FORM VI MATHEMATICS EXTENSION 2

Time allowed: 2 hours **Exam date:** 15th May 2002

Instructions:

All questions may be attempted.
All questions are of equal value.
Part marks are shown in boxes in the right margin.
All necessary working must be shown.
Marks may not be awarded for careless or badly arranged work.
Approved calculators and templates may be used.
A list of standard integrals is provided at the end of the examination paper.

Collection:

Each question will be collected separately.
Start each question in a new answer booklet.
If you use a second booklet for a question, place it inside the first. Don't staple.
Write your candidate number on each answer booklet.

Checklist:

SGS Examination booklets required.

QUESTION ONE (Start a new answer booklet)

Marks

(a) Find $\int \frac{\log x}{x}\,dx$. 1

(b) Find $\int \frac{1}{x^2+4x+13}\,dx$. 2

(c) Use partial fractions to find $\int \frac{5}{(x+2)(x-1)}\,dx$. 4

(d) By rationalising the numerator of the integrand, evaluate $\int_0^{\frac{1}{2}} \sqrt{\frac{1-x}{1+x}}\,dx$. 4

(e) Use integration by parts twice to find $\int_0^{\frac{\pi}{2}} x^2 \cos x\,dx$. 4

QUESTION TWO (Start a new answer booklet)

(a) Let $z = 1 - i\sqrt{3}$.

Marks

(i) Express $\frac{1}{z}$ with a real denominator. 2

(ii) On an Argand diagram, indicate the complex numbers $z, -z$ and $\overline{z}$. 2

(iii) Find $|z|$ and $\arg z$. 1

(iv) Write z^2 in modulus-argument form. 2

(b) (i) Expand $\left(\sqrt{3}(1-i)\right)^2$. 1

(ii) Hence solve the quadratic equation $z^2 - 2(1+i)z + 8i = 0$. 3

(c) Sketch the following loci on separate Argand diagrams:

(i) $|z-2i| \geq |z-2|$, 2

(ii) $\arg\left(\frac{z-1}{z+i}\right) = \frac{\pi}{4}$. 2

Exam continues next page ...

QUESTION THREE (Start a new answer booklet)

Marks

(a) Factorise $P(x) = x^3 + 3x^2 + x - 5$ into irreducible factors:

(i) over the real numbers, **2**

(ii) over the complex numbers. **2**

(b) Find the values of a and b, given that $(x-1)^2$ is a factor of $P(x) = x^5 + 2x^4 + ax^3 + bx^2$. **3**

(c) Find p and q, given that $\sqrt{3} + i$ is one root of $x^4 + px^2 + q = 0$, where p and q are real. **4**

(d) An aeroplane is flying horizontally, in the direction due East, at a constant altitude h metres and at a constant speed of 240 km/h. From a point on the ground, the bearing of the plane is 311° T, and 3 minutes later the bearing of the plane is 073° T and its angle of elevation then is 21°. Find the altitude h at which the plane is flying, correct to the nearest metre. **4**

Question Four is on the next page.

QUESTION FOUR (Start a new answer booklet)

Marks

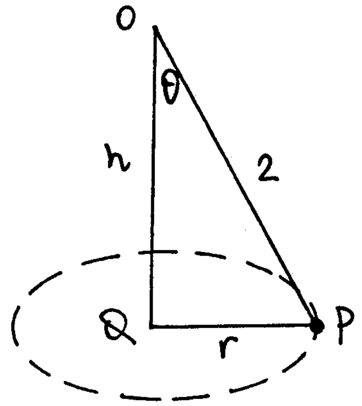
(a) The point P represents the complex number z, where $|z-2|=2$ and $0<\arg z<\frac{\pi}{2}$.

(i) Show that $|z^2-2z|=2\,|z|$. **1**

(ii) Sketch the locus of z and explain why $\arg(z-2)=2\arg(z)$. **2**

(iii) Find the value of k (a real number) if $\arg(z-2)=k\,\arg(z^2-2z)$. **3**

(b)



In the diagram above, a particle of mass 4 kg is attached at the end of a light string OP which is 2 metres long. The particle is moving in a horizontal circle with radius r metres whose centre Q is h metres vertically below O. The string makes an angle θ with the vertical and the tension in the string is T newtons.

