48\pi^2$

$w = 4\sqrt{3}\,\pi$

$\therefore$ avg. speed $= 2\sqrt{3}$ rev/sec.

7.



$T_2\sin(90-\theta) = T_2\cos\theta$

$T_1\sin\theta = T_2\cos\theta$, $T_1\cos\theta + T_2\sin\theta = mr\omega^2$

$\therefore T_1(\frac{3}{5}) = T_2(\frac{4}{5})$ $\quad T_1(\frac{4}{5}) + T_2(\frac{3}{5}) = 2\times\frac{1.5}{2}\times 16$

$\therefore 3T_1 = 4T_2$, $\quad 4T_1 + 3T_2 = 5\times 24$

$4T_1 + 3T_2 = 120$

$T_1 = \frac{4T_2}{3} \rightarrow \quad \therefore 4\cdot\frac{4T_2}{3} + 3T_2 = 120$

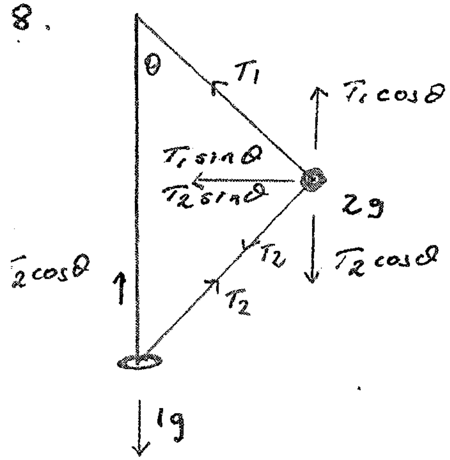
$16T_2 + 9T_2 = 360$

$T_2 = \frac{360}{25}$

$\therefore T_2 = 14.4\ N$

$T_1 = \frac{4}{3}\times 14.4\ N = 19.2\ N$

8.



$2g + T_2\cos\theta = T_1\cos\theta$

$\therefore 20 = T_1\cos\theta - T_2\cos\theta$

$\cos\theta = \cos 30 = \sqrt{3}/2$

$\therefore 20 = T_1(\frac{\sqrt3}{2}) - T_2(\frac{\sqrt3}{2})$

$40 = T_1\sqrt3 - T_2\sqrt3 \quad (1)$

$T_2\cos\theta = 1g = 10$

$\therefore T_2(\sqrt3/2) = 10$

$T_2 = 20/\sqrt3 \quad (2)$

Sub in (1)

$40 = T_1\sqrt3 - \frac{20}{\sqrt3}\cdot\sqrt3$

$\therefore 60 = T_1\sqrt3$

$T_1 = \frac{60}{\sqrt3} = 20\sqrt3$

$mr\omega^2 = T_1\sin\theta + T_2\sin\theta$

$\therefore 2\times1\times\omega^2 = 20\sqrt3\cdot\frac12 + \frac{20\sqrt3}{3}\times\frac12$

$\therefore 2\omega^2 = 10\sqrt3 + \frac{10\sqrt3}{3}$

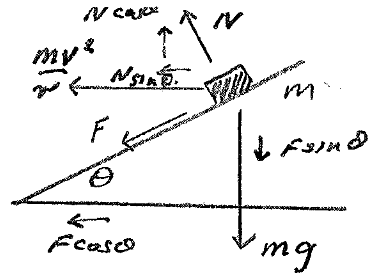
$= \frac{40\sqrt3}{3}$

$\therefore \omega^2 = \frac{20\sqrt3}{3}$

$\omega = \sqrt{\frac{20\sqrt3}{3}}$

$\therefore \omega \doteqdot 3.40$ rad/sec.

9.



