JASON SAUNDERS
PNC

SYDNEY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT
HALF-YEARLY EXAMINATIONS 2006

FORM V

MATHEMATICS EXTENSION 1

Examination date

Wednesday 17th May 2006

Time allowed

2 hours

Instructions

All eight questions may be attempted.
All eight questions are of equal value.
All necessary working must be shown.
Marks may not be awarded for careless or badly arranged work.
Approved calculators and templates may be used.
A list of standard integrals is provided at the end of the examination paper.

Collection

Write your candidate number clearly on each booklet.
Hand in the eight questions in a single well-ordered pile.
Hand in a booklet for each question, even if it has not been attempted.
If you use a second booklet for a question, place it inside the first.
Keep the printed examination paper and bring it to your next Mathematics lesson.

5A: WMP	5B: DNW	5C: DS
5D: REN	5E: KWM	5F: JNC
5G: PKH	5H: MLS	5I: TCW

Checklist

Folded A3 booklets: 8 per boy. A total of 1250 booklets should be sufficient.
Candidature: 137 boys.

Examiner

KWM

QUESTION ONE Use a separate writing booklet.

(a) Use your calculator to find $\sin 55^\circ 34'$ correct to two decimal places.

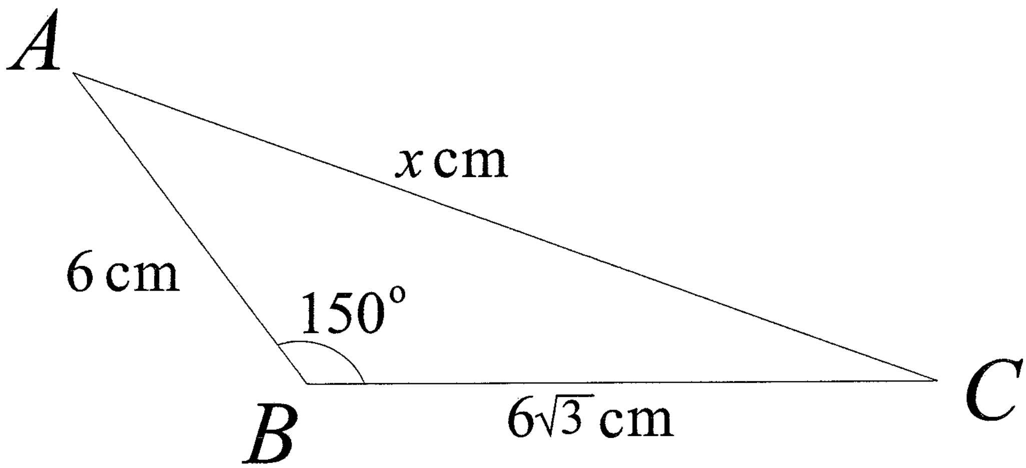
(b) Factorise $x^3 + 8$.

(c) Solve $|2x - 1| < 3$.

(d) Given that $g(x) = x^2$, find an expression for $g(h + 5)$.

(e) Calculate $\sum_{r=1}^{3}(r^2 + 1)$.

(f)



The diagram above shows $\triangle ABC$, where $\angle ABC = 150^\circ$, $AB = 6$ cm, $BC = 6\sqrt{3}$ cm and $AC = x$ cm.

(i) Find the exact value of x.

(ii) Find the exact area of $\triangle ABC$.

(g) Write $0{\cdot}4\dot{7}$ as a fraction in its lowest terms. Show full working.

QUESTION TWO Use a separate writing booklet.

(a) Solve $9^x = 27$.

(b) Express the fraction $\dfrac{3}{3-\sqrt{2}}$ with a rational denominator.

(c) Consider the arithmetic series $40 + 38 + 36 + \cdots$.

(i) Find the twentieth term of the series.

(ii) How many terms must be added for the sum of the series to be equal to zero?

(d) Solve the equation $\cos x = -\dfrac{1}{\sqrt{2}}$, for $0^\circ \le x \le 360^\circ$.

(e) Find the equation of the line in general form that passes through the point $(-1, 2)$ and has angle of inclination 135°.

(f) Write down the equation of the following curves.

(i) The reflection of $y = 2^x$ in the y-axis.

(ii) The curve $y = 2^x$ shifted 3 units to the right then reflected in the x-axis.

Exam continues overleaf ...

QUESTION THREE Use a separate writing booklet.

(a) Consider the function $y = -\sqrt{4 - x^2}$.

(i) Sketch the graph of the function, showing all intercepts with the axes.

(ii) Write down the domain of the function.

(iii) Write down the range of the function.

