SYDNEY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT
HALF-YEARLY EXAMINATIONS 2007

FORM III

MATHEMATICS

Examination date

Wednesday 16th May 2007

Time allowed

1 hour 30 minutes

Instructions

All eight questions may be attempted.
All eight questions are of equal value.
All necessary working must be shown.
Marks may not be awarded for careless or badly arranged work.
Calculators are not to be used.

Collection

Write your name, class and master clearly on the front.
Hand in all the writing paper in a single well-stapled bundle.
Keep the printed examination paper and bring it to your next Mathematics lesson.

3A: BDD	3B: MLS	3C: RCF/JNC	3D: JMR/KWM
3E: REP	3F: FMW	3G: SJE	3H: GJ
3I: LYL	3J: PKH		

Checklist

Writing paper required.
Candidature: 186 boys.

Examiner

REP

QUESTION ONE (13 marks) Start a new page.

(a) Write 803·7 in scientific notation.

(b) Simplify:

(i) $3u^3 - u^3$

(ii) $b + 2b - 6b + 6$

(iii) $5a^3 \times a^2$

(iv) $\sqrt{18}$

(c) Evaluate:

(i) $\sqrt{7} \times \sqrt{7}$

(ii) $\sqrt{0{\cdot}09}$

(d) Solve the equation $4n - 9 = 11$.

(e) Evaluate $7u - 2u^2$ when $u = -1$.

(f) Solve the inequation $x - 2 > -3$ and graph your solution on a number line.

(g) Express $\dfrac{3}{\sqrt{5}}$ with a rational denominator.

Exam continues next page ...

QUESTION TWO (13 marks) Start a new page.

(a) Simplify:

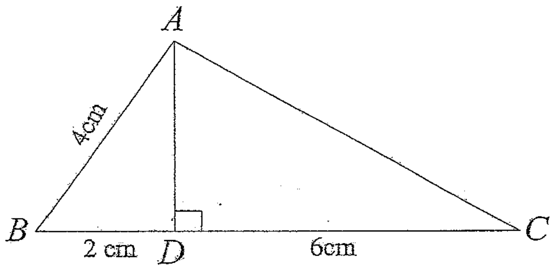
(i) $\dfrac{6y-y}{xy}$

(ii) $\dfrac{8}{u^2} \div \dfrac{r}{u^3}$

(iii) $u^5 \div u^3$

(iv) $\sqrt{63}-\sqrt{28}$

(b)



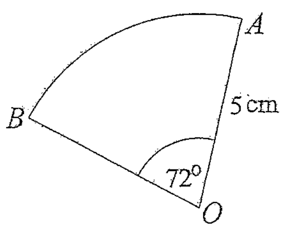
(i) Use Pythagoras' theorem to show that $AD = 2\sqrt{3}$ cm in the diagram above.

(ii) Hence find the area of triangle ABC. Leave your answer in surd form.

(c) Solve for x,

$$\frac{3-2x}{7} < 5.$$

(d)



In the diagram above, AOB is a sector of a circle with centre O and radius 5 cm. Given that $\angle AOB = 72°$, find the area of the sector. Use the approximation $\pi \doteqdot 3{\cdot}12$.

Exam continues overleaf ...

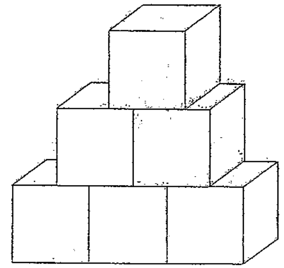
QUESTION THREE (13 marks) Start a new page.

(a) Evaluate:

(i) $16 \div 8^0$

(ii) $1000^{-\frac{2}{3}}$

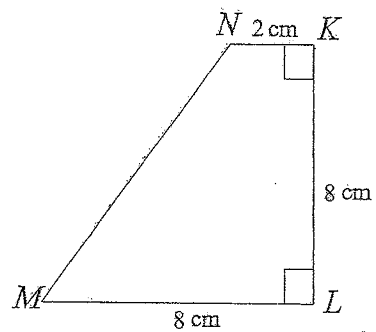
(b)



Find the surface area of the solid above, which is made up of cubes of side length 1 metre.

(c) Write the formula $V = 4A - B$ with A as the subject.

(d)



(i) Find the area of the trapezium $KLMN$ in the diagram above.

