SYDNEY GRAMMAR SCHOOL

2009 Assessment Examination

FORM VI
MATHEMATICS 2 UNIT

Thursday 28th May 2009

General Instructions

- Writing time — 40 minutes
- Write using black or blue pen.
- Board-approved calculators may be used.
- A list of standard integrals is provided at the end of the examination paper.
- All necessary working should be shown in every question.
- Start each question on a new leaflet.

Structure of the paper

- Total marks — 42
- All three questions may be attempted.
- All three questions are of equal value.

Collection

- Write your name, class and master clearly on each leaflet.
- Hand in the three questions in a single well-ordered pile.
- Hand in a leaflet for each question, even if it has not been attempted.
- If you use a second leaflet for a question, place it inside the first.
- The question papers will be collected separately.

6F: SJE	6G: FMW	6H: BDD	6P: KWM
6Q: JMR	6R: LYL	6S: RCF	

Checklist

- Writing leaflets: 3 per boy.
- Candidature — 101 boys

Examiner
KWM

QUESTION ONE (14 marks) Use a separate writing booklet. **Marks**

(a) Express 40° in radians. Leave your answer in terms of π. $\boxed{1}$

(b) Solve $\sin x = -\dfrac{\sqrt{3}}{2}$, for $0 \le x \le 2\pi$. $\boxed{2}$

(c) Differentiate the following: $\boxed{2}$

(i) $y = 3\sin 2x$

(ii) $y = \sin^2 x$

(d) The position x metres at time t seconds of a particle moving in a straight line is given by

$$x = t^2 - 8t + 12, \quad \text{where} \quad t \ge 0.$$

(i) Write down the initial position of the particle. $\boxed{1}$

(ii) At what time does the particle first pass through the origin? $\boxed{2}$

(iii) Differentiate to find the velocity v as a function of time t. $\boxed{1}$

(iv) When is the particle stationary, and where is it at this time? $\boxed{2}$

(v) Find the distance the particle travels in the first 3 seconds. $\boxed{1}$

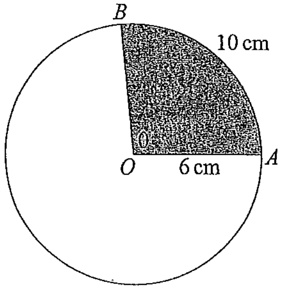
(e) Evaluate the definite integral $\boxed{2}$

$$\int_0^{\frac{\pi}{2}} \sec^2 \tfrac{x}{2}\, dx.$$

Exam continues next page ...

QUESTION TWO (14 marks) Use a separate writing booklet. **Marks**

(a)



A circle has radius 6 cm. An arc AB of length 10 cm subtends an angle θ at the centre of the circle.

(i) Find the exact size of angle θ in radians. 1

(ii) Calculate the area of the shaded sector AOB. 1

(b) Consider the wave function $y = 4\sin\frac{\pi}{3}$.

(i) Write down the amplitude of the wave. 1

(ii) Find the period of the wave. 1

(c) Find $\dfrac{dy}{dx}$ for each function:

(i) $y = e^{\cos x}$ 1

(ii) $y = \ln(\sin x)$ 1

(d) Find the gradient of the tangent to the curve $y = \tan x$ at $x = \frac{\pi}{3}$. 2

(e) A ball is projected upwards with a velocity of 25 m/s from a fixed point situated 30 m above the ground. Using the ground as the origin, take $g = 10\,\text{m/s}^2$ and upwards as positive, so that the acceleration function is $a = -10$.

(i) Find the velocity function v and the displacement function x, in terms of time t. 2

(ii) Find the time taken for the ball to reach its highest point. 1

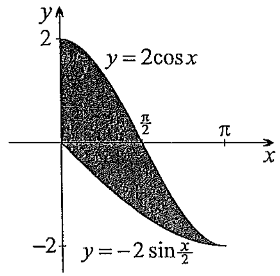
(iii) At what speed will the ball strike the ground? 3

Exam continues overleaf ...

