SYDNEY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT
HALF-YEARLY EXAMINATIONS 2006

FORM III

MATHEMATICS

Examination date

Wednesday 24th May 2006

Time allowed

1 hour 30 minutes

Instructions

All eight questions may be attempted.
All eight questions are of equal value.
All necessary working must be shown.
Marks may not be awarded for careless or badly arranged work.
Calculators are not to be used.

Collection

Write your name, class and master clearly on the front.
Hand in all the writing paper in a single well-stapled bundle.
Keep the printed examination paper and bring it to your next Mathematics lesson.

3A: FMW	3B: KWM	3C: SJE/MLS	3D: LYL/TCW
3E: JCM	3F: GJ	3G: PKH	3H: DS
3I: REP	3J: REN		

Checklist

Writing paper required.
Candidature: 186 boys.

Examiner

SJE/DS

QUESTION ONE (12 marks) Start a new page.

(a) Write 13·43449 correct to 3 decimal places.

(b) Simplify:

(i) $2p^2 - p^2$

(ii) $12a - 4a - 3a + 3$

(iii) $2b^4 \times b$

(iv) $\sqrt{45}$

(c) Evaluate:

(i) $\sqrt{5} \times \sqrt{5}$

(ii) $\sqrt{0{\cdot}81}$

(d) Solve the equation $2m - 7 = 25$.

(e) Evaluate $2x^2 - 5x$ when $x = -2$.

(f) Solve the inequation $x - 3 > -1$ and graph your solution on a number line.

(g) Express $\dfrac{2}{\sqrt{3}}$ with a rational denominator.

QUESTION TWO (12 marks) Start a new page.

(a) Simplify:

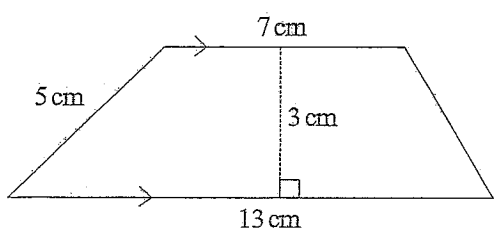
(i) $\dfrac{8a + a}{ab}$

(ii) $\dfrac{3}{x} \div \dfrac{y}{x^2}$

(iii) $7^{2b} \div 7^b$

(iv) $\sqrt{50} - \sqrt{8}$

(b)



Find the area of the trapezium above.

Exam continues next page ...

(c) Solve for x:

$$\frac{4-x}{12} \geq 7$$

(d) Find A and n if $Ax^n = (2x^3)^5$.

(e) A liquid container in the shape of a rectangular prism has dimensions 8 cm, 15 cm and 20 cm. How many litres will it hold?

QUESTION THREE (12 marks) Start a new page.

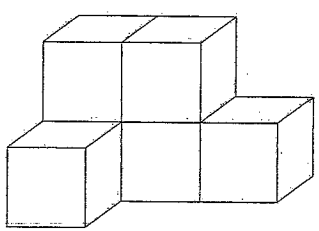
(a) Evaluate:

(i) 3×5^0

(ii) $100^{\frac{3}{2}}$

(iii) $\left(\frac{1}{3}\right)^{-2}$

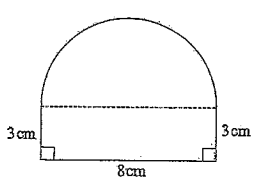
(b)



Find the surface area of the solid above which is made up of cubes of side 1 cm.

(c) Write the formula $C = 3P + Q$ with P as the subject.

(d)



The figure above is made up from a semi-circle and a rectangle. Find the perimeter, leaving your answer in terms of π.

(e) Expand and simplify:

(i) $(a-7)^2$

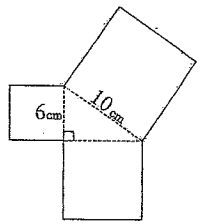
(ii) $(3x+2y)(3x-2y)$

(iii) $(2\sqrt{5}-\sqrt{3})(\sqrt{3}+\sqrt{5})$

Exam continues overleaf ...

