SYDNEY GRAMMAR SCHOOL

2010 Assessment Examination

FORM VI
MATHEMATICS EXTENSION 2

Wednesday 3rd February 2010

General Instructions
- Writing time — Period 3
- Write using black or blue pen.
- Board-approved calculators may be used.
- A list of standard integrals is provided at the end of the examination paper.
- All necessary working should be shown in every question.
- Start each question on a new leaflet.

Structure of the paper
- Total marks — 36
- All three questions may be attempted.
- All three questions are of equal value.

Collection
- Write your name, class and master clearly on each leaflet and on the tear-off sheet.
- Hand in the three questions in a single well-ordered pile.
- Hand in a leaflet for each question, even if it has not been attempted.
- If you use a second leaflet for a question, place it inside the first.
- Bundle the tear-off sheet with the question it belongs to.
- Write your name on the question paper and place it inside your leaflet for Question One.

6A: REP	6B: PKH	6C: BDD
6D: FMW	6E: KWM	6F: MLS

Checklist
- Writing leaflets: 3 per boy.
- Candidature — 90 boys

Examiner
PKH

QUESTION THREE (12 marks) Use a separate writing booklet. **Marks**

(a) Sketch the locus of the points z which simultaneously satisfy $|z-2i| \le 4$ and $\mathrm{Im}(z) \ge 2$. **3**

(b) (i) Sketch the locus of the points z which satisfy $\arg(z+6i) = \frac{\pi}{3}$. **2**

(ii) Find in modulus-argument form the point on the locus in part (i) for which $|z|$ is least. **2**

(c) Let $z_1 = i$ and $z_2 = \dfrac{1+i}{\sqrt{2}}$.

(i) Show the vectors z_1, z_2 and $z_1 + z_2$ on an Argand diagram. **1**

(ii) Hence find $\arg(z_1 + z_2)$ giving clear geometric reasons. **2**

(iii) Hence find the exact value of $\cot \frac{\pi}{8}$ without using a calculator. **2**

END OF EXAMINATION

Tear-off pages follow ...

QUESTION ONE (12 marks) Use a separate writing booklet. Marks

(a) If $z = 3 - 2i$ find:

(i) $\text{Im}(z)$ 1

(ii) $\overline{z}$ 1

(iii) $|z|$ 1

(b) Write $\dfrac{1+i}{3-i}$ in the form $a + ib$ where a and b are rational. 2

(c) Write the complex number $\sqrt{3} - i$ in modulus-argument form. 2

(d) Use implicit differentiation to find the gradient of the tangent to the curve $x^2 - 4y^2 = 9$ at the point $(5, 2)$. 2

(e) (i) Given that $1 + i$ is a solution to the equation $z^2 + iz + a = 0$, find a. 2

(ii) Find the other root of the equation in part (i). 1

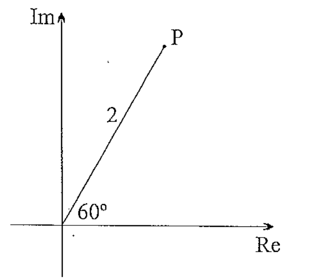
Exam continues next page ...

QUESTION TWO (12 marks) Use a separate writing booklet. Marks

(a) Prove that $(\cos\theta + i\sin\theta)^2 = \cos 2\theta + i\sin 2\theta$. 2

(b) Find the solutions to the equation $z^2 = -5 + 12i$. 3

(c)



In the diagram above, the point P represents the complex number

$$z = 2(\cos 60^\circ + i\sin 60^\circ).$$

Carefully plot the following points on the tear-off sheet provided on page 5.

(i) A represents iz, 1

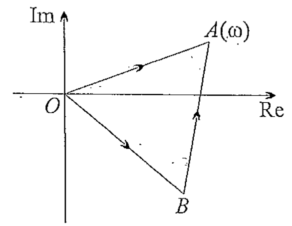
(ii) B represents $\overline{z}$, 1

(iii) C represents $-z$, 1

(iv) D represents z^2, 1

(v) E represents a square root of z where $\text{Re}(z)$ is negative. 1

(d) 2



Referring to the diagram above, point A represents the complex number ω. The triangle OAB is equilateral, with point B in the fourth quadrant. If $\phi = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$, what complex number is represented by the vector BA? Write your answer in terms of ω and ϕ.

Exam continues overleaf ...