QUESTION THREE (14 marks) Use a separate writing booklet. **Marks**

(a) Calculate the exact area enclosed by the hyperbola $y = \dfrac{2}{x+1}$, the x-axis and the lines $x = 1$ and $x = 3$. 2

(b) 3



The diagram above shows the curves $y = 2\cos x$ and $y = -2\sin\frac{x}{2}$ sketched for $0 \le x \le \pi$. Find the shaded area.

(c) Consider the function $y = 2x - 3\ln x$.

(i) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. 2

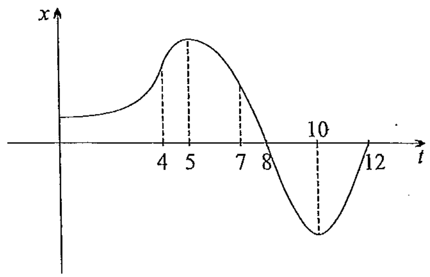
(ii) Hence show that the curve $y = 2x - 3\ln x$ has a minimum turning point at $x = \frac{3}{2}$. 2

(d) The velocity of a particle moving in a straight line is given by 3

$$v = 1 + \frac{8}{2t+1}.$$

Show that the distance travelled by the particle in the first 4 seconds is $4 + 8\log_e 3$.

(e) 2



A particle moves along a straight line. The graph above shows the distance x of the particle from a fixed point (the origin) at time t. Sketch the graph of the velocity v as a function of time for $0 \le t \le 12$.

Exam continues next page ...

QUESTION 1

a) $180^\circ = \pi$

$40^\circ = \dfrac{\pi}{180} \times 40$

$40^\circ = \dfrac{2\pi}{9}$ radians ✓

b) $\sin x = -\dfrac{\sqrt{3}}{2}$ ✓ | ✓

$x = \pi + \dfrac{\pi}{3}$ or $x = 2\pi - \dfrac{\pi}{3}$

$x = \dfrac{4\pi}{3}$ ✓ or $x = \dfrac{5\pi}{3}$ ✓

c) (i) $y = 3\sin 2x$

$y' = 6\cos 2x$ ✓

(ii) $y = \sin^2 x$

$y' = 2\sin x\cos x$ ✓

(d) $x = t^2 - 8t + 12$

(i) initial position $(t=0)$

$x = 12.$ ✓

12 units to the right of the origin.

(ii) put $x = 0$

$t^2 - 8t + 12 = 0$

$(t-6)(t-2) = 0$ ✓

$t = 2$ or $t = 6$

The particle first passes through the origin at $t = 2$s. ✓

(iii) $\dot{x} = 2t - 8$ ✓

(iv) $\dot{x} = 0$

$2t - 8 = 0$

$2t = 8$

$t = 4$s ✓

$x = 16 - 32 + 12$

$x = -4$

The particle is stationary after 4s, it is 4 units left of the origin. ✓

(v)

