



SYDNEY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT
HALF-YEARLY EXAMINATIONS 2005

FORM IV

MATHEMATICS

Examination date

Monday 16th May 2005

Time allowed

2 hours

Instructions

- All eight questions may be attempted.
- All eight questions are of equal value.
- All necessary working must be shown.
- Marks may not be awarded for careless or badly arranged work.
- Approved calculators and templates may be used.

Collection

- Write your name, class and master clearly on the front.
- Hand in all the writing paper in a single well-stapled bundle.
- Keep the printed examination paper and bring it to your next Mathematics lesson.

4A: TCW	4B: KWM	4C: JCM
4D: DS	4E: JMR	4F: REP
4G: FMW	4H: JNC	4I: GJ

Checklist

- Writing paper required.
- Candidature: 192 boys.

Examiner

JNC

QUESTION ONE Start a new page.

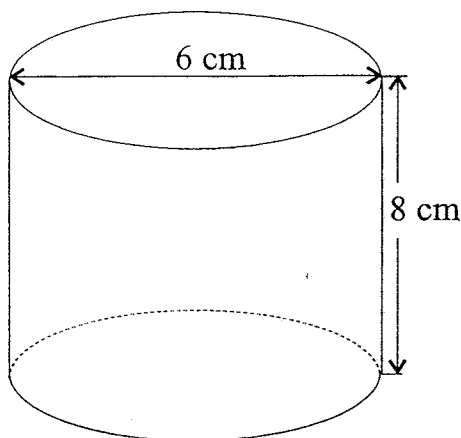
- (a) Expand and simplify $(2x + 1)(x - 3)$.
- (b) Write down the volume of a cube of edge 8 cm.
- (c) Write down the gradient of the line $y = 1 - 2x$.
- (d) Factorise $x^2 + 3x - 10$.
- (e) Find the simple interest earned if \$ 5000 is invested at 4% p.a. for 5 years.
- (f) Find the midpoint of the interval joining $A(-3, 8)$ and $B(-5, -1)$.
- (g) What is the probability of choosing the ace of hearts from a normal deck of 52 playing cards?
- (h) Expand and simplify $(x - 4)^2$.
- (i) Write down the coordinates of the centre and the length of the radius of the circle with equation $(x - 2)^2 + (y + 4)^2 = 1$.
- (j) Write down the solutions of the quadratic equation $2x(x + 5) = 0$.

QUESTION TWO Start a new page.

- (a) Write down the equation of the line with gradient 3 and y -intercept $(0, -6)$.
- (b) Max buys a stereo system with a marked price of \$2135. He pays for it by making 24 equal monthly payments of \$117.45.
 - (i) Calculate the total amount that Max pays for the stereo.
 - (ii) How much interest did Max pay?
 - (iii) Find the annual rate of interest as a percentage of the original price. Give your answer correct to the nearest whole percent.
- (c) Solve the following quadratic equations by the method specified. In each case leave your answers as exact values.
 - (i) Solve $x^2 - 3x - 28 = 0$ by factorising.
 - (ii) Solve $2x^2 + x - 13 = 0$ by the quadratic formula.
 - (iii) Solve $x^2 + 6x + 8 = 0$ by completing the square.

QUESTION THREE Start a new page.

(a)



In the diagram above, the cylindrical can has a diameter of 6 cm and height of 8 cm.

- (i) Find the area of the curved surface of the can. Give your answer correct to the nearest square centimetre.
- (ii) Find the volume of the can. Give your answer correct to the nearest cubic centimetre.

(b) (i) Copy and complete the table below of values for the graph of $y = \frac{2}{x}$.

x	-4	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2	4
y								

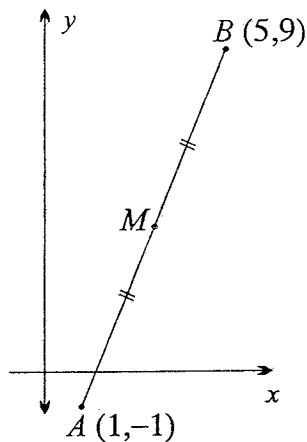
- (ii) Write down the equation of the horizontal asymptote.
 - (iii) Sketch the graph of $y = \frac{2}{x}$ using a scale of 1 unit = 1 cm on both axes.
- (c) In a raffle, 100 people bought 3 tickets each, 30 people bought 2 tickets each and 15 people bought 1 ticket each.
- (i) If Stan was one of those who bought 1 ticket, what is the probability that he wins the raffle?
 - (ii) What is the probability that someone who bought 2 tickets wins the raffle?
- (d) Write down the equation of the axis of symmetry of the parabola $y = (x - a)^2$.