NAME:_ QUESTION NUMBER: 1

9

ai) $z = 3-2i$

$Im(z) = -2$

ii) $\bar{z} = 3+2i$

iii) $|z| = \sqrt{3^2+2^2}$

$= \sqrt{13}$

b) $\frac{1+i}{3-i} \times \frac{3+i}{3+i} = \frac{3+i+3i-1}{10}$

$= \frac{2+4i}{10}$

$= \frac{1}{5} + \frac{2}{5}i$

c) $z = \sqrt{3} - i$

~~mod~~ $z = \sqrt{3+1}$

$= 2$

arg $z = \tan^{-1} \frac{1}{\sqrt{3}}$

$= \frac{\pi}{6}$

$z = 2 \text{cis} \left(-\frac{\pi}{6}\right)$

d) $x^2 - 4y^2 = 9$

implicitly differentiated $2x - 8yy' = 9$

$-8yy' = 9-2x$

$y' = \frac{9-2x}{-8y}$

At pt. $(5,2)$

$y' = \frac{9-2(5)}{-8(2)}$

$= \frac{-1}{-16}$

$= \frac{1}{16}$

e(ii) if $1+i$ is a solution then $-(1+i)$ is a solution.

i) $(1+i)^2 + i(1+i) + a = 0$

$\cancel{1} + 2i - \cancel{1} + i - 1 + a = 0$

$a = 1-3i$

NAME:_ QUESTION NUMBER: 2

10

a) ~~$(\cos\theta + i\sin\theta)^2$~~ LHS $= (\cos\theta + i\sin\theta)^2$

$= \cos^2\theta + 2\cos\theta\sin\theta - \sin^2\theta$

$= \cos 2\theta + i\sin 2\theta$

$= $ RHS

b) let $z = x+iy$

$(x+iy)^2 = -5+12i$

$x^2 + 2xiy - y^2 = -5+12i$

(1) $x^2 - y^2 = -5$ and (2) $2xy = 12$

$y = \frac{6}{x}$

sub into (1)

$x^2 - \frac{36}{x^2} = -5$

$x^4 + 5x^2 - 36 = 0$

$(x^2+9)(x^2-4) = 0$

$x^2 = 4$

$x = \pm 2$

$y = \pm 3$

solutions: $\pm(2+3i)$

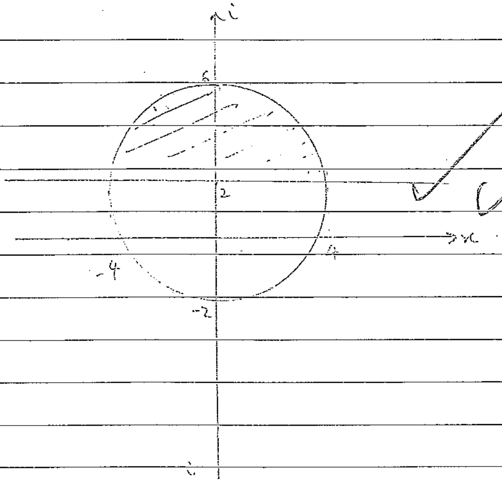
d) $\phi = \text{cis}\frac{\pi}{3}$

$\vec{BA} =$ ~~$\text{cis}\frac{2\pi}{3}$~~

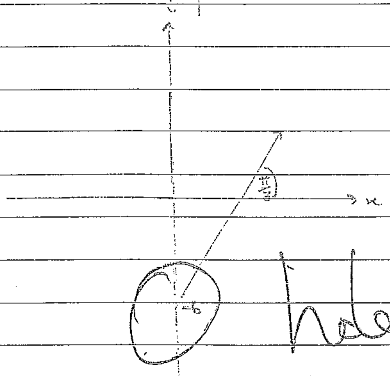
$w\phi$ how?

NAME:__ (5) QUESTION NUMBER: 3

a)



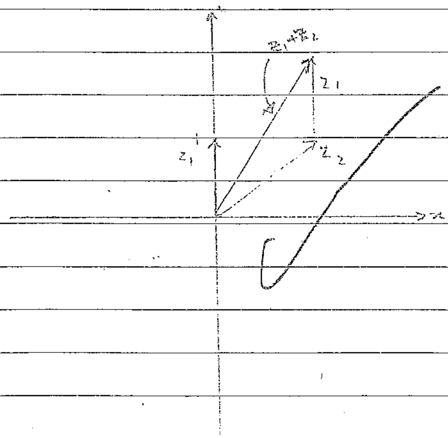
b i)



ii) $2\operatorname{cis}\frac{\pi}{3}$

hole? X

c i)



ii) $z_1 \parallel z_1$

$\arg(z_1) = \cancel{\pi/2}$

$\arg(z_2) = \tan^{-1} \dfrac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$

$= 45$

NAME: . ..... CLASS: MLS... MASTER: MLS..

DETACH THIS SHEET AND BUNDLE IT WITH THE REST OF QUESTION TWO.

QUESTION TWO

(c)