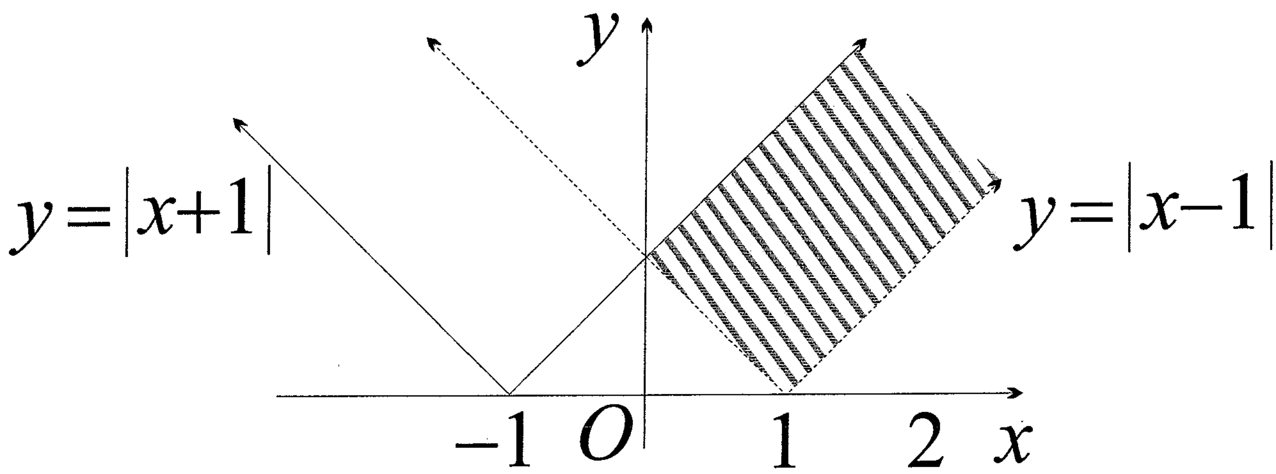
(b) Consider the function $h(x) = \dfrac{2x+1}{3}$.

(i) Find $h^{-1}(x)$.

(ii) Find $h\left(h^{-1}(1)\right)$.

(c) Solve $x^2 + x - 6 \geq 0$.

(d)



The diagram above shows the graphs of $y = |x+1|$ and $y = |x-1|$ and the region between these graphs in the first quadrant is shaded. NOTE: $y = |x-1|$ is shown with a broken line.

(i) Use the diagram to solve the inequation $|x+1| < |x-1|$.

(ii) Select one of the letters below that describes the shaded region.

A. $|x+1| < y$ and $y \geq |x-1|$
B. $y \leq |x+1|$ and $y < |x-1|$
C. $y \leq |x+1|$ and $y > |x-1|$
D. $y < |x+1|$ and $y > |x-1|$

Exam continues next page ...

QUESTION FOUR Use a separate writing booklet.

(a) Find, in general form, the equation of the perpendicular bisector of the interval joining the points $P(1,-2)$ and $Q(3,4)$.

(b) Given the points $A(-5,4)$ and $B(4,1)$, find the coordinates of the point P that divides the interval AB in the ratio 2:1.

(c) (i) Find the centre and radius of the circle $x^2+6x+y^2-2y+6=0$.

(ii) Find the perpendicular distance from the point $(-3,1)$ to the line $5x+12y+29=0$.

(iii) Explain why the line $5x+12y+29=0$ is a tangent to the circle $x^2+6x+y^2-2y+6=0$.

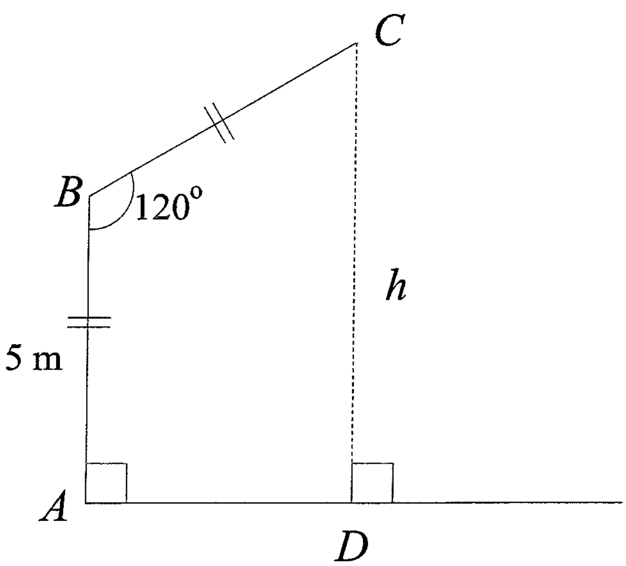
(d) (i) Find an expression for the gradient of the line $3x-4y+2+k(x+y+1)=0$.

(ii) Without finding the point of intersection, find the equation of the line passing through the point of intersection of the lines $3x-4y+2=0$ and $x+y+1=0$, and parallel to the line $4x-3y-1=0$. Write your answer in general form.