(ii) The trapezium is the cross-section of a steel girder 2 metres long. Find the volume of the girder. Leave your answer in cm^3.

(e) Solve:

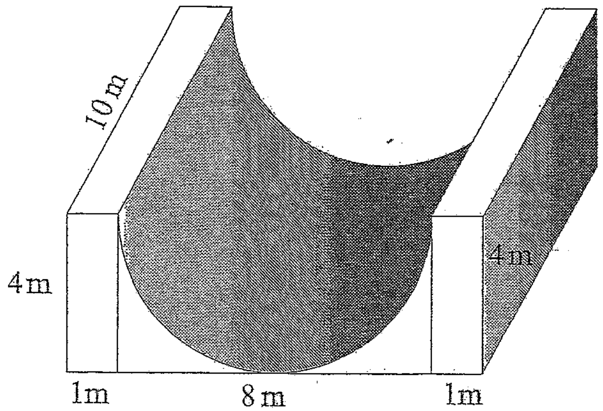
(i) $2(3+x) = 3x$

(ii) $\dfrac{1-x}{2} + \dfrac{x+3}{5} = 0$

Exam continues next page ...

QUESTION FOUR (13 marks) Start a new page.

(a)



In a skateboard park there is a solid with dimensions (in metres) of $10 \times 4 \times 10$. A piece of the solid in the shape of half a cylinder of diameter 8 metres has been removed, as shown in the diagram above.

(i) Find the area of the curved surface. Use the approximation $\pi \doteqdot 3{\cdot}1$.

(ii) Find the volume of the solid. Use the approximation $\pi \doteqdot 3{\cdot}1$.

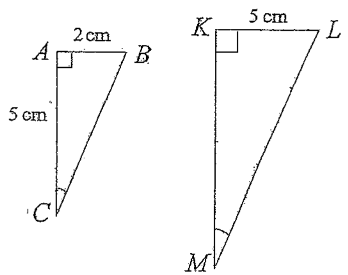
(b) Expand and simplify:

(i) $(2a-b)(2a+b)$

(ii) $(3\sqrt{7}-\sqrt{2})(\sqrt{7}+3\sqrt{2})$

(c) Suppose $u = at - x$. Find the value of a when $u = 10$, $t = 3$ and $x = 8$.

(d)



In the diagram above, triangles ABC and KLM are both right-angled and $\angle ACB = \angle KML$. $AB = 2$ cm, $AC = 5$ cm and $KL = 5$ cm.

(i) Explain why $\triangle ABC \,|||\, \triangle KLM$.

(ii) Find the length of KM.

Exam continues overleaf ...

QUESTION FIVE (13 marks) Start a new page.

(a) Write in simplest form, without negative indices:

(i) $(3x)^{-3}$

(ii) $(m^{-1}n^2)^2$

(b) Simplify $\dfrac{3}{\sqrt{2}-1} + \dfrac{2}{\sqrt{2}+1}$.

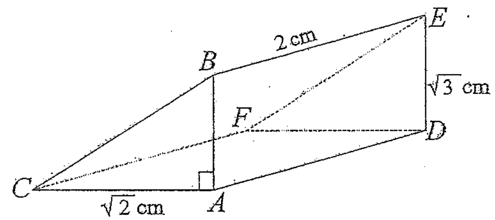
(c) Simplify $(x+y)^2 - (x-y)^2$.

(d) If $s = \frac{1}{5}$ and $t = -3$, evaluate:

(i) $10st^2$

(ii) $s^{-1} - t^{-2}$

(e)



A goldsmith designed a small pendant in the shape of a triangular prism as shown.

(i) Use Pythagoras' theorem to show that $CB = \sqrt{5}$ cm.

(ii) Find the surface area of the pendant. Leave your answer in surd form.

(iii) Find the volume of the pendant. Leave your answer in surd form.

Exam continues next page ...

QUESTION SIX (13 marks) Start a new page.

(a) Byron has been asked to solve the inequality

$$5 - x < 4$$

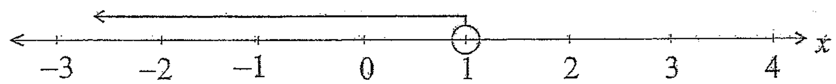
and to display his solution on a number line. His solution is as follows:

$$5 - x < 4$$

So $\quad -x < 4 - 5$

So $\quad -x < -1$

So $\quad x < 1$



Is Byron's solution correct? If not, show using a specific value for x that his solution is incorrect.