(i) Draw a diagram of the forces acting on the particle, clearly indicating all the forces acting on the particle. **1**

(ii) If the particle is moving at $\sqrt{g}$ metres per second, show, by resolving forces vertically and horizontally, that $T=\dfrac{8g}{r^2}$, where g is the acceleration due to gravity and $r=QP$. **2**

(iii) Taking $g=10\,\text{m/s}^2$, show that the tension in the string is $T \doteqdot 51{\cdot}2$ newtons and that its inclination to the vertical is $38^\circ 40'$ (correct to the nearest minute). **6**

QUESTION FIVE (Start a new answer booklet)

An object is dropped from a lookout on top of a high cliff.

Take the acceleration due to gravity to be $10\,\mathrm{m/s^2}$.

(a) At first, air resistance causes a deceleration of magnitude $\dfrac{v}{10}$, where v m/s is the speed of the object t seconds after it is dropped.

Marks

(i) Taking downwards as positive, explain why its equation of motion is **[1]**

$$\ddot{x} = 10 - \frac{v}{10},$$

where x is the distance that the object has fallen in the first t seconds.

(ii) Show that $\dfrac{dv}{dx} = \dfrac{100 - v}{10v}$, and hence show that the speed V of the object when it is 40 metres below the lookout satisfies the equation **[4]**

$$V + 100 \log_e \left(1 - \frac{V}{100}\right) + 4 = 0.$$

(b) After the object has fallen 40 metres and reached this speed V, a very small parachute opens, and air resistance now causes a deceleration to its motion of magnitude $\dfrac{v^2}{10}$.

(i) Taking downwards as positive, write an expression for the new acceleration $\ddot{x}$ of the object, where x now is the distance that the object has fallen in the first t seconds after the parachute opens. **[1]**

(ii) Show that $v^2 = 100 - (100 - V^2)e^{-\frac{1}{5}x}$, and hence find the terminal velocity of the object. **[4]**

(iii) Show that t seconds after the parachute opens, **[4]**

$$t = \frac{1}{2} \log_e \frac{(v+10)(V-10)}{(v-10)(V+10)}.$$

(iv) Given that the solution to the equation in part (ii) of part (a) is $V \doteqdot 25{\cdot}7\,\mathrm{m/s}$, how long after the parachute opens does the particle reach 105% of its terminal velocity? **[1]**

QUESTION SIX (Start a new answer booklet)

Marks

(a) (i) Use the substitution $t = \tan\frac{x}{2}$ to show that $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{1+\frac{1}{2}\sin x} = \frac{2\pi}{3\sqrt{3}}$. $\boxed{3}$

(ii) Show that $\displaystyle\int_0^{2a} f(x)\,dx = \int_0^{a} \Big(f(x) + f(2a-x)\Big)\,dx.$ $\boxed{2}$

(iii) Hence evaluate $\displaystyle\int_0^{\pi} \frac{x\,dx}{1+\frac{1}{2}\sin x}$. $\boxed{2}$

(b) (i) Let $\theta = \tan^{-1}x + \tan^{-1}y$. Show that $\boxed{1}$

$$\tan\theta = \frac{x+y}{1-xy}.$$

(ii) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, show that $\boxed{3}$

$$xy + yz + zx = 1.$$

(iii) Let $\psi_n = \tan^{-1}x_1 + \tan^{-1}x_2 + \cdots + \tan^{-1}x_n$, where $n \geq 1$. $\boxed{4}$
Show, by mathematical induction or otherwise, that

$$\tan\psi_n = -\frac{\operatorname{Im}(w_n)}{\operatorname{Re}(w_n)},$$

where $w_n = (1-ix_1)(1-ix_2)\cdots(1-ix_n)$.

JNC

Question 1

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$\frac{1}{2}$ YEARLY

a) $\int \frac{\log x}{x}\,dx = \frac{1}{2}(\log x)^2 + c$

b) $\int \frac{1}{x^2+4x+13}\,dx = \int \frac{1}{(x+2)^2+9}\,dx$

$= \frac{1}{3}\tan^{-1}\left(\frac{x+2}{3}\right) + c$

c) Let $\frac{5}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$,

then $5 = A(x-1) + B(x+2)$

$\left.\begin{array}{l} A + B = 0 \\ -A + 2B = 5 \end{array}\right\}$ $A = -\frac{5}{3}$ and $B = \frac{5}{3}$

$\int \frac{5}{(x+2)(x-1)}\,dx = \frac{5}{3}\int \frac{-1}{x+2} + \frac{1}{x-1}\,dx$

