

**SYDNEY GRAMMAR SCHOOL**

5G 2006

**Algebra, Numbers and Functions**

15th February

1. Which of the following are irrational numbers  $\pi$ ,  $\frac{22}{7}$ ,  $-1.5$ ,  $0.\dot{3}$ ,  $\tan 30^\circ$ .
2. Simplify  $(3\sqrt{6})^2$ .
3. Simplify  $\frac{6\sqrt{18}}{2\sqrt{2}}$ .
4. Find  $0.2\dot{3}\dot{7}$  as a simplified fraction.
5. Simplify  $\sqrt{54} + 2\sqrt{12} - \sqrt{24} - \sqrt{27}$ .
6. Find the value of  $c$  and  $d$  if  $(2\sqrt{3} + 4)^2 = c + d\sqrt{2}$  where  $c$  and  $d$  are rational.
7. Simplify  $\frac{\sqrt{2}}{1-\sqrt{2}} - \frac{\sqrt{2}}{1+\sqrt{2}}$ .
8. Find the value of  $a$  and  $b$  if  $\frac{2}{\sqrt{5}-1} = a + \sqrt{b}$  where  $a$  and  $b$  are rational.
9. On separate axes sketch the following relations.
 

(a) $y = 3x$	(d) $y = 9 - x^2$
(b) $2x + 3y = 6$	(e) $(x-3)^2 + y^2 = 9$
(c) $y = 3^{-x}$	(f) $y = \frac{4}{x}$
10. Sketch the function  $y = \sqrt{9-x^2}$ . State its domain and range.
11. Sketch the parabola  $y = x^2 - 6x + 8$  showing the roots,  $y$ -intercept and vertex.
12. Sketch the hyperbola  $y = 2 + \frac{4}{x-1}$  showing the asymptotes and intercepts with the axes.
13. Find the inverse function of  $f(x) = \frac{x}{2x-1}$ . State the domain and range of the inverse function.
14. On separate axes sketch:
 

(a) $y = \sqrt{x}$ ,	(c) $y = \sqrt{x+4}$ ,
(b) $y = -\sqrt{x}$ ,	(d) $y = -\sqrt{x-4}$ ,

Solutions to Test

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- $\pi$  and  $\tan 30^\circ = \frac{1}{\sqrt{3}}$  are irrational ✓
- $(3\sqrt{6})^2 = 9 \times 6 = 54$  ✓
- $\frac{6\sqrt{18}}{2\sqrt{2}} = 3\sqrt{9} = 9$  ✓✓

4. Let  $x = 0.237$

$$\begin{aligned} 1000x &= 237 \cdot \frac{37}{100} \\ 10x &= 2 \cdot 37 \\ 990x &= 235 \\ x &= \frac{235}{990} = \frac{47}{198} \end{aligned}$$

✓✓✓

5.  $\begin{aligned} \sqrt{54} + 2\sqrt{12} - \sqrt{24} - \sqrt{27} \\ = \sqrt{9}\sqrt{6} + 2\sqrt{4}\sqrt{3} - \sqrt{4}\sqrt{6} - \sqrt{9}\sqrt{3} \\ = 3\sqrt{6} + 4\sqrt{3} - 2\sqrt{6} - 3\sqrt{3} \\ = \sqrt{6} + \sqrt{3} \end{aligned}$

✓✓✓

6.  $(2\sqrt{3} + 4)^2 = c + d\sqrt{3}$

$$\begin{aligned} 12 + 16\sqrt{3} + 16 &= 28 + 16\sqrt{3} = c + d\sqrt{3} \\ c = 28 \text{ and } d = 16 \end{aligned}$$

✓✓

7.  $\frac{\sqrt{2}}{1-\sqrt{2}} - \frac{\sqrt{2}}{1+\sqrt{2}}$

$$\begin{aligned} &= \frac{\sqrt{2} + \sqrt{4} - \sqrt{2}(1-\sqrt{2})}{1-2} \\ &= \frac{\sqrt{2} + 2 - \sqrt{2} + 2}{-1} \\ &= -4. \quad \checkmark \end{aligned}$$

So  $a = \frac{1}{2}, b = \frac{5}{4}$

✓

11.  $y = x^2 - 6x + 8$   
 $y = (x-4)(x-2)$  ✓

Roots where  $x = 2, 4$

Vertex where  $x = 3$  ✓  
 $y = 9 - 18 + 8 = -1$

12.  $y = 2 + \frac{4}{x-1}$

when  $x = 0, y = 2 - 4 = -2$  ✓

when  $y = 0, \frac{4}{x-1} = -2$   
 $4 = -2x + 2$   
 $x = -1$  ✓

13. Let  $y = \frac{x}{2x-1}$

Swap  $x$  and  $y$ :  $x = \frac{y}{2y-1}$  ✓

$$\begin{aligned} 2xy - x &= y \\ 2xy - y &= x \\ y(2x-1) &= x \\ y &= \frac{x}{2x-1} \end{aligned}$$

✓

So  $f^{-1}(x) = \frac{x}{2x-1}$

$D: \text{all } x \neq \frac{1}{2}$

$R: \text{all } y \neq \frac{1}{2}$  ✓