(b) The smallest of three consecutive integers, plus three times the next, plus five times the next, is 724. Form an equation and solve it to find the three integers.

(c) A father is now four times as old as his son, and in 16 years time he will be twice as old as his son. By forming an equation and solving it, find the age of the son.

(d) Solve $\sqrt[3]{3+y} = 2$.

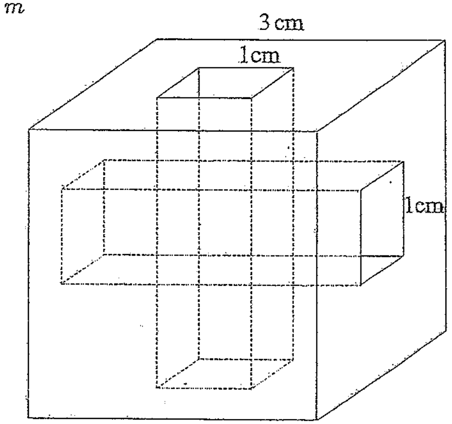
(e) For the formula $y = \dfrac{x}{3-x}$, find x if $y = 8$.

QUESTION SEVEN (13 marks) Start a new page.

(a) Make m the subject of the formula

$$\frac{3}{m} - 7n = 2r\,.$$

(b)



The cube above has two square holes, one vertically from top to bottom, the other horizontally from left to right. Find the volume of the resulting solid.

(c) The speed of light, usually denoted by c, is approximately given by $c = 3 \times 10^8$ metres per second.

(i) Using the value of c above, find how many metres a beam of light will travel in one hour. Leave your answer in scientific notation.

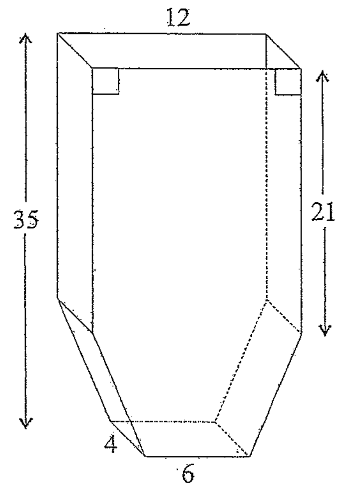
(ii) The very distant star Thalis is approximately $2{\cdot}16 \times 10^{25}$ kilometres from Earth. How many hours would it take for a radio signal which travels at the speed of light to reach Thalis from Earth? Leave your answer in scientific notation.

(d) The running-rail of a race course is 400 metres along the straight section and has a semi-circle of diameter 100 metres at each end. Two horses are travelling around the course. The first is travelling along the running-rail and for all practical purposes covers a distance equal to the length of the running-rail. The second horse is 20 metres out from the first horse for one complete circuit of the course. How much further does the second horse travel? Leave your answer in terms of π.

(e) Solve $\sqrt{\sqrt[3]{\sqrt[4]{x}}} = 2^{\frac{1}{12}}$.

QUESTION EIGHT (13 marks) Start a new page.

(a)

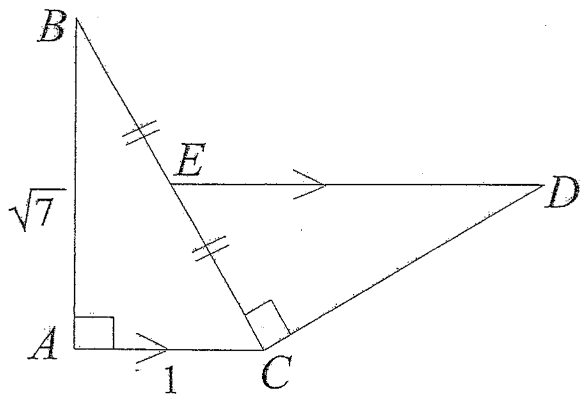


The figure above is a vase. All measurements are in centimetres.

(i) Show that the capacity of the vase is 1·512 litres.

(ii) If 1·25 litres of water is poured into the vase, how far from the top will the water reach? Give your answer correct to the nearest millimetre.