$= \frac{5}{3}\ln\left|\frac{x-1}{x+2}\right|$

d) $\int_0^{\frac{1}{2}} \sqrt{\frac{1-x}{1+x}}\,dx = \int_0^{\frac{1}{2}} \frac{1-x}{\sqrt{1-x^2}}\,dx$

$= \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}\,dx$

$= \left[\sin^{-1}x\right]_0^{\frac{1}{2}} + \left[\sqrt{1-x^2}\right]_0^{\frac{1}{2}}$

$= \frac{\pi}{6} + \frac{\sqrt{3}}{2} - 1$

e) $\int_0^{\frac{\pi}{2}} x^2\cos x\,dx = \left[x^2\sin x\right]_0^{\frac{\pi}{2}} - 2\int_0^{\frac{\pi}{2}} x\sin x\,dx$

$= \frac{\pi^2}{4} - 2\left\{\left[-x\cos x\right]_0^{\pi/2} + \int_0^{\frac{\pi}{2}}\cos x\,dx\right\}$

$= \frac{\pi^2}{4} - 2\left\{0 + \left[\sin x\right]_0^{\frac{\pi}{2}}\right\}$

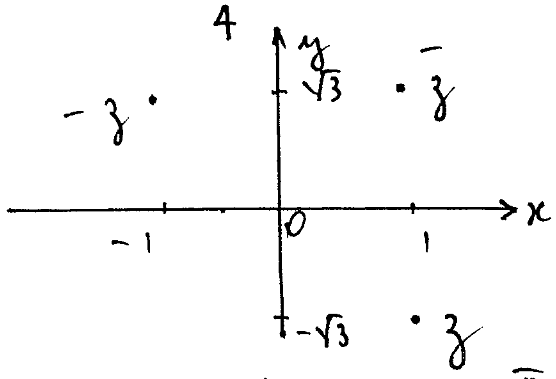
$= \frac{\pi^2}{4} - 2$

Question 2

a) (i) $\dfrac{1}{z} = \dfrac{1}{(1 - i\sqrt{3})} \dfrac{(1 + i\sqrt{3})}{(1 + i\sqrt{3})}$

$= \dfrac{1 + i\sqrt{3}}{4}$

(ii)



(iii) $|z| = 2$ and $\arg z = -\dfrac{\pi}{3}$

(iv) $z = 2\left(\cos\left(-\dfrac{\pi}{3}\right) + i\sin\left(-\dfrac{\pi}{3}\right)\right)$

$\therefore\ z^2 = 4\left(\cos\left(\dfrac{-2\pi}{3}\right) + i\sin\left(\dfrac{-2\pi}{3}\right)\right)$

b) (i) $\left(\sqrt{3}(1-i)\right)^2 = 3(1-i)^2$

$= 3(-2i)$

$= -6i$

(ii) $\Delta = 4(1+i)^2 - 8i$

$= 4(1 + 2i - 1) - 8i$

$= -24i$

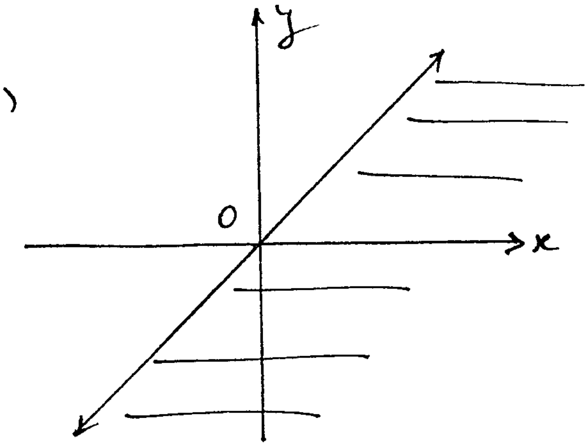
$= 4\left(\sqrt{3}(1-i)\right)^2$, from (i)