QUESTION FOUR Start a new page.

- (a) Find the gradient of any line that is parallel to the line $3x + 2y - 12 = 0$.
- (b) Find in simplest form the gradient of the interval joining $A(2a, -2b)$ and $B(4b, -a)$.
- (c) Alan bought a new car valued at \$50 000. It depreciates in value at the rate of 18% per annum.
- What is the car worth after 2 years?
 - How much value will the car have lost after 4 years? Give your answer correct to the nearest dollar.
- (d) Consider the parabola $y = x^2 - 2x - 8$.
- Find the x -intercepts of the parabola.
 - Find the coordinates of the vertex of the parabola.
 - Sketch the graph of $y = x^2 - 2x - 8$.
 - What is the maximum value of the expression $8 + 2x - x^2$?

QUESTION FIVE Start a new page.

(a)

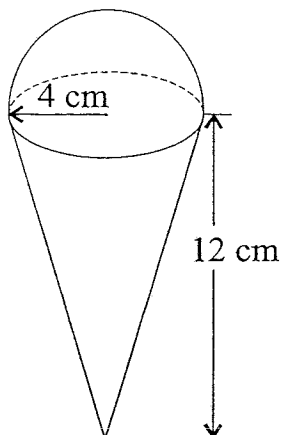


In the diagram above, M is the midpoint of the interval joining $A(1, -1)$ and $B(5, 9)$.

- Show that the length of the interval AB is $2\sqrt{29}$.
- Write down the coordinates of M .
- Write down the equation of the circle with diameter AB .
- Find the gradient of AB .
- Find the equation of the line that is parallel to AB and passes through the origin. Give your answer in general form.

Exam continues next page ...

(b)



In the diagram above, a hemi-spherical blob of ice cream of radius 4 centimetres is placed on top of a waffle cone of height 12 centimetres.

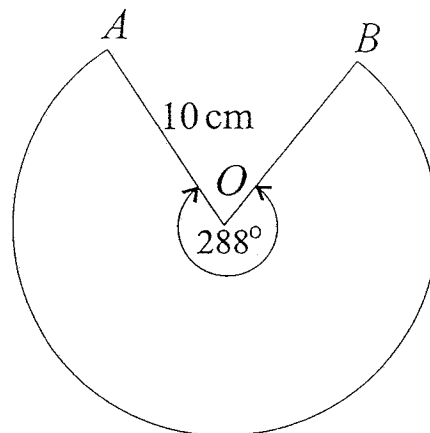
- (i) Find the volume of ice cream. Give your answer correct to the nearest cubic centimetre.
- (ii) The ice cream melts into the cone and the liquid has the same volume as the solid blob of ice cream. It fills the cone to a depth of H centimetres, and has a surface of radius R centimetres.

Given that $\frac{H}{R} = 3$, find the depth of the melted ice cream, giving your answer correct to the nearest millimetre.

QUESTION SIX Start a new page.

- (a) A different letter of the alphabet was written on each of 26 cards. One of these cards is then drawn at random. What is the probability that the card drawn is:
 - (i) a letter of the word *mathematics*?
 - (ii) either a letter of the word *trick* or a letter of the word *quick*?
- (b)
 - (i) If x is an odd integer, write down an expression for the next odd integer.
 - (ii) Form a quadratic equation and solve it to find two consecutive odd positive integers such that the sum of their squares is 394.

(c)



The diagram above shows a sector of a circle with centre O , radius 10 centimetres and an angle of 288° at the centre.

- (i) Show that the length of major arc AB is 16π centimetres.
- (ii) The sector is cut out and the edges OA and OB are joined so that a cone is formed. Show that the radius of the base of the cone is 8 centimetres.
- (iii) Find the exact volume of the cone in part (ii).

QUESTION SEVEN Start a new page.

- (a) A shell is fired from ground level. After t seconds, its height h in metres is given by $h = 50t - 5t^2$.
 - (i) How long is the shell in the air?
 - (ii) What is the maximum height reached?
 - (iii) Draw a graph of the parabola $h = 50t - 5t^2$, for $0 \leq t \leq 10$.
 - (iv) A second, similar shell is fired from the edge of a cliff, which is 375 metres above sea-level. How long will it take for the shell to hit the water after it is fired?
- (b) A cylinder has a radius of r centimetres and a height of h centimetres.
 - (i) Find the volume, in terms of r and h , of the cylinder if its radius is increased by 25% and the height is increased by 10% .
 - (ii) By what percentage has the volume of the cylinder increased?

QUESTION EIGHT Start a new page.

(a) (i) Simplify $\frac{1}{n} - \frac{1}{n+1}$.