QUESTION FOUR (12 marks) Start a new page.

(a)



The diagram above shows a right-angled triangle with squares drawn on each side. The hypotenuse is 10 cm and the length of one of the shorter sides is 6 cm. Find the perimeter of this 9-sided figure.

(b) Write each of the following expressions in the form x^n:

(i) $\frac{1}{\sqrt{x}}$

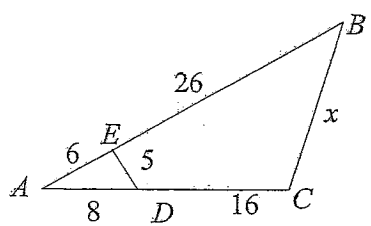
(ii) $\sqrt[3]{x^4}$

(c) If $v = \frac{2\pi r}{t}$, find r when $v = 11$ and $t = 4$. Use $\pi = \frac{22}{7}$.

(d) Express $\frac{3}{\sqrt{8}} + \frac{5}{\sqrt{2}}$ in simplest form with a rational denominator.

(e) Solve $\frac{2a-5}{a+2} = 5$.

(f)



In the diagram above $\triangle ABC \,|||\, \triangle ADE$.

(i) Which angle in $\triangle AED$ is equal to $\angle ABC$?

(ii) Find the value of x.

Exam continues next page ...

QUESTION FIVE (12 marks) Start a new page.

(a) Write in simplest form, without negative indices:

(i) $(5y)^{-2}$

(ii) $(p^2q^{-1})^{-1}$

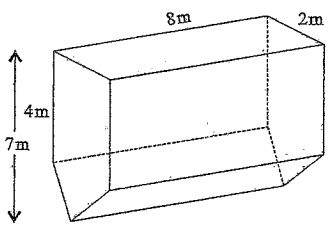
(b) Solve $1 - \dfrac{x-1}{2} = 6$.

(c) If $a = \frac{1}{3}$ and $b = -2$, evaluate:

(i) $3ab^2$

(ii) $a^{-1} + b^{-1}$

(d)



A large holding tank, which has dimensions as indicated in the diagram, is used to load grain into the carriages of a goods train. What volume can it hold when full?

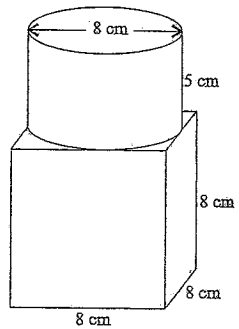
(e) Expand and simplify: $(1+x)(1-x) - (1-x)^2$.

(f) Simplify $(p^{\frac{1}{4}} \times p^{\frac{1}{12}})^2$, leaving your answer in index form.

Exam continues overleaf ...

QUESTION SIX (12 marks) Start a new page.

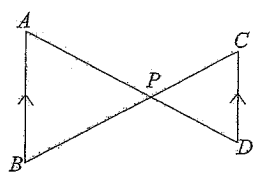
(a)



A cylinder of diameter 8 cm and height 5 cm sits completely on top of a cube with sides 8 cm in length. Find the surface area, leaving your answer in terms of π.

(b) Express $\dfrac{3+\sqrt{2}}{1+\sqrt{2}}$ in simplest form with a rational denominator.

(c)



Prove that the triangles in the diagram above are similar.

(d) Half the sum of three consecutive even numbers is 237. Form an equation and solve it to find the numbers.

(e) Write $\dfrac{x+y}{x^{-1}+y^{-1}}$ in simplest form without negative indices.

Exam continues next page ...

QUESTION SEVEN (12 marks) Start a new page.

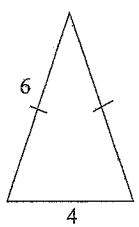
(a) Make r the subject of the formula $V = \frac{4}{3}\pi r^3$.