So $z = \dfrac{2(1+i) \pm 2\sqrt{3}(1-i)}{2}$

$z = (1 + \sqrt{3}) + i(1 - \sqrt{3})$ or

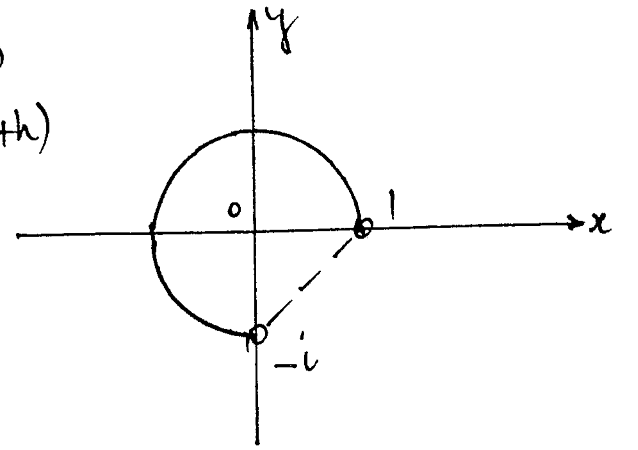
$(1 - \sqrt{3}) + i(1 + \sqrt{3})$.

(c)(i)



(ii)

(both)



(c) -1 per error of omission or commission.

Question 3

a) Possible zeros are $\pm 1, \pm 5$

$P(1) = 0 \Rightarrow (x-1)$ is a factor

$$\begin{array}{r} x^2 + 4x + 5 \\ x-1 \,\big)\, \overline{x^3 + 3x^2 + x - 5} \\ \underline{x^3 - x^2} \qquad\qquad \\ 4x^2 + x \qquad \\ \underline{4x^2 - 4x} \qquad \\ 5x - 5 \\ \underline{5x - 5} \\ \cdot \quad \cdot \end{array}$$

$P(x) = (x-1)(x^2 + 4x + 5)$

Solving $x^2 + 4x + 5 = 0$

gives $x = \dfrac{-4 \pm 2i}{2}$

$= -2 \pm i$

$P(x) = (x-1)(x + 2 - i)(x + 2 - i)$

$P'(x) = 5x^4 + 8x^3 + 3ax^2 + 2bx$

so $P'(1) = 13 + 3a + 2b = 0$

and $P(1) = 3 + a + b = 0$

$13 + 3a + 2b = 0$

$6 + 2a + 2b = 0$

$\therefore a = -7$

and $b = 4$

If $\sqrt{3} + i$ is a factor then

$(\sqrt{3}+i)^4 + p(\sqrt{3}+i)^2 + q = 0$

$(2 + 2\sqrt{3}i)^2 + p(2 + 2\sqrt{3}i) + q = 0$

$-8 + 8\sqrt{3}i + p(2 + 2\sqrt{3}i) + q = 0$

Equating real & imaginary parts:

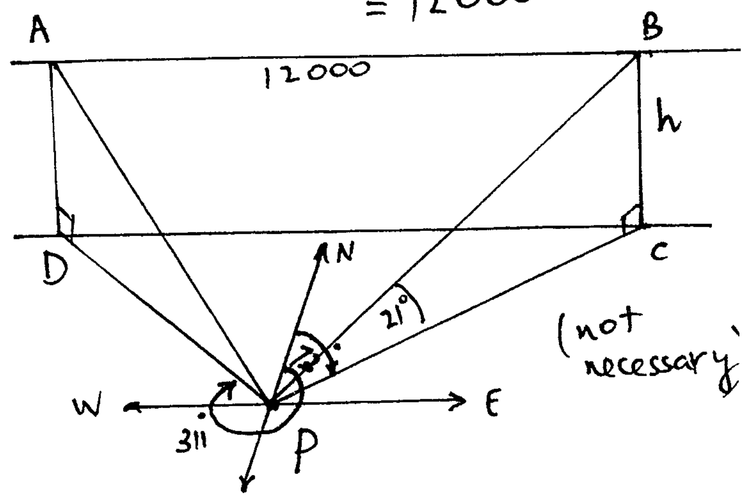
$-8 + 2p + q = 0$

$8\sqrt{3} + 2\sqrt{3}p = 0$

$\therefore p = -4$ and $q = 16$.

d) Distance travelled $= 240000 \times \dfrac{3}{60}$

$= 12000$



(not necessary)

In ΔPCB,

$PC = h \tan 69°$ (or $h \cot 21°$)

$\angle CDP = 41°$ and $\angle DPC = 122°$

In ΔPDC:

$\dfrac{h \tan 69°}{\sin 41°} = \dfrac{12000}{\sin 122°}$

$\therefore h = \dfrac{12000 . \sin 41°}{\tan 69° . \sin 122°}$

$= 3564$ m (nearest metre)