(ii) Find the sum of the series $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{100 \times 101}$.

(b) An office has two envelope-filling machines. Working together, they can fill a batch of envelopes in 2 hours. Working separately, it will take the second machine 1 hour longer than the first machine to fill a batch of envelopes. How long will it take each machine to fill a batch of envelopes by themselves? Give your answer correct to the nearest minute.

(c) Michael invested a sum of money $\$P$ on January 1st, 1975. The account received interest, which was compounded annually. On January 1st, 1985 there was $\$44\,771.20$ in the account and on January 1st 2005 the amount of money in the account had grown to $\$143\,587.28$.

(i) Find the rate of interest that Michael received on his investment. Give your answer correct to the nearest whole percent per annum.

(ii) Find the amount $\$P$ that Michael invested in 1975?

(iii) In what year will the account have more than $\$250\,000$ in it?

END OF EXAMINATION

FORM IV 2005 HALF-YEARLY

QUESTION ONE

a) $(2x+1)(x-3) = 2x^2 - 5x - 3$ ✓

b) $V = 512 \text{ cm}^3$ ✓

c) $m = -2$ ✓

d) $x^2 + 3x - 10 = (x+5)(x-2)$ ✓

e) Interest = $5000 \times 0.04 \times 5$
 $= \$1000$ ✓✓

f) $M = \left(\frac{-3-5}{2}, \frac{8-1}{2} \right)$ ✓✓

$= \left(-4, \frac{7}{2} \right)$ ✓✓

g) $P(\text{ace hearts}) = \frac{1}{52}$ ✓

h) $(x-4)^2 = x^2 - 8x + 16$ ✓

i) centre = $(2, -4)$ ✓

radius = 1 ✓

j) $2x(x+5) = 0$

Either $2x = 0$ or $x+5 = 0$ ✓✓

$\therefore x = 0$ or -5 ✓✓

Question TWO

a)

$$y = 3x - 6 \quad \checkmark \checkmark$$

b) (i) Total amount = 24×117.45
 $= \$2818.80 \quad \checkmark$

(ii) Interest = $2818.80 - 2135$
 $= \$683.80 \quad \checkmark$

(iii) Rate = $\frac{341.90}{2135} \times \frac{100}{1} \% \text{ (p.a.)} \quad \checkmark$
 $= 16\% \quad \checkmark$

c) (i) $x^2 - 3x - 28 = 0$

$$(x - 7)(x + 4) = 0 \quad \checkmark$$

$$\therefore x = -4 \text{ or } 7 \quad \checkmark \checkmark$$

(ii) $2x^2 + x - 13 = 0$

$$x = \frac{-1 \pm \sqrt{1 + 4 \times 2 \times 13}}{2 \times 2} \quad \checkmark$$

$$\therefore x = \frac{-1 \pm \sqrt{105}}{4} \quad \checkmark$$

(-1 for approximation)

(iii) $x^2 + 6x + 8 = 0$

$$x^2 + 6x + 9 = 1 \quad \checkmark$$

$$(x + 3)^2 = 1 \quad \checkmark$$

$$x + 3 = \pm 1 \quad \checkmark$$

$$\therefore x = -4 \text{ or } -2 \quad \checkmark$$

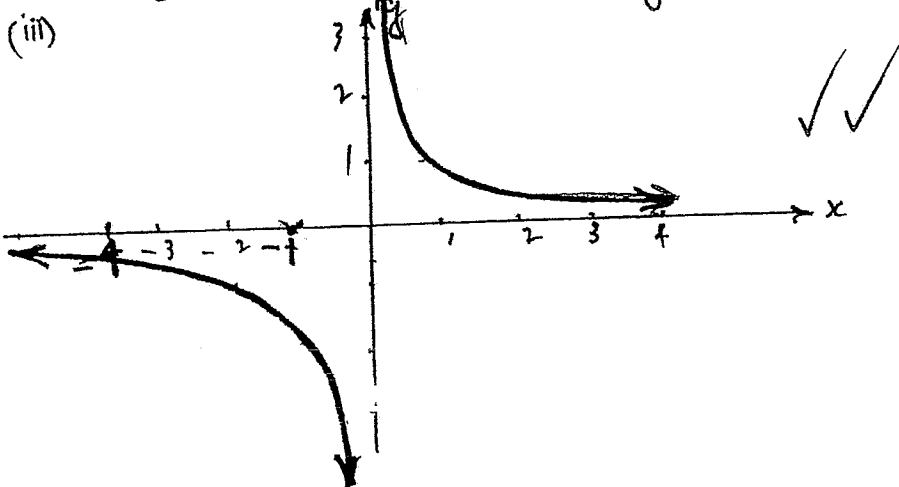
QUESTION THREE

a) (i) Area = $2\pi rh$ } ✓
 $= 2\pi \times 3 \times 8$ } ✓
 $= 151 \text{ cm}^2$ ✓