(b)



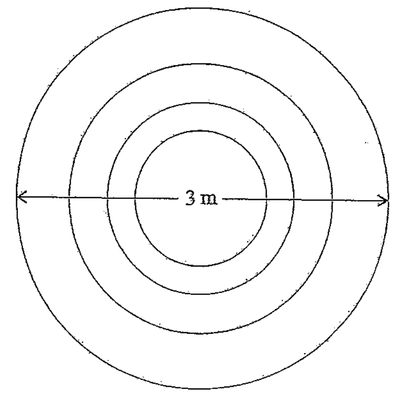
In the diagram above, $\triangle ABC$ and $\triangle CDE$ have right angles at A and C respectively. E is the mid-point of BC. ED is parallel to AC. $AB = \sqrt{7}$ and $AC = 1$.

(i) Prove $\triangle ABC \parallel\!| \triangle CDE$.

(ii) Hence show that $CD = \sqrt{14}$.

Exam continues overleaf ...

(c)



The target above is to be constructed so that there are four concentric circles, the largest having a diameter of 3 metres. The other circles are to be such that the area of the central circle is equal to the area between successive circles. What are the radii of the four circles? Where necessary leave you answers in surd form.

(d) Given that $\dfrac{1}{\sqrt{7}+\sqrt{5}-\sqrt{3}}$ can be expressed in the form

$$\frac{A\sqrt{7}+B\sqrt{5}+C\sqrt{105}+D\sqrt{3}}{59}$$

where A, B, C, and D are integers, find A, B, C and D.

Form III, 2007 Half-Yearly : *Solutions* 16th May

EACH QUESTION IS OUT OF 13

1. (a) $8{\cdot}037 \times 10^2$ $\boxed{\surd}$

(b) (i) $2u^3$ $\boxed{\surd}$

(ii) $6 - 3b$ $\boxed{\surd}$

(iii) $5a^5$ $\boxed{\surd}$

(iv) $3\sqrt{2}$ $\boxed{\surd}$

(c) (i) 7 $\boxed{\surd}$

(ii) 0·3 $\boxed{\surd}$

(d) $4n - 9 = 11$

So $4n = 20$

So $n = 5$ $\boxed{\surd}$

(e) $7 \times -1 - 2 \times (-1)^2$

$= -7 - 2$

$= -9$ $\boxed{\surd\surd}$

(f) $x - 2 > -3$

So $x > -1$ $\boxed{\surd}$

$\boxed{\surd}$

(g) $\dfrac{3\sqrt{5}}{5}$ $\boxed{\surd}$

2. (a) (i) $\dfrac{6y - y}{xy} = \dfrac{5y}{xy}$

$= \dfrac{5}{x}$ $\boxed{\surd\surd}$

(ii) $\dfrac{8u}{r}$ $\boxed{\surd}$

(iii) u^2 $\boxed{\surd}$

(iv) $\sqrt{63} - \sqrt{28} = 3\sqrt{7} - 2\sqrt{7}$

$= \sqrt{7}$ $\boxed{\surd\surd}$

(b) (i) In $\triangle ADB$:

$AD^2 + 2^2 = 4^2$

So $AD^2 = 16 - 4$

So $AD = \sqrt{12}$

So $AD = 2\sqrt{3}$ cm $\boxed{\surd\surd}$

(ii) Area $\triangle ABC = \frac{1}{2} \times 8 \times 2\sqrt{3}$

$= 8\sqrt{3}\,\text{cm}^2$ $\boxed{\surd}$

(c) $\dfrac{3 - 2x}{7} < 5$

So $3 - 2x < 35$

So $-2x < 32$

So $x > -16$ $\boxed{\surd\surd}$

(d) $A = \dfrac{72}{360} \times \pi \times 5^2$

$= \dfrac{1}{5} \times 3{\cdot}12 \times 5^2$

$= 3{\cdot}12 \times 5$

So $A = 15{\cdot}6\,\text{cm}^2$

$\overset{\text{or}}{=} \frac{78}{5} \overset{\text{or}}{=} 15\frac{3}{5}$ $\boxed{\surd\surd}$