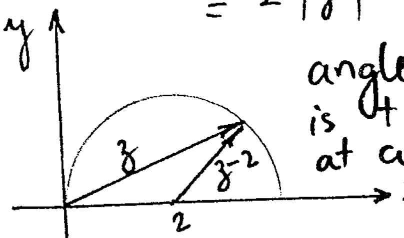
Question 4

a) (i) $|z^2 - 2z| = |z(z-2)|$

$= |z| \cdot |z-2|$

$= 2|z|$

(ii)



angle at centre is twice angle at circumference.

iii) $\arg(z-2) = k \arg(z^2 - 2z)$

$\arg(z-2) = k\{\arg z + \arg(z-2)\}$

$(1-k)\arg(z-2) = k \arg z$

$(1-k) \cdot 2\arg z = k \arg z$

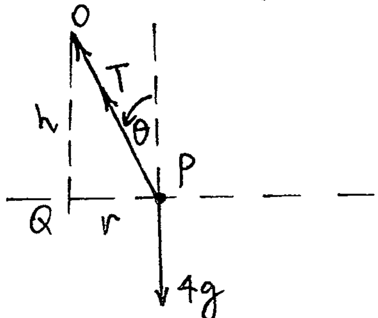
(angle at centre is twice angle at circumference)

$\therefore 2(1-k) = k$

$3k = 2$

$\therefore k = \frac{2}{3}$.

b) (i)



ii) Resolving :

$T\cos\theta = 4g$ — ①

$T\sin\theta = 4r\omega^2$ — ②

Now $T\sin\theta = 4.(2\sin\theta)\frac{v^2}{r^2}$

since $v = \sqrt{g}$;

$T = \frac{8g}{r^2}$ as required.

(iii) From ① and ② :

$\tan\theta = \frac{r\omega^2}{g}$

$= \frac{v^2}{rg}$

since $v = \sqrt{g}$, $\tan\theta = \frac{1}{r}$.

But $\tan\theta = \frac{r}{h}$

$\therefore \frac{1}{r} = \frac{r}{h}$

$r^2 = h$. — ③

By Pythagoras :

$h^2 + r^2 = 4$

$r^2 = 4 - h^2$.

So $h = 4 - h^2$ (substituting ③).

$h^2 + h - 4 = 0$

$\therefore h = \frac{-1 \pm \sqrt{17}}{2}$

but $h > 0$, $\therefore h = \frac{-1+\sqrt{17}}{2}$

So $r^2 = \frac{-1+\sqrt{17}}{2}$

$\therefore T = \frac{8g}{r^2} = 51.2$ N

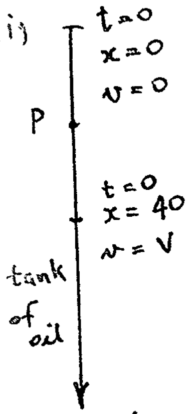
Also;
$\cos\theta = \frac{4g}{T}$

$= \frac{r^2}{2}$

$\therefore \theta = 38°40'$

uestion 5

(a)(i)



$\ddot{x} = g - \frac{v}{10}$

ie $\ddot{x} = 10 - \frac{v}{10}$

(ii) $v \cdot \frac{dv}{dx} = 10 - \frac{v}{10}$

$10 \cdot \frac{dv}{dx} = \frac{100 - v}{v}$

$\frac{1}{10} \cdot \frac{dx}{dv} = \frac{v}{100 - v}$

$\frac{x}{10} = \int \frac{v}{100 - v}\, dv$

$-\frac{x}{10} = \int \frac{100 - v}{100 - v} - \frac{100}{100 - v}\, dv$

$-\frac{x}{10} = v + 100 \ln(100 - v) + c$

When $t = 0$, $x = 0$, $v = 0 \Rightarrow c = -100 \ln 100$

$\therefore -\frac{x}{10} = v + 100 \ln\left(\frac{100 - v}{100}\right)$

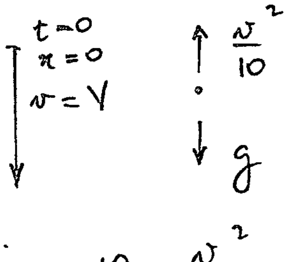
When $x = 40$, $v = V$

$-4 = V + 100 \ln\left(\frac{100 - V}{V}\right)$

ie, $V + 100 \ln\left(1 - \frac{V}{100}\right) + 4 = 0$

[or $\int_0^{40} -\frac{dx}{10} = \int_0^{V} 1 - \frac{100}{100 - v}\, dv$]