(ii) Volume = $\pi r^2 h$ } ✓
 $= \pi \times 9 \times 8$ } ✓
 $= 226 \text{ cm}^3$ ✓

b) (i) x -4 -2 -1 - $\frac{1}{2}$ - $\frac{1}{2}$ 1 2 4 } ✓✓
 y - $\frac{1}{2}$ -1 -2 -4 4 2 1 $\frac{1}{2}$ ✓✓

(ii) Horizontal asymptote is $y=0$ ✓



c) (i) No. of tickets sold = $3 \times 100 + 2 \times 30 + 1 \times 15$
 $= 375$ ✓

$P(\text{stan wins}) = \frac{1}{375}$ ✓

(ii) $P(\text{two ticket person}) = \frac{30 \times 2}{375}$ ✓
 $= \frac{4}{25}$ ✓

d) Axis of symmetry is $x=a$. ✓

QUESTION FOUR

a) $3x + 2y - 12 = 0$

$2y = -3x + 12$

$y = -\frac{3}{2}x + 6$

$\therefore m = -\frac{3}{2}$

b) A $(2a, -2b)$ B $(4b, -a)$

$m = \frac{-a + 2b}{4b - 2a}$

$= \frac{(-a + 2b)}{2(-a + 2b)}$

$= \frac{1}{2}$

(c)(i) Value after 2 years = $50000(1 - 0.18)^2$
= \$33620

(ii) Value after 4 years = $50000(1 - 0.18)^4$
= \$22606

It has lost \$27394 in value

d)(i) $y = x^2 - 2x - 8$

$(x - 4)(x + 2) = 0$

$\therefore x = -2, 4$

or $(-2, 0)$ and $(4, 0)$ are intercepts

(ii) $x = \frac{4 - 2}{2}$

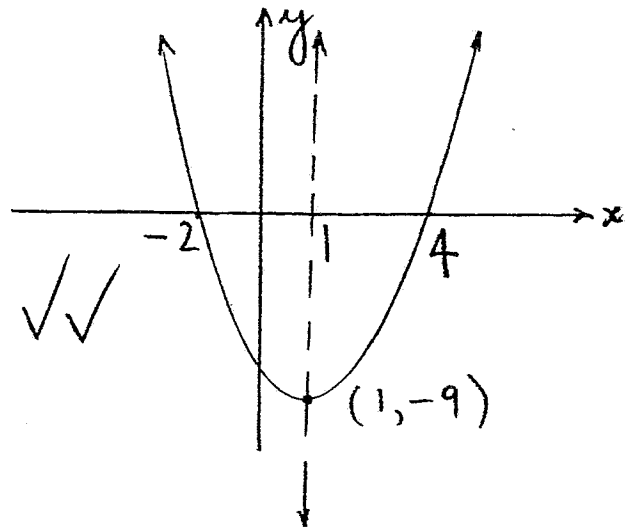
$= 1$

and $y = 1 - 2 - 8$

$= -9$

Vertex is $(1, -9)$

(iii)



(iv) Maximum value of $8 + 2x - x^2$ is 9.

QUESTION FIVE

$$\begin{aligned} \text{a) (i) } AB &= \sqrt{(9-(-1))^2 + (5-1)^2} \\ &= \sqrt{100 + 16} \\ &= \sqrt{116} \\ &= 2\sqrt{29} \end{aligned}$$

$$\text{(ii) } M = (3, 4)$$

$$\text{(iii) } (x-3)^2 + (y-4)^2 = 29$$

$$\text{(iv) } m = \frac{10}{4} = \frac{5}{2}$$

$$\text{(v) } y = \frac{5}{2}x$$

$$5x - 2y = 0$$

$$\begin{aligned} \text{b) (i) } V &= \frac{1}{2} \times \left(\frac{4}{3} \pi r^3 \right) \\ &= \frac{2}{3} \times \pi \times 4^3 \\ &= 134 \text{ cm}^3 \end{aligned}$$

$$\text{(ii) } V_{\text{cone}} = \frac{1}{3} \pi R^2 H$$

$$\frac{1}{3} \pi R^2 H = 134 \text{ and } R = \frac{H}{3}$$

$$\frac{1}{3} \pi \left(\frac{H}{3} \right)^2 \cdot H = 134$$

$$H = \sqrt[3]{\frac{134 \times 27}{\pi}}$$

$$= 10.481 \dots$$

Height of ice cream is 10.48 cm which is 10.5 cm (or 105 mm) to nearest mm.