3. (a) (i) $16 \div 8^0 = 16 \div 1$

$= 16$ $\boxed{\surd}$

(ii) $1000^{-\frac{2}{3}} = \dfrac{1}{(\sqrt[3]{1000})^2}$

$= \dfrac{1}{10^2}$

$= \dfrac{1}{100}$ $\boxed{\surd\surd}$

(b) $A = 6 \times 2 + 3 \times 2 + 3 \times 2$

$= 24\,\text{m}^2$ $\boxed{\surd\surd}$

(c) $V = 4A - B$

So $4A = V + B$

So $A = \dfrac{V + B}{4}$ $\boxed{\surd}$

(d) (i) $A = \frac{1}{2}(2 + 8) \times 8$

$A = 40\,\text{cm}^2$ $\boxed{\surd}$

(ii) $V = 40 \times 200$

So $V = 8000\,\text{cm}^3$ $\boxed{\surd\surd}$

Deduct one mark for no/wrong units

(e) (i) $2(3 + x) = 3x$

So $6 + 2x = 3x$

So $x = 6$ $\boxed{\surd\surd}$

(ii) $\dfrac{1 - x}{2} + \dfrac{x + 3}{5} = 0$

So $5(1 - x) + 2(x + 3) = 0$

So $5 - 5x + 2x + 6 = 0$

So $-3x = -11$

So $x = \frac{11}{3} \overset{\text{or}}{=} 3\frac{2}{3}$ $\boxed{\surd\surd}$

4. (a) (i) $A = \frac{1}{2} \times 2 \times \pi \times r \times h$

$= 3{\cdot}1 \times 4 \times 10$

So $A = 124\,\text{m}^2$ $\boxed{\surd\surd}$

(ii) $V =$ V. of prism $-$ V. of half cylinder

$= 10 \times 4 \times 10 - \frac{1}{2} \times \pi \times 4^2 \times 10$

$= 400 - 8 \times 31$

$= 400 - 248$

$V = 152\,\text{m}^3$ $\boxed{\surd\surd\surd}$

(b) (i) $(2a - b)(2a + b)$

$= 4a^2 - b^2$ $\boxed{\surd\surd}$

(ii) $(3\sqrt{7} - \sqrt{2})(\sqrt{7} + 3\sqrt{2})$

$= 21 + 9\sqrt{14} - \sqrt{14} - 6$

$= 15 + 8\sqrt{14}$ $\boxed{\surd\surd}$

(c) $10 = 3a - 8$

So $3a = 18$

So $a = 6$ $\boxed{\surd}$

(d) (i) AA test. OR as there are 2 pairs of equal angles. $\boxed{\surd}$

(ii) In similar triangles the corresponding sides are proportional.

So $\dfrac{KM}{5} = \dfrac{5}{2}$

So $KM = \dfrac{5}{2} \times 5$

So $KM = 12{\cdot}5 \overset{\text{or}}{=} \frac{25}{2}$ cm. $\boxed{\surd\surd}$

5. (a) (i) $\dfrac{1}{27x^3}$ $\boxed{\checkmark}$

(ii) $m^{-2}n^4$

$= \dfrac{n^4}{m^2}$ $\boxed{\checkmark}$

(b) $\dfrac{3}{\sqrt{2}-1} + \dfrac{2}{\sqrt{2}+1}$

$= \dfrac{3(\sqrt{2}+1)+2(\sqrt{2}-1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$

$= \dfrac{3\sqrt{2}+3+2\sqrt{2}-2}{2-1}$

$= 5\sqrt{2}+1$ $\boxed{\checkmark\checkmark}$

(c) $(x+y)^2-(x-y)^2$

$= (x+y+x-y)(x+y-x+y)$

$= 2x \times 2y$

$= 4xy$ $\boxed{\checkmark\checkmark}$

(d) (i) $10 \times \frac{1}{5} \times (-3)^2$

$= 18$ $\boxed{\checkmark}$

(ii) $\left(\frac{1}{5}\right)^{-1} - (-3)^{-2}$

$= 5 - \frac{1}{9}$

$= 4\frac{8}{9} \overset{\text{or}}{=} \frac{44}{9}$ $\boxed{\checkmark\checkmark}$

(e) (i) $CB^2 = (\sqrt{2})^2 + (\sqrt{3})^2$

$= 5$

So $CB = \sqrt{5}\,\text{cm}$ $\boxed{\checkmark}$

(ii) S.A. $= 2 \times \frac{1}{2}\sqrt{3} \times \sqrt{2} + 2\sqrt{3} + 2\sqrt{2} + 2 \times \sqrt{5}$