(b) 6.022×10^{23} atoms of silver have a mass of 108 g.

(i) Write this number of atoms correct to 2 significant figures. Leave your answer in scientific notation.

(ii) Using your answer in part (i), find the mass, in kilograms, of one atom of silver. Express your answer in scientific notation.

(c)



The isosceles triangle above has dimensions as marked.

(i) A formula for the area of any triangle is $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{a+b+c}{2}$ and a, b and c are the side lengths. Calculate the area using this formula.

(ii) Verify your answer to part (i) by using the standard formula for the area of a triangle.

(d) Solve this problem by forming and then solving an equation.

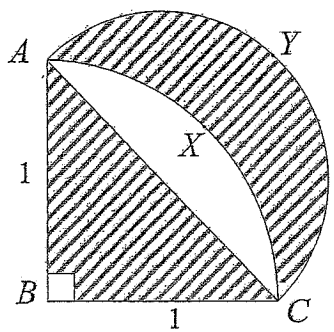
Two cyclists A and B leave at 9 a.m., riding towards each other from two towns 140 km apart on a straight road. A rides at 40 km/h and B rides at 30 km/h. If they meet after T hours, find where and when they meet.

Exam continues overleaf ...

QUESTION EIGHT (12 marks) Start a new page.

(a) Solve the equation $2^{x-1} = 8^{x+1}$.

(b)

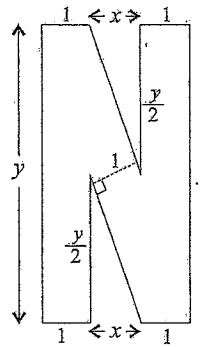


In the diagram above, the outer arc AYC is a semicircle with diameter AC. The inner arc AXC is a quarter of a circle, centre B. Show that the area of the two shaded regions is the same.

(c) (i) Show that $x^2 + y^2 = (x+y)^2 - 2xy$.

(ii) Hence, or otherwise, show that if $x = \frac{-1+\sqrt{5}}{2}$ and $y = \frac{-1-\sqrt{5}}{2}$ then $x^2 + y^2 = 3$.

(d)



Consider the diagram of the letter N above with dimensions as marked.

(i) Find the area of the letter N in terms of x and y.

(ii) Use areas to show that $x^2 = \frac{y^2}{y^2-4}$.

END OF EXAMINATION

Marks

Question 1

(a) 13.434 ✓

(b) (i) $2\rho^2 - \rho^2 = \rho^2$ ✓

(ii) $12a - 4a - 3a + 3 = 5a + 3$ ✓

(iii) $2b^4 \times b = 2b^5$ ✓

(iv) $\sqrt{45} = 3\sqrt{5}$ ✓

(c) (i) $\sqrt{5} \times \sqrt{5} = 5$ ✓

(ii) $\sqrt{0.81} = 0.9$ ✓

(d) $2m - 7 = 25$

$2m = 32$

$m = 16$ ✓

(e) $2x^2 - 5x$, $x = -2$

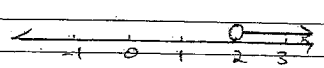
$= 2(-2)^2 - 5(-2)$

$= 8 + 10$

$= 18$ ✓

(f) $x - 3 > -1$

$x > 2$ ✓



✓

(g) $\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ ✓

Marks

Question 2

a) (i) $\frac{8a + a}{ab} = \frac{9}{b}$ ✓

(ii) $\frac{3}{x} \div \frac{y}{x^2} = \frac{3}{x} \times \frac{x^2}{y}$

$= \frac{3x}{y}$ ✓

(iii) $7^{2b} \div 7^b = 7^b$ ✓

(iv) $\sqrt{50} - \sqrt{8} = 5\sqrt{2} - 2\sqrt{2}$

$= 3\sqrt{2}$ ✓

(b) Area $= \frac{1}{2} \times 3(13 + 7)$

$= 30 \text{ cm}^2$ ✓✓ (1 mark for units)