rounding penalty

QUESTION SIX

$$\begin{aligned} \text{a) (i) } P(\text{mathematics}) &= \frac{8}{26} \\ &= \frac{4}{13} \\ \text{(ii) } P(\text{quick or trick}) &= \frac{7}{26} \end{aligned}$$

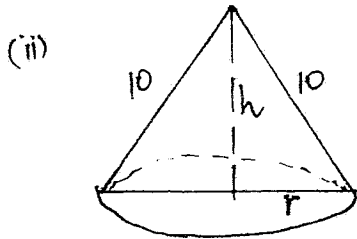
$$\begin{aligned} \text{b) (i) } &x + 2 \\ \text{(ii) } &x^2 + (x + 2)^2 = 394 \\ &x^2 + x^2 + 4x + 4 = 394 \\ &2x^2 + 4x - 390 = 0 \\ &x^2 + 2x - 195 = 0 \\ &(x + 15)(x - 13) = 0 \\ &\therefore x = 13 \text{ or } -15 \end{aligned}$$

So the numbers are 13 and 15

$$\begin{aligned} \text{c) (i) Length AB} &= \frac{288}{360} \times 2\pi \times 10 \\ &= 16\pi \text{ cm} \end{aligned}$$

Must have a conclusion here.

(1 for $\frac{288}{360}$
1 for $2\pi \times 10$)



$$\begin{aligned} 2\pi r &= 16\pi \\ \therefore r &= 8 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{(iii) } r^2 + h^2 &= 10^2 \\ h^2 + 64 &= 100 \\ h^2 &= 36 \\ h &= 6 \end{aligned}$$

$$\begin{aligned} \text{Volume} &= \frac{1}{3} \pi \times 8^2 \times 6 \\ &= 128\pi \text{ cm}^3 \end{aligned}$$

QUESTION SEVEN

a) (i) $h = 5t(10-t)$

(1 for writing)

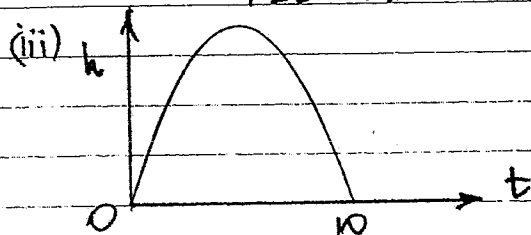
When $h=0$, $t=0$ or 10

So shell is in air for 10 seconds. ✓✓

(ii) Max height when $t=5$:

$$h = 5 \times 5 (10-5)$$

$$= 125 \text{ m}$$



✓✓

1 for label axes

1 for scale

(iv) $50t - 5t^2 = -375$ ✓

$$t^2 - 10t - 75 = 0$$

$$(t-15)(t+5) = 0$$
 ✓

$$\therefore t = -5 \text{ or } 15$$

It will take the shell 15 seconds to hit water. ✓

(b) (i) After increase:

$$V = \pi \left(\frac{5r}{4}\right)^2 \times \frac{11h}{10}$$
 ✓✓

$$= \pi r^2 h \times \frac{55}{32}$$
 ✓

(ii) % increase = $\left(\frac{55}{32} - 1\right) \times \frac{100}{1} \%$ ✓

$$= 71.875 \%$$
 ✓

OR

(i) $V = \pi (1.25r)^2 \times 1.1h$

$$= 1.71875 \times \pi r^2 h$$

(ii) % increase = $(1.71875 - 1) \times \frac{100}{1} \%$

$$= 71.875 \%$$

(c)

$$(i) \quad P \left(1 + \frac{r}{100}\right)^{10} = 44771.20 \quad \checkmark$$

$$P \left(1 + \frac{r}{100}\right)^{30} = 143587.28 \quad \checkmark$$

$$\left(1 + \frac{r}{100}\right)^{20} = 3.207134944 \quad \checkmark$$

$$1 + \frac{r}{100} = 1.0599999 \quad \checkmark$$

$$\therefore r = 6\% \text{ p.a.} \quad \checkmark$$

$$(ii) \quad P \times 1.06^{10} = 44771.20$$

$$\therefore P = \$25000 \quad \checkmark$$

$$(iii) \quad 25000 \times 1.06^n > 250000$$

$$1.06^n > 10 \quad \checkmark$$

By calculator or trial and error:

$$1.06^{39} = 9.7 \dots \quad \text{and} \quad 1.06^{40} = 10.28 \dots \quad \checkmark$$

On 1st January 2015 the account will have more than \$250000 in it.