$= \sqrt{6} + 2\sqrt{3} + 2\sqrt{2} + 2\sqrt{5}\,\text{cm}$ $\boxed{\checkmark\checkmark}$

(iii) $V = 2 \times \frac{1}{2}\sqrt{6}$

So $V = \sqrt{6}\,\text{cm}^3$ $\boxed{\checkmark}$

6. (a) No. Byron is not correct.
There is an error from the penultimate line to the last line.
The direction of the inequality should be reversed.
A possible numerical example is $x = 0$.
If $x = 0$ we get $5 - 0 < 4$ which is not true. $\boxed{\checkmark\checkmark}$

(b) Let the smallest integer $= x$.

So $x + 3(x+1) + 5(x+2) = 724$

So $x + 3x + 3 + 5x + 10 = 724$

So $9x = 711$

So $x = 79$

So the integers are 79, 80 and 81.

$\boxed{\checkmark\checkmark\checkmark}$

(c) Let the son's age be s.

So $4s + 16 = 2(s+16)$

So $4s + 16 = 2s + 32$

So $2s = 16$

So $s = 8$

So the son is 8. $\boxed{\checkmark\checkmark\checkmark}$

(d) $\sqrt[3]{3+y} = 2$

So $3 + y = 8$

So $y = 5$ $\boxed{\checkmark\checkmark}$

(e) $8 = \dfrac{x}{3-x}$

So $8(3-x) = x$

So $24 - 8x = x$

So $9x = 24$

So $x = \frac{8}{3} \overset{\text{or}}{=} 2\frac{2}{3}$ $\boxed{\checkmark\checkmark\checkmark}$

7. (a) $\dfrac{3}{m} - 7n = 2r$

So $\dfrac{3}{m} = 2r + 7n$

So $\dfrac{m}{3} = \dfrac{1}{2r+7n}$

So $m = \dfrac{3}{2r+7n}$ $\boxed{\checkmark\checkmark}$

(b) $V = 3^3 - 2 \times 1^2 \times 3 + 1^3$

So $V = 27 - 6 + 1$

So $V = 22\,\text{cm}^3$ $\boxed{\checkmark\checkmark}$

(c) (i) $d =$ speed $\times$ time

$= 3 \times 10^8 \times 60 \times 60$

$= 3 \times 3600 \times 10^8$

$= 3 \times 3{\cdot}6 \times 10^3 \times 10^8$

$= 10{\cdot}8 \times 10^{11}$

So $d = 1{\cdot}08 \times 10^{12}\,\text{m}$ $\boxed{\checkmark\checkmark}$

(ii) $t =$ distance $\div$ speed

$= \dfrac{2{\cdot}16 \times 10^{25} \times 1000}{3 \times 10^8}$

$= \dfrac{2{\cdot}16 \times 10^{(28-8)}}{3}$

$= 0{\cdot}72 \times 10^{20}$

$= 7{\cdot}2 \times 10^{19}$ seconds

$= \dfrac{7{\cdot}2 \times 10^{19}}{60 \times 60}$ hours

$= \dfrac{7{\cdot}2 \times 10^{19}}{3{\cdot}6 \times 10^3}$

$= \dfrac{7{\cdot}2}{3{\cdot}6} \times 10^{(19-3)}$

So $t = 2 \times 10^{16}$ hours $\boxed{\checkmark\checkmark\checkmark}$

(d) The difference only occurs on the bends.
So the difference $= 140\pi - 100\pi$
So the second horse travels 40π m further. $\boxed{\checkmark\checkmark}$

(e) $\sqrt{\sqrt[3]{\sqrt[4]{x}}} = 2^{\frac{1}{12}}$

So $\left(\left(x^{\frac{1}{4}}\right)^{\frac{1}{3}}\right)^{\frac{1}{2}} = 2^{\frac{1}{12}}$

So $x^{\frac{1}{24}} = 2^{\frac{1}{12}}$

So $\left(x^{\frac{1}{24}}\right)^{24} = \left(2^{\frac{1}{12}}\right)^{24}$

So $x = \left(2^{\frac{24}{12}}\right)$

So $x = 2^2$

So $x = 4$ $\boxed{\checkmark\checkmark}$

8. (a) (i) $V =$ area cross-section $\times 4$

$$= \left(\tfrac{1}{2}(6+12)\times 14 + 21 \times 12\right)\times 4$$
$$= (9\times 14 + 252)\times 4$$
$$= 378 \times 4$$
$$= 1512\,\text{cm}^3$$

So $V = 1{\cdot}512$ litres $\boxed{\checkmark\checkmark}$

(ii) Volume of water in trapezoid $= 126 \times 4 = 504\,\text{cm}^3$
So the volume of water in the rectangular prism $= 1250 - 504 = 746\,\text{cm}^3$
Let $x =$ the depth of the water in the rectangular prism.