(c) $\frac{4 - x}{12} \geq 7$

$4 - x \geq 84$ ✓

$-x \geq 80$

$x \leq -80$ ✓

(d) $Ax^n = (2x^3)^5$

$= 32x^{15}$ (1 mark for correctly expanding but not expressing what A and n are)

$\therefore A = 32$ ✓

$n = 15$ ✓

(e) Volume $= 15 \times 8 \times 20 \text{ cm}^3$

$= 2400 \text{ cm}^3$ ✓

$= 2.4 \text{ L}$ ✓

Marks

Question 3

(a) (i) $3 \times 5^0 = 3$ ✓

(ii) $100^{3/2} = 1000$ ✓

(iii) $\left(\frac{1}{3}\right)^{-2} = 3^2$
$= 9$ ✓

(b) Surface Area $= (4+4) + (3+3) + (5+5)$
$= 24 \text{ cm}^2$ ✓

(c) $C = 3P + Q$
$3P = C - Q$ ✓
$P = \frac{C-Q}{3}$ ✓

(d) Perimeter $= (3+8+3) + \frac{1}{2}\pi 8$
$= (14 + 4\pi)$ cm ✓✓ (1 mark for perimeter of each part)

(e) (i) $(a-7)^2 = a^2 - 14a + 49$ ✓

(ii) $(3x+2y)(3x-2y) = 9x^2 - 4y^2$ ✓

(iii) $(2\sqrt{5} - \sqrt{3})(\sqrt{3} + \sqrt{5}) = 2\sqrt{15} + 10 - 3 - \sqrt{15}$ ✓
$= \sqrt{15} + 7$ ✓

Marks

Question 4

a) Length of 3rd side $= \sqrt{10^2 - 6^2}$
$= 8$ cm ✓

Perimeter $= 3(10+6+8)$
$= 72$ cm ✓

b) (i) $\frac{1}{\sqrt{x}} = x^{-1/2}$ ✓

(ii) $\sqrt[3]{x^4} = x^{4/3}$ ✓

c) $V = \frac{2\pi r}{t}$

Rearranging, $r = \frac{Vt}{2\pi}$
$= \frac{11 \times 4}{2 \times \frac{22}{7}}$ ✓ (1 mark for: $11 = \frac{2 \times \frac{22}{7} r}{4}$)
$= \frac{44}{\frac{44}{7}}$
$= 7$ ✓

(d) $\frac{3}{\sqrt{8}} + \frac{5}{\sqrt{2}} = \frac{3+10}{2\sqrt{2}}$ ✓ (1 mark for correctly rationalising both terms)
$= \frac{13}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{13\sqrt{2}}{4}$ ✓

(e) $\frac{2a-5}{a+2} = 5$
$2a - 5 = 5(a+2)$
$2a - 5 = 5a + 10$ ✓
$-3a = 15$
$a = -5$ ✓

(f) (i) $\angle ADE$ ✓

(ii) $\frac{x}{32} = \frac{5}{8}$
$x = \frac{160}{8}$
$= 20$ ✓

Marks

Question 5

(a) (i) $(5y)^{-2} = \frac{1}{25y^2}$ ✓

(ii) $(p^2q^{-1})^{-1} = p^{-2}q^1$

$= \frac{q}{p^2}$ ✓

(b) $1 - \frac{x-1}{2} = 6$

$2 - (x-1) = 12$

$2 - x + 1 = 12$ ✓ (1 mark for correct working with only one arithmetic error)

$-x = 9$

$x = -9$ ✓

(c) (i) $3ab^2 = 3(\frac{1}{3})(-2)^2$

$= 4$ ✓

(ii) $a^{-1} + b^{-1} = (\frac{1}{3})^{-1} + (-2)^{-1}$

$= 3 + (-\frac{1}{2})$

$= 2\frac{1}{2}$ or $\frac{5}{2}$ ✓

(d) Volume $= Ah$

$= [(4\times2) + (\frac{1}{2}\times2\times3)] \times 8$ ✓ (1 mark for correct working with 1 incorrect substitution or 1 arithmetic error)