(b)(i)



$\ddot{x} = 10 - \frac{v^2}{10}$

ie, $\ddot{x} = \frac{100 - v^2}{10}$

(ii) $v \cdot \frac{dv}{dx} = \frac{100 - v}{10}$

$\frac{dx}{dv} = \frac{10v}{100 - v^2}$

$x = \int \frac{10v}{100 - v^2}\, dv$

$x = -5 \ln(100 - v^2) + c$

when $x = 0$, $v = V \Rightarrow c = 5 \ln(100 - V^2)$

$\therefore x = -5 \ln\left[\frac{100 - v^2}{100 - V^2}\right]$

$v^2 = 100 - (100 - V^2) e^{-\frac{x}{5}}$

As $x \to \infty$, $v^2 = 100$, so

terminal velocity $= 10$ m/s. $(v > 0)$

(iii) $\frac{dv}{dt} = \frac{100 - v^2}{10}$

$t = \int \frac{10}{100 - v^2}\, dv$

Consider; $\frac{1}{100 - v^2} = \frac{a}{10 + v} + \frac{b}{10 - v}$

$a = b = \frac{1}{20}$

so $t = \frac{10}{20} \int \frac{1}{10 + v} + \frac{1}{10 - v}\, dv$

$t = \frac{1}{2}\left[\ln(10 + v) - \ln(10 - v)\right] + c$

When $t = 0$, $v = V \Rightarrow 2c = -\ln\left(\frac{10 + V}{10 - V}\right)$

$\therefore t = \frac{1}{2}\left[\ln\left(\frac{10 + v}{10 - v}\right) - \ln\left(\frac{10 + V}{10 - V}\right)\right]$

$= \frac{1}{2} \ln\left[\left(\frac{10 + v}{10 - v}\right)\left(\frac{10 - V}{10 + V}\right)\right]$

$= \frac{1}{2} \ln\left[\left(\frac{v + 10}{v - 10}\right)\left(\frac{V - 10}{V + 10}\right)\right]$

(iv) 105% of terminal velocity is 10.5

and $V = 25.7$ so

$t = \frac{1}{2} \ln\left[\left(\frac{20.5}{0.5}\right)\left(\frac{15.7}{35.7}\right)\right] \doteqdot 1.4(46$ sec.

Question 6

Let $t = \tan \frac{x}{2}$,

hen $dx = \frac{2\,dt}{1+t^2}$. When $x=0, t=0$

d when $x = \frac{\pi}{2}$, $t = 1$.

$$\int_0^{\frac{\pi}{2}} \frac{dx}{1+\frac{1}{2}\sin x} = \int_0^1 \frac{2\,.\,dt}{(1+t^2).\left(1+\frac{1}{2}.\frac{2t}{1+t^2}\right)}$$

$$= \int_0^1 \frac{2\,dt}{(t^2+t+1)}$$

$$= \int_0^1 \frac{2\,dt}{(t+\frac{1}{2})^2 + \frac{3}{4}}$$

$$= 2\,\frac{2}{\sqrt{3}}\tan^{-1}\frac{2(t+\frac{1}{2})}{\sqrt{3}}\Big]_0^1$$

$$= \frac{4}{\sqrt{3}}\left[\tan^{-1}\frac{3}{\sqrt{3}} - \tan^{-1}\frac{1}{\sqrt{3}}\right]$$

$$= \frac{4}{\sqrt{3}}\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$$

$$= \frac{2\pi}{3\sqrt{3}}$$

) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_a^{2a} f(x)\,dx.$

onsider $\int_a^{2a} f(x)\,dx$.

et $x = 2a - t$, then $dx = -dt$

nd when $x = a$, $t = a$ and when

$x = 2a$, $t = 0$. So

$$\int_a^{2a} f(x)\,dx = \int_a^0 f(2a-t).dt$$

$$= \int_0^a f(2a-t)\,dt$$

$$= \int_0^a f(2a-x)\,dx.$$

$$\therefore \int_0^{2a} f(x)\,dx$$

$$= \int_0^a \left[f(x) + f(2a-x)\right] dx$$

(iii) $\int_0^{\pi} \frac{x\,dx}{1+\frac{1}{2}\sin x}$

$$= \int_0^{\frac{\pi}{2}} \frac{x}{1+\frac{1}{2}\sin x} + \frac{\pi - x}{1+\frac{1}{2}\sin x}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\pi}{1+\frac{1}{2}\sin x}\,dx$$

$$= \pi\,\frac{2\pi}{3\sqrt{3}} \quad \text{(from part (a))}$$

$$= \frac{2\pi^2}{3\sqrt{3}}$$

6 b)

