SYD. GRAMMAR - YR11 - EXT. 1.

Due 22 July.

1. Differentiate with respect to x:

   (a) $y = 5x(x-1)$,

   (b) $g(x) = 2\sqrt{x}$,

   (c) $y = \dfrac{1}{(5-2x)^4}$,

   (d) $f(x) = \dfrac{2-x^2}{2-x}$,

   (e) $y = x^2(2x-4)^5$.

2. Find the equation of the tangent to the curve $y = 8x^3 - 6x + 4$ at the point where $x = -1$.

3. Consider the function $y = \dfrac{x}{\sqrt{x-1}}$:

   (a) Show that $\dfrac{dy}{dx} = \dfrac{x-2}{2(\sqrt{x-1})^3}$.

   (b) Find the point at which the tangent to the curve is parallel to the x-axis.

4. Evaluate the following limits:

   (a) $\lim\limits_{x \to (-1)} (x^2 - x)$,

   (b) $\lim\limits_{x \to (-1)} \dfrac{x^2-1}{x+1}$,

   (c) $\lim\limits_{x \to \infty} \dfrac{2x^3 - 6x + 2}{4 - 3x^3}$.

5. Differentiate $f(x) = x^2 - 2x$ from first principles.

6. A spherical balloon of radius r is inflating so that its volume is increasing at a constant rate of 100 $(\text{cm})^3/\text{s}$.

   (a) Show that $\dfrac{dV}{dt} = 4\pi r^2 \dfrac{dr}{dt}$.

   (b) At what rate is the radius increasing when the radius is 4 cm?

7. Sketch an example of a <u>function</u> which is <u>not differentiable</u> at $x = 1$.

8. Given $f(x) = \begin{cases} x, & \text{for } x \leq 0 \\ -x, & \text{for } x > 0 \end{cases}$

   (a) sketch the curve,

   (b) find $\lim_{x \to 0^+} f(x)$, $\lim_{x \to 0^-} f(x)$ and $f(0)$,

   (c) draw a conclusion about continuity at $x = 0$,

   (d) can you write the function in an easier way?

9. (a) Find the equation of the tangent to the curve $y = 6x^2 - 3x + 24$ at the point on the curve where $x = a$.

   (b) Hence find any points on the curve $y = 6x^2 - 3x + 24$ where the tangent passes through the origin.

Introductory Calculus Assignment

a) $y = 5x(x-1)$ $= 5x^2 - 5x$ is quicker

$u \quad v$

$$\frac{dy}{dx} = vu' + uv'$$
$$= 5(x-1) + 5x(1)$$
$$= 5x - 5 + 5x$$
$$= 10x - 5$$ ✓

b) $g(x) = 2x^{-1/2}$ try again

$$\frac{dy}{dx} = (2)(-1)x^{-2}$$
$$= -2x^{-2}$$
$$= \frac{-2}{x^2}$$

c) $y = \frac{1}{(5-2x)^4}$

$$= (5-2x)^{-4}$$ ✓

$$\frac{dy}{dx} = -4(5-2x)^{-5} \cdot -2$$
$$= 8(5-2x)^{-5}$$ ✓

d) $f(x) = \frac{2-x^2}{2-x}$

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$
$$= \frac{(2-x)(-2x) - (2-x^2).(-1)}{(2-x)^2}$$
$$= \frac{-2x(2-x) + (2-x^2)}{(2-x)^2}$$ ✓
$$= \frac{-4x + 2x^2 + 2 - x^2}{4 - 4x + x^2}$$ ✓
$$= \frac{x^2 - 4x + 2}{x^2 - 4x - 4}$$

e) $y = x^2(2x-4)^5$

$$\frac{dy}{dx} = vu' + uv'$$
$$= (2x-4)^5 . 2x + x^2 . 5(2x-4)^4 . 2$$
$$= 2x(2x-4)^5 + 10x^2(2x-4)^4$$
$$= (2x-4)^4\left[2x(2x-4) + 10x^2\right]$$ ✓
$$= (2x-4)^4\left[4x^2 - 8x + 10x^2\right]$$
$$= (2x-4)^4\left[14x^2 - 8x\right]$$
$$= 2x(2x-4)^4(7x-4)$$ ✓

2. $\frac{dy}{dx} = 8x^3 - 6x + 4$

$$= 24x^2 - 6$$

$\therefore$ m at $x = -1$

$$m = 24(-1)^2 - 6$$
$$= 18.$$ ✓

Eq^n. of tangent is ... continue.