$= 88 \text{ m}^3$ ✓

(e) $(1+x)(1-x) - (1-x)^2 = 1 - x^2 - (1 - 2x + x^2)$ ✓

$= 1 - x^2 - 1 + 2x - x^2$

$= 2x - 2x^2$ ✓

(f) $(p^{1/4} \times p^{1/12})^2 = p^{1/2} \times p^{1/6}$ ✓ (or $(p^{1/3})^2$)

$= p^{2/3}$ ✓

Marks

Question 6

a) Surface area $= (8^2 \times 6) + 2\pi 4 \times 5$ ✓ (1 mark for correctly calculating the curved surface $2\pi rh$ or the surface area of the cube)

$= 64 \times 6 + 40\pi$

$= 384 + 40\pi \text{ cm}^2$ ✓

b) $\frac{3+\sqrt{2}}{1+\sqrt{2}} = \frac{3+\sqrt{2}}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$

$= \frac{3 - 3\sqrt{2} + \sqrt{2} - 2}{1-2}$ ✓

$= \frac{1 - 2\sqrt{2}}{-1}$

$= 2\sqrt{2} - 1$ ✓

c) In the triangles APB and DPC

$\angle BAP = \angle CDP$ (alternate angles, AB||CD) ✓

$\angle APB = \angle DPC$ (vertically opposite angles) ✓

$\therefore \triangle APB \,|||\, \triangle DPC$ (AA similarity test) ✓

Must have reasons to get the mark

Vertices must be in corresponding order

d) Let the numbers be $x, x+2, x+4$

$\frac{1}{2}(x + x + 2 + x + 4) = 237$ ✓

$\frac{1}{2}(3x + 6) = 237$

$3x + 6 = 474$

$3x = 468$

$x = 156$ ✓

$\therefore$ The numbers are 156, 158, 160 ✓

e) $\frac{x+y}{x^{-1}+y^{-1}} = \frac{x+y}{\frac{1}{x}+\frac{1}{y}}$

$= \frac{x+y}{\frac{y+x}{xy}}$ ✓

$= \frac{xy(x+y)}{y+x}$

$= xy$ ✓

Marks

Question 7

(a) $V = \frac{4}{3}\pi r^3$

$r^3 = \frac{3V}{4\pi}$ ✓ (1 mark for trying to find the cube root)

$r = \sqrt[3]{\frac{3V}{4\pi}}$ ✓

(b) (i) 6.0×10^{23} ✓

(ii) Let x be the mass of 1 atom

$x(6.0 \times 10^{23}) = 108\,g$

$x = \frac{108\,g}{6 \times 10^{23}}$ ✓

$= 18 \times 10^{-23}\,g$

$= 1.8 \times 10^{-25}\,kg$ ✓

(c) (i) $s = \frac{6+6+4}{2}$

$= 8$ ✓

Area $= \sqrt{8(8-6)(8-6)(8-4)}$

$= \sqrt{8 \times 16}$

$= 4\sqrt{8}$

$= 8\sqrt{2}$ square units ✓

(ii) height of triangle $= \sqrt{6^2 - 2^2}$

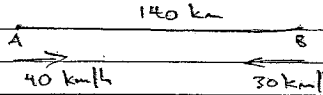
$= \sqrt{32}$ ✓

So Area $= \frac{1}{2} \times 4 \times \sqrt{32}$

$= 2\sqrt{32}$ ✓

$= 8\sqrt{2}$ square units

(d)