QUESTION FIVE Use a separate writing booklet.

(a) Given that $\cot\theta=\frac{20}{21}$ and θ is a reflex angle, find the exact value of $\sec\theta$.

(b)



The diagram above shows a vertical crane AB of height 5 metres. The jib BC of the crane is also of length 5 metres and makes an angle of 120° with the crane. Calculate the exact height of the point C above the horizontal ground.

(c) Solve $\tan^2\theta-3\sec\theta+3=0$, for $0^\circ\le\theta\le 360^\circ$.

(d) Prove the identity $\dfrac{\cos x}{1-\tan x}+\dfrac{\sin x}{1-\cot x}=\sin x+\cos x$.

Exam continues overleaf ...

QUESTION SIX Use a separate writing booklet.

(a) Solve the inequation $\dfrac{x+1}{x-2} > 3$.

(b) The sum of a series is defined by $S_n = 3n^2 + 10n$ for all values of n. Find an expression for the nth term T_n.

(c) The sum of the first and second terms of a geometric sequence is 40, while the sum of the fourth and fifth terms is 135. Find the first three terms of the sequence.

(d) The houses of Peter's street are consecutively numbered from 1 to 49, and Peter lives in the house numbered n.

(i) Write down an expression for the sum of the house numbers before Peter's house.

(ii) Write down an expression for the sum of the house numbers after Peter's house.

(iii) The sum of the numbers of the houses before Peter's house is equal to the sum the numbers of the houses after it. Form an equation involving n and hence find the number of Peter's house.

QUESTION SEVEN Use a separate writing booklet.

(a) Consider the curve $y = \dfrac{x-1}{(x-2)(x-3)}$.

(i) Write down the equations of the vertical asymptotes.

(ii) Find the x and y intercepts.

(iii) Investigate the behaviour of the function as $x \to \infty$ and as $x \to -\infty$ to find the horizontal asymptote.

(iv) For what values of x is $y > 0$?

(v) Sketch the curve.

(b) Use mathematical induction to prove that for all positive integer values of n,

$$1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{n}{3}(4n^2 - 1).$$

Exam continues next page ...

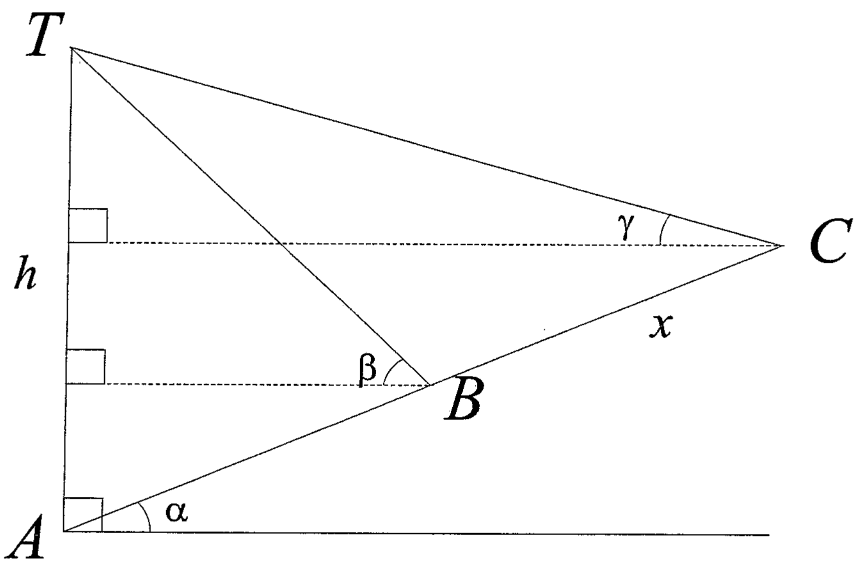
QUESTION EIGHT Use a separate writing booklet.

(a) (i) Show that the series $\frac{1}{2\sqrt{2}}+\frac{2-\sqrt{2}}{4}+\frac{3\sqrt{2}-4}{4}+\cdots$ is geometric.

(ii) Explain why the series has a limiting sum, and show that this limiting sum is $\frac{\sqrt{2}+1}{4}$.

(iii) How many terms of the series must be added so that the sum differs from the limiting sum by less than one millionth?

(b)



ABC is a straight line sloping upward from A at an angle α to the horizontal. At B the angle of elevation to the top T of a vertical mast at A is β, and at C the angle of elevation of T is γ. Given that the distance BC is x, prove that the height h of the mast is given by

$$h=\frac{x\sin(\alpha+\beta)\sin(\alpha+\gamma)}{\sin(\beta-\gamma)\cos\alpha}.$$

(c) Evaluate $\sum_{n=0}^{\infty}\frac{3^n-2^n}{6^n}$.

END OF EXAMINATION

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STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$