3a) $y = \frac{x}{(x-1)^{1/2}}$ $\frac{u}{v}$

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$
$$= \frac{1(x-1)^{1/2} - x\left[\frac{1}{2}(x-1)^{-1/2} . 1\right]}{(x-1)}$$ ✓
$$= \frac{(x-1)^{1/2} - \frac{x}{2}(x-1)^{-1/2}}{(x-1)}$$

$$= \frac{(x-1)^{-\frac{1}{2}}\left[(x-1) - \frac{x}{2}\right]}{(x-1)}$$

$$= \frac{(x-1)^{-\frac{1}{2}}\left[\frac{2x-2-x}{2}\right]}{x-1}$$

$$= \frac{(x-1)^{-\frac{1}{2}}\left[\frac{x-2}{2}\right]}{x-1} \checkmark$$

$$= \frac{(x-2)}{2}\left[(x-1)^{-3/2}\right]$$

$$= \frac{x-2}{2(\sqrt{x-1})^3} \checkmark$$

b) parallel to x-axis

$\therefore m = 0$

$\therefore \frac{dy}{dx} = 0$

$$\frac{x-2}{2(\sqrt{x-1})^3} = 0$$

$x - 2 = 0$

$x = 2$ ✓

$\therefore y = \frac{2}{\sqrt{2-1}}$

$= \frac{2}{1}$

$= 2$

$\therefore (2, 2)$ ✓

4a) $\lim_{x \to (-1)} (x^2 - x)$

$= (-1)^2 - (-1)$

$= 1 + 1$

$= 2$ ✓

b) $\lim_{x \to (-1)} \frac{x^2 - 1}{x+1}$

$= \lim_{x \to (-1)} \frac{(-1-1)\cancel{(x+1)}}{\cancel{(x+1)}}$ ✓

$= -1 - 1$

$= -2$ ✓

c) $\lim_{x \to \infty} \frac{2x^3 - 6x + 2}{4 - 3x^3}$

$$= \lim_{x \to \infty} \frac{\cancel{x^3}\left(2 - \frac{6}{x^2} + \frac{2}{x^3}\right)}{\cancel{x^3}\left(\frac{4}{x^3} - 3\right)}$$

$= \frac{2}{-3}$ ✓

5. $f(x) = x^2 - 2x$

$$y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{(x+h)^2 - 2(x+h) - x^2 + 2x}{h} \checkmark$$

$$=" \frac{\cancel{x^2} + 2xh + h^2 \cancel{-2x} - 2h \cancel{-x^2} \cancel{+2x}}{h} \checkmark$$

$$=" \frac{h^2 - 2xh - 2h}{h} \quad \lim_{h \to 0} h - 2x - 2$$

$= -2x - 2$

(6) Let V = Volume of a sphere

$$= \frac{4}{3}\pi r^3$$

$$\therefore \frac{dV}{dr} = \frac{4}{3}\pi . 3r^2 \qquad \text{and} \qquad \frac{dV}{dt} = 100 \text{ cm}^3/\text{s}$$

$$= 4\pi r^2$$

(a) $$L.H.S. = \frac{dV}{dt}$$

$$= \frac{dV}{dr} \times \frac{dr}{dt}$$

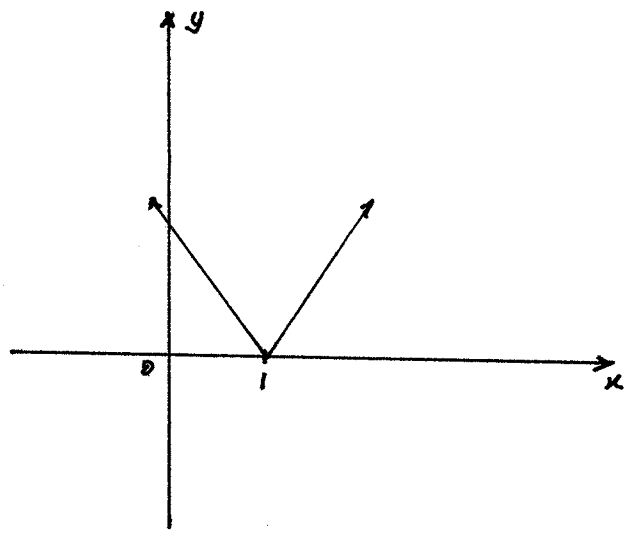
$$= 4\pi r^2 . \frac{dr}{dt}$$

(b) $$\therefore \quad 100 = 4\pi r^2 . \frac{dr}{dt}$$

$$\therefore \frac{100}{4\pi(4)^2} = \frac{dr}{dt}$$

$$\therefore \underline{\frac{25}{16\pi} = \frac{dr}{dt}}$$

(7)