t	0	1	2	3
x	12	5	0	-3

The particle travels 15m in the first 3 seconds. ✓

(e) $\displaystyle\int_0^{\frac{\pi}{2}} \sec^2 \frac{x}{2}\, dx$

$= \left[2\tan\dfrac{x}{2}\right]_0^{\pi/2}$ ✓

$= 2\left(\tan\dfrac{\pi}{4} - \tan 0\right)$

$= 2$ ✓

14

QUESTION 2

a) (i) $r\theta = l$

$6\theta = 10$

$\theta = \dfrac{5}{3}$ radians ✓

(ii) $A_{\text{sector}} = \dfrac{1}{2}r^2\theta$

$= \dfrac{1}{2} \times 36 \times \dfrac{5}{3}$

$= 30$ cm^2 ✓

b) $y = 4\sin\dfrac{x}{3}$

(i) amplitude $= 4$ ✓

(ii) period $T = \dfrac{2\pi}{n}$

$T = \dfrac{2\pi}{1/3}$

$T = 6\pi$ ✓

c) (i) $y = e^{\cos x}$

$\dfrac{dy}{dx} = -\sin x\, e^{\cos x}$ ✓

(ii) $y = \ln(\sin x)$

$\dfrac{dy}{dx} = \dfrac{\cos x}{\sin x}$ ✓

$= \cot x$

(d) $y = \tan x$

$\dfrac{dy}{dx} = \sec^2 x$ ✓

at $x = \dfrac{\pi}{3}$

gradient $= \sec^2\dfrac{\pi}{3}$

$= 4.$ ✓

(e) (i) $\ddot{x} = -10$

$\dot{x} = -10t + c_1$

. when $t = 0$, $\dot{x} = 25$

$\dot{x} = -10t + 25$ ✓

$x = -5t^2 + 25t + c_2$

. when $t = 0$, $x = 30$

$x = -5t^2 + 25t + 30$ ✓

(ii) highest point $\dot{x} = 0$

$-10t + 25 = 0$

$10t = 25$

$t = 2\frac{1}{2}$ s. ✓

(iii) put $x = 0$.

$-5t^2 + 25t + 30 = 0$

$t^2 - 5t - 6 = 0$

$(t+1)(t-6) = 0$

hits the ground at $t = 6$s. ✓

$\dot{x} = -10t + 25$

$\dot{x} = -60 + 25$

$\dot{x} = -35$ m/s. ✓

The ball hits the ground at 35 m/s. ✓

14

QUESTION 3

(a)

$$A = \int_1^3 \frac{2}{x+1}\,dx$$

$$A = \Big[2\ln(x+1)\Big]_1^3 \checkmark$$

$$= 2\ln 4 - 2\ln 2$$

$$= \ln 4 \text{ sq units} \checkmark$$

(b)

$$A = \int_0^{\pi} 2\cos x + 2\sin\frac{x}{2}\,dx \checkmark$$

$$= \Big[2\sin x - 4\cos\frac{x}{2}\Big]_0^{\pi} \checkmark$$

$$= (0) - (-4)$$

$$= 4 \text{ sq units} \checkmark$$

(c) $y = 2x - 3\ln x$

(i) $$\frac{dy}{dx} = 2 - \frac{3}{x} \checkmark$$

$$\frac{d^2y}{dx^2} = \frac{3}{x^2} \checkmark$$

(ii) $$\frac{dy}{dx} = 0: \quad 2 - \frac{3}{x} = 0$$

$$2x - 3 =$$

$$2x = 3$$

$$x = \frac{3}{2}$$

stationary point at $x = \frac{3}{2}$ ✓

$$\frac{d^2y}{dx^2}: \quad \frac{3}{x^2} = \frac{3}{\frac{9}{4}}$$

at $x = 3/2$ $$= \frac{4}{3} \checkmark$$

$\frac{d^2y}{dx^2} > 0$, $\therefore$ a minimum.

(d) $$v = 1 + \frac{8}{2t+1}$$

distance travelled $$= \int_0^4 1 + \frac{8}{2t+1}\,dt$$

$$= \Big[t + 4\ln(2t+1)\Big]_0^4 \checkmark$$

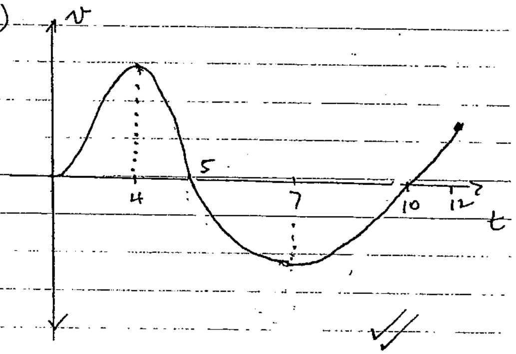
$$= (4 + 4\ln 9) - (0) \checkmark$$

$$= 4 + 4\ln 3^2$$

$$= 4 + 8\ln 3 \checkmark$$

(as required)

(e)



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