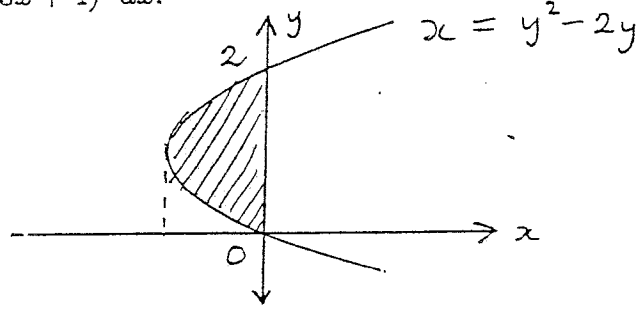
1. If $\frac{dy}{dx} = 6x^2 - 4x + 1$, find an expression for y in terms of x if $y = -1$ when $x = 0$.

2. Evaluate the following:

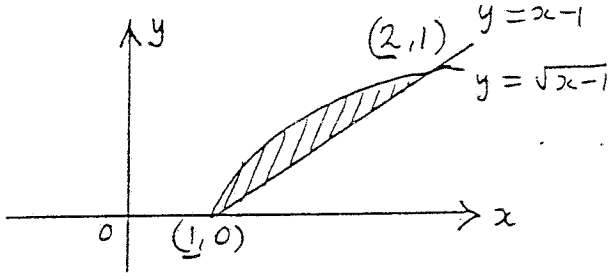
   (a) $\int_0^2 (5x^2 - 1)^2 \, dx$,

   (b) $\int_{-1}^3 (\frac{1-x}{x^3}) \, dx$.

3. Find $\int (3x + 4)^7 \, dx$.

4. 

   In the diagram above find the shaded area.

5. 

   The diagram shows the area enclosed between the curves $y = x - 1$ and $y = \sqrt{x - 1}$. If this enclosed area is rotated about the x-axis, find the volume of the solid formed.

6. A rectangular sheet of cardboard measures 16 cm by 6 cm. Equal squares, sides x cm, are cut out at the corners of the rectangle and the sides are turned up to form an open box.

   (a) Show that the volume, $V cm^3$, of the box is given by $V = 4x^3 - 44x^2 + 96x$.

   (b) Find the maximum volume of the box.

7. A sphere is formed by rotating the circle $x^2 + y^2 = 1$ about the x-axis. The sphere is cut into two portions by a vertical plane distant h from the centre of the sphere. If the volumes of the two portions are in the ratio 2 : 1, show that h is a root of the equation $3h^3 - 9h + 2 = 0$.

(1) $y = 2x^3 - 2x^2 + x + c$

$-1 = 0 - 0 + 0 = c$

$y = 2x^3 - 2x^2 + x - 1$ (2)

(2) (a) $\int_0^2 (25x^4 - 10x^2 + 1)\,dx$

$= [5x^5 - \frac{10}{3}x^3 + x]_0^2$

$= 5\times 2^5 - \frac{10}{3}\times 2^3 + 2 - 0$

$= 135\frac{1}{3}$

(b) $\int_{-1}^{3} (x^{-3} - x^{-2})\,dx$

$= \left[-\frac{1}{2x^2} + \frac{1}{x}\right]_{-1}^{3}$ (4)

$= -\frac{1}{18} + \frac{1}{3} + \frac{1}{2} + 1$

$= 1\frac{7}{9}$

(3) $\frac{(3x+4)^8}{24} + c$ (2)

(4) $A = \int_0^2 (y^2 - 2y)\,dy$

$= \left[\frac{1}{3}y^3 - y^2\right]_0^2$ (3)

$= -\frac{4}{3}$ units^2

(5) $V = \pi\int_1^2 (x-1) - (x-1)^2\,dx$

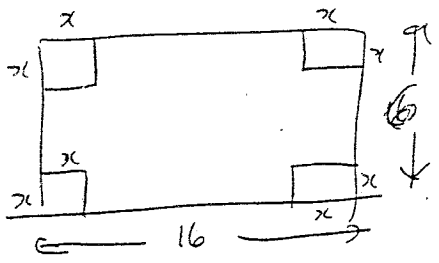
$= \pi\int_1^2 (-x^2 + 3x - 2)\,dx$

$= \pi\left[-\frac{1}{3}x^3 + \frac{3}{2}x^2 - 2x\right]_1^2$ (3)

$= \pi\left[-\frac{8}{3} + 6 - 4 + \frac{1}{3} - \frac{3}{2} + 2\right]$

$= \frac{\pi}{6}$ units^3

(6)



a) $V = x(16 - 2x)(6 - 2x)$

$V = 4x^3 - 44x^2 + 96x$

(b) $\frac{dV}{dx} = 12x^2 - 88x + 96$

$3x^2 - 22x + 24 = 0$ at max

$(3x - 4)(x - 6) = 0$

$x = \frac{4}{3}$ ($x = 6$ too large) (5)

$\frac{d^2V}{dx^2} = 24x - 88$

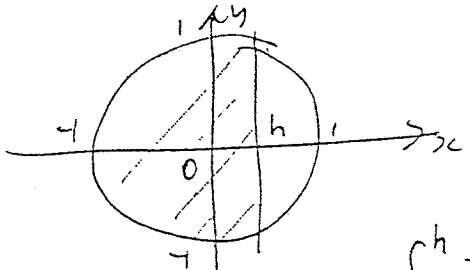
when $x = \frac{4}{3}$, $\frac{d^2V}{dx^2} = 24\times\frac{4}{3} - 88 = -56$

Since $\frac{d^2V}{dx^2} < 0$, max value exists

$V_{max} = 4\times\left(\frac{4}{3}\right)^3 - 44\left(\frac{4}{3}\right)^2 + 96\times\frac{4}{3}$

$= 59\frac{7}{27}$ units^3

(7)



Vol. of larger portion $= \int_{-1}^{h} \pi(1 - x^2)\,dx$

$= \pi\left[x - \frac{1}{3}x^3\right]_{-1}^{h}$

$= \pi(h - \frac{1}{3}h^3 + 1 - \frac{1}{3})$

$= \pi(h - \frac{1}{3}h^3 + \frac{2}{3})$ (6)

Vol. of smaller $= \int_h^1 \pi(1 - x^2)\,dx$

$= \pi\left[x - \frac{1}{3}x^3\right]_h^1$

$= \pi(1 - \frac{1}{3} - h + \frac{1}{3}h^3)$

$= \pi(\frac{2}{3} - h + \frac{1}{3}h^3)$

$\pi(h - \frac{1}{3}h^3 + \frac{2}{3}) = \pi(\frac{4}{3} - 2h + \frac{2}{3}h^3)$

$h^3 - 3h + \frac{2}{3} = 0$

$3h^3 - 9h + 2 = 0$

25

(1) +C

(2)

(3) +C

(4)

(5)

(6) $x = 6$? $\frac{d^2V}{dx^2}$?

(7) ?