(i) $\tan\theta = \tan(\tan^{-1}x + \tan^{-1}y)$

$$= \frac{\tan(\tan^{-1}x) + \tan(\tan^{-1}y)}{1 - \tan(\tan^{-1}x)\tan(\tan^{-1}y)}$$

$$= \frac{x+y}{1-xy}$$

ii) $\tan(\tan^{-1}x + \tan^{-1}y + \tan^{-1}z)$

$$= \frac{\tan(\tan^{-1}x + \tan^{-1}y) + \tan(\tan^{-1}z)}{1 - \tan(\tan^{-1}x + \tan^{-1}y).\tan(\tan^{-1}z)}$$

$$= \frac{\frac{x+y}{1-xy} + z}{1 - \left(\frac{x+y}{1-xy}\right) z}$$

$$= \frac{x+y+(1-xy)z}{(1-xy)} \quad \frac{(1-xy)}{1-xy-(x+y)z}$$

$$= \frac{x+y+z-xyz}{1-(xy+yz+zx)}$$, which is

undefined if $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$

So $1-(xy+yz+zx) = 0$

ie/ $xy + yz + zx = 1$.

iii) When $n=1$, $\psi_1 = \tan^{-1}x_1$ and $\tan\psi_1 = x_1$.

Also $\tan\psi_1 = -\frac{Im(\omega_1)}{Re(\omega_1)}$, when $\omega_1 = 1 - ix_1$.

$$= -\frac{(-x_1)}{1}$$

$= x_1$ and the result holds for $n=1$

Assume that $\tan\psi_k = -\frac{m(\omega_k)}{e(\omega_k)}$ for $\omega_k = (1-ix_1)(1-ix_2)\ldots(1-ix_k)$.

Let k be a positive integer for which the result is true.

ie $$\tan \psi_k = -\frac{\operatorname{Im}(\omega_k)}{\operatorname{Re}(\omega_k)}.$$

We now prove the result for $n = k+1$.

ie/ $$\tan \psi_{k+1} = -\frac{\operatorname{Im}(\omega_{k+1})}{\operatorname{Re}(\omega_{k+1})}$$

$$\begin{aligned}
LHS &= \tan\left(\psi_k + \tan^{-1} x_{k+1}\right) \\
&= \frac{\tan\psi_k + x_{k+1}}{1 - x_{k+1}\tan\psi_k} \\
&= \frac{-\dfrac{\operatorname{Im}(\omega_k)}{\operatorname{Re}(\omega_k)} + x_{k+1}}{1 + x_{k+1}\dfrac{\operatorname{Im}(\omega_k)}{\operatorname{Re}(\omega_k)}} \\
&= \frac{-\operatorname{Im}(\omega_k) + x_{k+1}\operatorname{Re}(\omega_k)}{\operatorname{Re}(\omega_k) + x_{k+1}\operatorname{Im}(\omega_k)} \\
&= \frac{-\operatorname{Im}(\omega_k - i x_{k+1}\omega_k)}{\operatorname{Re}(\omega_k - i x_{k+1}\omega_k)} \\
&= -\frac{\operatorname{Im}(\omega_k(1 - i x_{k+1}))}{\operatorname{Re}(\omega_k(1 - i x_{k+1}))} \\
&= RHS.
\end{aligned}$$

It follows by mathematical induction that the result is true for all positive integers n.

Alternately ;

For $i = 1, 2, 3, \ldots, n$, $\arg(1 - ix_i) = \tan x_i$

because $1 - ix$ is always in quadrant 1 or 2.

Hence $\arg \omega_n = \arg(1-ix_1) + \arg(1-ix_2) + \ldots + \arg(1-ix_n)$

$$= -\tan^{-1} x_1 - \tan^{-1} x_2 - \ldots - \tan^{-1} x_n$$

$$= -\psi_n .$$

Thus $\tan \psi_n = -\tan(-\psi_n)$ since $\tan\theta$ is odd

$$= -\tan(\arg \omega_n)$$

$$= -\frac{\operatorname{Im}(\omega_n)}{\operatorname{Re}(\omega_n)}, \text{ as required.}$$