$mg + F\sin\theta = N\cos\theta$

$\therefore mg = N\cos\theta - F\sin\theta \quad (1)$

$\frac{mv^2}{r} = N\sin\theta + F\cos\theta \quad (2)$

(1) x cos, (2) x sin

$mg\cos\theta = N\cos^2\theta - F\sin\theta\cos\theta$

$\frac{mv^2}{r}\sin\theta = N\sin^2\theta + F\sin\theta\cos\theta$

adding: $N = mg\cos\theta + \frac{mv^2}{r}\sin\theta$

(1) x sin, (2) x cos

$mg\sin\theta = N\cos\theta\sin\theta - F\sin^2\theta$

$\frac{mv^2}{r}\cos\theta = N\sin\theta\cos\theta + F\cos^2\theta$

Subtracting: $F = \frac{mv^2}{r}\cos\theta - mg\sin\theta.$

$r = 1000$, $v = 20$, $g = 10$, $F = 0$:

$\tan\theta = \frac{v^2}{rg} = \frac{400}{1000\times10} = \frac{1}{25}$

$\therefore \theta \doteqdot 2^\circ 17'$

iv) at $v = 0$, $N = mg\cos\theta$

$= 1200\times10\times\cos 2^\circ 17'$

$\doteqdot 11990$ Newtons

$F = |-mg\sin\theta|$

$\doteqdot 480$ Newtons

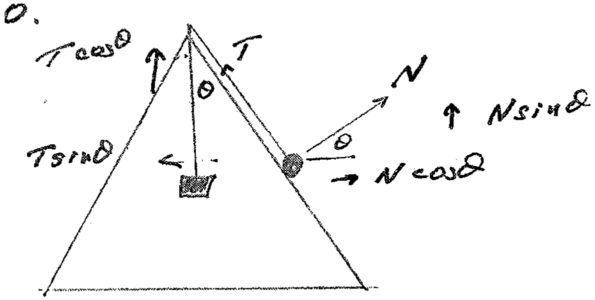
1) at $V = 30$,

$F = 1200\left(\frac{900}{1000}\cos 2°17' - 10\sin 2°17'\right)$

$\doteqdot 601\ N$

Since F is positive, the force of friction is down the slope.

0.



$mr\omega^2 = T\sin\theta - N\cos\theta$

$mg = T\cos\theta + N\sin\theta$

$m = 3,\ r = 1,\ \omega = 2$

$3 \times 4 = T\left(\frac{1}{2}\right) - N\left(\sqrt{3}/2\right)$

$\therefore 24 = T - N\sqrt{3}$ (1)

$3 \times 10 = T\left(\frac{\sqrt{3}}{2}\right) + N\left(\frac{1}{2}\right)$

$\therefore 60 = T\sqrt{3} + N$ (2)

$\therefore 60\sqrt{3} = 3T + N\sqrt{3}$ (2a)

Adding:

$24 + 60\sqrt{3} = 4T$

& $T = 6 + 15\sqrt{3}$

Sub into (1)

$24 = 6 + 15\sqrt{3} - N\sqrt{3}$

$18 = 15\sqrt{3} - N\sqrt{3}$

$\therefore N = 15 - 6\sqrt{3}$

c) $T = Mg$

$\therefore M = \frac{6 + 15\sqrt{3}}{10}$

d) Strings Reversed

$r_2 = 1/2,\ T = 6 + 15\sqrt{3}$

$m = 3,\ N = ?,\ \omega_2 = ?$

$3 \times \frac{1}{2} \times \omega^2 = T\left(\frac{1}{2}\right) - N\left(\sqrt{3}/2\right)$

$3\omega^2 = T - N\sqrt{3}$

& $3 \times 10 = T\left(\sqrt{3}/2\right) + N\left(\frac{1}{2}\right)$

$60 = T\sqrt{3} + N$

If $T = 6 + 15\sqrt{3}$,

$N = 60 - \sqrt{3}(6 + 15\sqrt{3})$

$= 15 - 6\sqrt{3}$

$\therefore 3\omega^2 = 6 + 15\sqrt{3} - \sqrt{3}(15 - 6\sqrt{3})$

$= 6 + 15\sqrt{3} - 15\sqrt{3} + 18$

$= 24$

$\therefore \omega^2 = 8$

$\therefore \omega = 2\sqrt{2}$ rad/sec.