After T hours A has travelled $40T$ km

B has travelled $30T$ km

$40T + 30T = 140$ ✓

$T = 2$ hours

So they meet at 11:00 am ✓

A travels 80 km, B travels 60 km

So they meet 80 km from A (or 60 km from B) ✓

Marks

Question 8

(a) $2^{x-1} = 8^{x+1}$

$2^{x-1} = 2^{3(x+1)}$

So $x - 1 = 3(x+1)$ ✓

$x - 1 = 3x + 3$

$-2x = 4$

$x = -2$ ✓

(b) Area of $\triangle ABC = \frac{1}{2} \times 1 \times 1$

$= \frac{1}{2}$ square unit ✓ 1st mark for area of triangle

Area of Segment AXC (white area)

$= \frac{1}{4}\pi(1)^2 - \frac{1}{2}$

$= \left(\frac{\pi}{4} - \frac{1}{2}\right)$ square units ✓ 2nd mark for either:

- area of Segment or
- area of semi-circle or
- area of quadrant

Area of Semi-Circle $= \frac{1}{2}\pi\left(\frac{\sqrt{2}}{2}\right)^2$

$= \frac{\pi}{4}$

So Area of Outer Shaded Area

$= \frac{\pi}{4} - \left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= \frac{1}{2}$ square unit

$=$ Area $\triangle ABC$ ✓ 3rd mark for area of outer shaded area AND showing that it is equal to area of $\triangle ABC$

(c) (i) $x^2 + y^2 = (x+y)^2 - 2xy$

RHS $= x^2 + 2xy + y^2 - 2xy$

$= x^2 + y^2$

$=$ LHS ✓

(ii) From (i) $x^2 + y^2 = (x+y)^2 - 2xy$

$x^2 + y^2 = \left(\frac{-1+\sqrt{5}}{2} + \frac{-1-\sqrt{5}}{2}\right)^2$

$-2\left(\frac{-1+\sqrt{5}}{2}\right)\left(\frac{-1-\sqrt{5}}{2}\right)$ ✓

$= \left(\frac{-2}{2}\right)^2 - \left(\frac{1-5}{2}\right)$

$= 1 - (-2)$

$= 3$ ✓

Alternate solution - see over

Marks

Question 8 (continued)

(c) (ii) $x^2+y^2 = \left(\frac{-1+\sqrt{5}}{2}\right)^2 + \left(\frac{-1-\sqrt{5}}{2}\right)^2$ ✓

$= \left(\frac{1-2\sqrt{5}+5}{4}\right) + \left(\frac{1+2\sqrt{5}+5}{4}\right)$ ✓

$= \frac{6-2\sqrt{5}+6+2\sqrt{5}}{4}$

$= \frac{12}{4}$

$= 3$ ✓

Alternate solution from previous page.

d) (i) Area using alter rectangle and subtracting triangles $= (2+x)y - 2\left(\frac{1}{2}\frac{y}{2}x\right)$ ✓

$= 2y + xy - \frac{xy}{2}$

$= 2y + \frac{xy}{2}$ ✓

Only 2 marks for this part

OR Area using two rectangles and a parallelogram $= y + y +$ Area of Parallelogram

Base of Parallelogram $= \sqrt{x^2+\left(\frac{y}{2}\right)^2}$ ✓

Area of Parallelogram $= \sqrt{x^2+\left(\frac{y}{2}\right)^2} \times 1$

So Area of N $= 2y + \sqrt{x^2+\left(\frac{y}{2}\right)^2}$ ✓

(ii) Calculate the area by the alternate method (see (i) above) ✓

1 mark for correct alternate expression for the area from that determined in (i)

Equating these areas,

$2y + \frac{xy}{2} = 2y + \sqrt{x^2+\left(\frac{y}{2}\right)^2}$

$+\frac{xy}{2} = \sqrt{x^2+\left(\frac{y}{2}\right)^2}$

$\frac{x^2y^2}{4} = x^2 + \frac{y^2}{4}$

$x^2\left(\frac{y^2}{4} - 1\right) = \frac{y^2}{4}$

$x^2(y^2-4) = y^2$

$x^2 = \frac{y^2}{y^2-4}$ ✓