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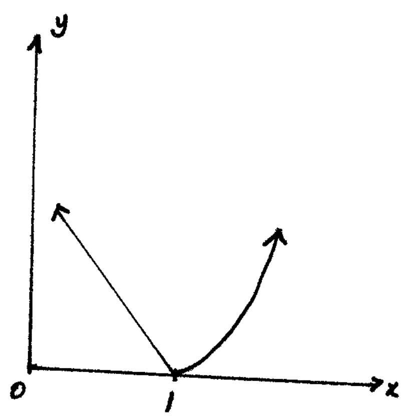
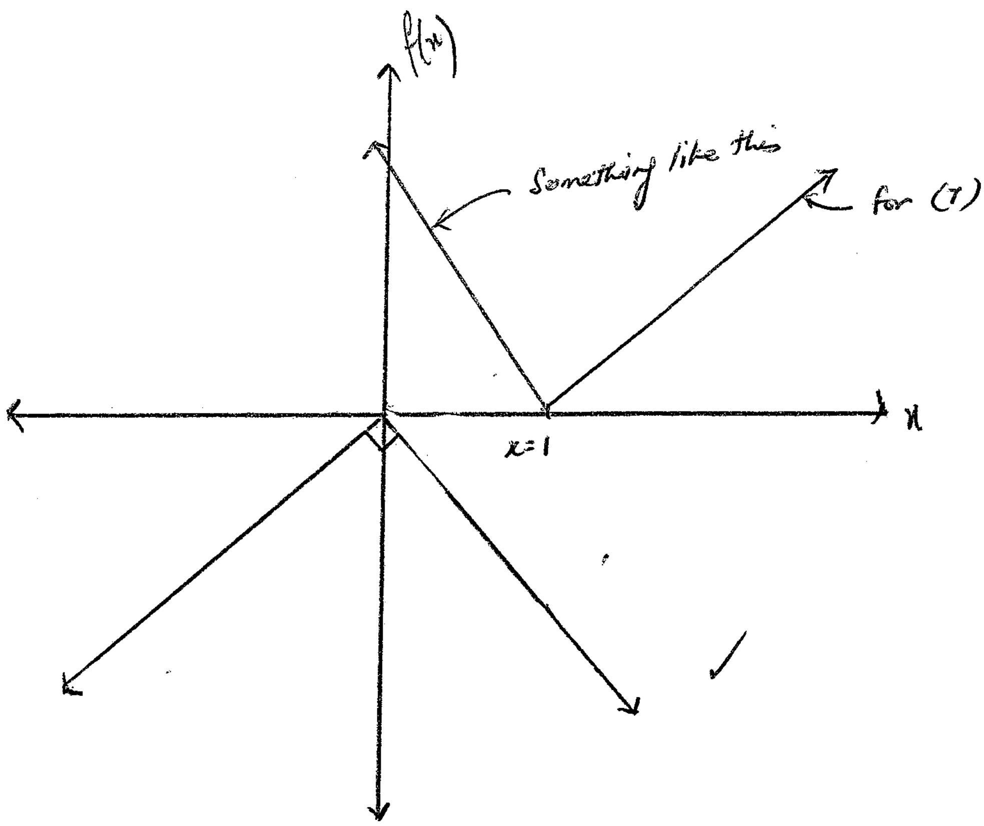


8(a)



b) $\lim_{x\to 0^+} f(x)$, $\lim_{x\to 0^-} f(x)$, $f(0) = 0$ ✓

$= 0$ ✓ $= 0$ ✓

c) The graph is continuos. ✓

d) $y = -|x|$ ✓

9a) $\frac{dy}{dx} = 12x - 3$

when $x = a$

$m = 12a - 3$ ✓

$y = 6a^2 - 3a + 24$

$y - y_1 = m(x - x_1)$

$y - 6a^2 + 3a - 24 = (12a - 3)(x - a)$ ✓

$y - 6a^2 \cancel{+3a} - 24 = 12ax - 12a^2 - 3x \cancel{+3a}$ ✗

$y = 12ax \cancel{-12a^2} - 3x + 6x^2 + 24$
$-6a^2$

b) 0,0

$\therefore 0 = 12a(0) \overset{-6a^2}{\cancel{-12a^2}} - 3(0) + 6(0)^2 + 24 \Rightarrow 6a^2 = 24$

$a^2 = 4$

$a = \pm 2.$

$0 = -12a^2 + 24$

$12a^2 - 24 = 0$

$a^2 - 2 = 0$ ✓

$a^2 = 2$

$a = \pm\sqrt{2}$ ✓

$\therefore x = 2$ or $x = -2$

$y = 6(4) - 6 + 24$; $y = 24 + 6 + 24$

$= -6$; $= 54$

$(2, -6)$ or $(-2, 54)$

$\therefore y = 6(\sqrt{2})^2 - 3\sqrt{2} + 24$ or $y = 6(-\sqrt{2})^2 - 3(-\sqrt{2}) + 24$

$= 12 - 3\sqrt{2} + 24$; $= 12 + 3\sqrt{2} + 24$

$= 36 - 3\sqrt{2}$ ✓ ; $= 36 + 3\sqrt{2}$ ✓

$\therefore \begin{cases} x = \sqrt{2} \\ y = 36 - 3\sqrt{2} \end{cases}$ ✓ or $\begin{cases} x = -\sqrt{2} \\ y = 36 + 3\sqrt{2} \end{cases}$ ✓