So $48x = 746$

So $x = 746 \div 48 \approx 15{\cdot}54$
So the height from the top that the surface of the water is: $21 - 15{\cdot}54 = 5{\cdot}46$
So the height from the top is $5{\cdot}5$ cm
($\overset{\text{or}}{=}$ 55 mm) to the nearest mm. $\boxed{\checkmark\checkmark\checkmark}$

(b) (i) In Δ's ABC and CDE
(i) $\angle BAC = \angle DCE$ (given)
(ii) $\angle BCA = \angle DEC$ (alternate $\angle$'s, $AC \parallel ED$)
So $\triangle ABC \,|||\, \triangle CDE$ (AA test) $\boxed{\checkmark}$

(ii) In $\triangle ABC$ $BC^2 = (\sqrt{7})^2 + 1^2$ (Pythagoras)
So $BC^2 = 8$
So $BC = 2\sqrt{2}$
So $EC = \sqrt{2}$ (E is mid-point of BC)
Now $\dfrac{CD}{AB} = \dfrac{CE}{AC}$ (corresponding sides are proportional)
So $\dfrac{CD}{\sqrt{7}} = \dfrac{\sqrt{2}}{1}$
So $CD = \sqrt{7}\times\sqrt{2}$
So $CD = \sqrt{14}$ $\boxed{\checkmark\checkmark}$

(c) Area of the largest circle $= \pi \times \left(\frac{3}{2}\right)^2 = \frac{9}{4}\pi\,\text{m}^2$.

So the area of the inner circle is $\frac{9}{16}\pi\,\text{m}^2$.
Let the inner circle have radius r_1, the second circle r_2 and the third circle r_3.
So $\pi r_1^2 = \dfrac{9}{16}\pi$
so $r_1^2 = \dfrac{9}{16}$
So $r_1 = \dfrac{3}{4}$ m

Area of the second circle $= \dfrac{9}{8}\pi\,\text{m}^2$
This gives $r_2 = \dfrac{3\sqrt{2}}{4}$ m
Similarly $r_3 = \dfrac{3\sqrt{3}}{4}$ m

So the 4 radii in ascending order are: $\dfrac{3}{4}$, $\dfrac{3\sqrt{2}}{4}$, $\dfrac{3\sqrt{3}}{4}$ and $\dfrac{3}{2}$ metres. $\boxed{\checkmark\checkmark}$

(d)
$$\frac{1}{\sqrt{7}+\sqrt{5}-\sqrt{3}} = \frac{\sqrt{7}+\sqrt{5}+\sqrt{3}}{\left((\sqrt{7}+\sqrt{5})-\sqrt{3}\right)\left((\sqrt{7}+\sqrt{5})+\sqrt{3}\right)}$$
$$= \frac{\sqrt{7}+\sqrt{5}+\sqrt{3}}{(\sqrt{7}+\sqrt{5})^2-3}$$
$$= \frac{\sqrt{7}+\sqrt{5}+\sqrt{3}}{7+2\sqrt{5}\sqrt{7}+5-3}$$
$$= \frac{\sqrt{7}+\sqrt{5}+\sqrt{3}}{2\sqrt{35}+9}$$
$$= \frac{(\sqrt{7}+\sqrt{5}+\sqrt{3})(2\sqrt{35}-9)}{(2\sqrt{35}+9)(2\sqrt{35}-9)}$$
$$= \frac{14\sqrt{5}+10\sqrt{7}+2\sqrt{105}-9\sqrt{7}-9\sqrt{5}-9\sqrt{3}}{140-81}$$
$$= \frac{\sqrt{7}+5\sqrt{5}+2\sqrt{105}-9\sqrt{3}}{59}$$

So $A = 1$, $B = 5$, $C = 2$ and $D = -9$ $\boxed{\checkmark\checkmark\checkmark}$