SYDNEY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT
YEARLY EXAMINATIONS 2004

FORM III

MATHEMATICS

Examination date

Friday 5th November 2004

Time allowed

2 hours

Instructions

All ten questions may be attempted.
All ten questions are of equal value.
All necessary working must be shown.
Marks may not be awarded for careless or badly arranged work.
Approved calculators and templates may be used.

Collection

Write your name, class and master clearly on the front and on the tear-off sheet.
Hand in all the writing paper in a single well-stapled bundle.
Bundle the tear-off sheet with the question it belongs to.
Keep the printed examination paper and bring it to your next Mathematics lesson.

3A: DNW	3B: MLS	3C: LYL	3D: PKH/REP
3E: GJ/JCM	3F: BDD/DS	3G: JMR	3H: KWM
3I: TCW	3J: REN		

Checklist

Writing paper required.
Candidature: 187 boys.

Examiner

JMR

QUESTION ONE Start a new page.

(a) Use your calculator to find $\sin 21°$, correct to three decimal places.

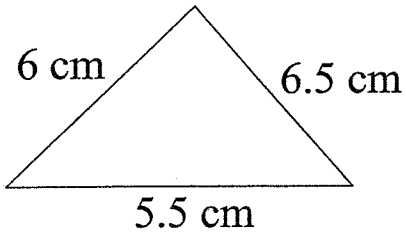
(b) Find the coordinates of the midpoint of the interval joining the points $A(4, 8)$ and $B(8, 0)$.

(c) Factorise $3a + 15$.

(d) Simplify $\sqrt{45}$.

(e) Evaluate 2^x when $x = 4$.

(f)



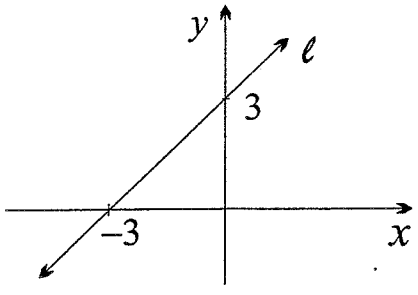
Find the perimeter of the triangle drawn above.

(g) Expand and simplify:

(i) $(x + 3)^2$

(ii) $(x + 6)(x - 6)$

(h)



Write down the gradient of the line ℓ in the diagram above.

(i) Given the scores 8, 7, 9, 13 and 8:

(i) find the mode,

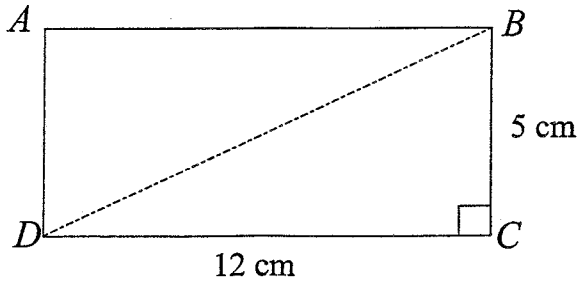
(ii) find the median,

(iii) find the mean.

Exam continues next page ...

QUESTION TWO Start a new page.

(a)



Find the length of the diagonal BD of the rectangle drawn above.

(b) Solve for x:

(i) $3x - 8 = 4$

(ii) $3(x + 1) = 5$

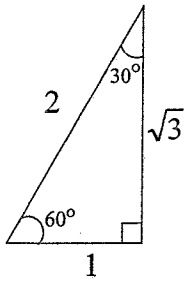
(c) Factorise fully:

(i) $h^2 - g^2$

(ii) $4ab + 8a^2$

(iii) $x^2 + 3x + 2$

(d)



Using the triangle above, give exact values for:

(i) $\sin 60°$

(ii) $\cos 30°$

(iii) $\tan 60°$

Exam continues overleaf ...

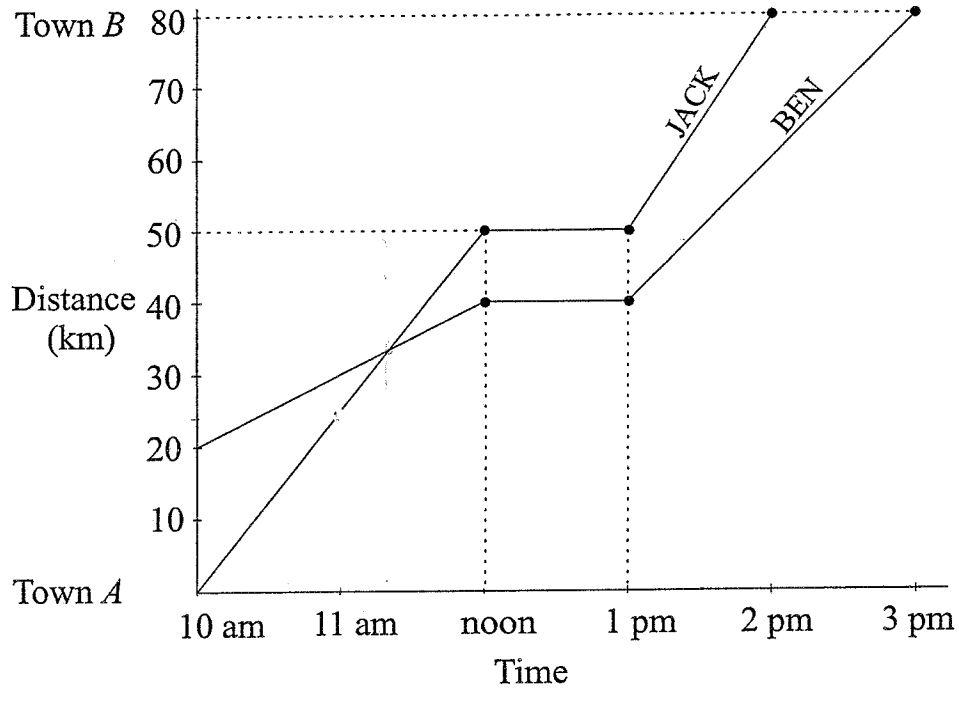
QUESTION THREE Start a new page.

(a) Solve each pair of simultaneous equations using any appropriate method.

(i) $x + y = 14$
$2x - y = 1$

(ii) $2x + y = 7$
$x + 2y = 11$

(b)



The travel graph above shows the journeys of Jack and Ben on the only road between Town A and Town B. The towns are 80 km apart.

(i) How far is Ben from Town A when Jack is leaving Town A?

(ii) How far is Jack from Town B at 1 pm?

(iii) Who reaches Town B first?

(iv) Use the graph to estimate the time at which Jack and Ben meet on their journey.

(v) Calculate Jack's average speed in kilometres per hour for the first hour of his journey.

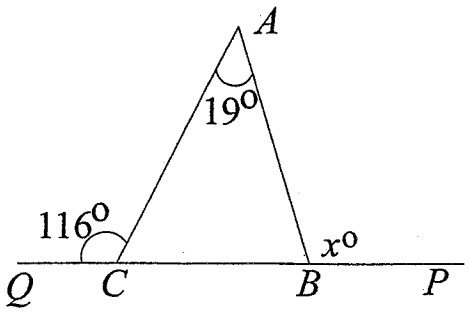
(vi) Calculate Ben's average speed in kilometres per hour for his entire journey.

Exam continues next page ...

QUESTION FOUR Start a new page.

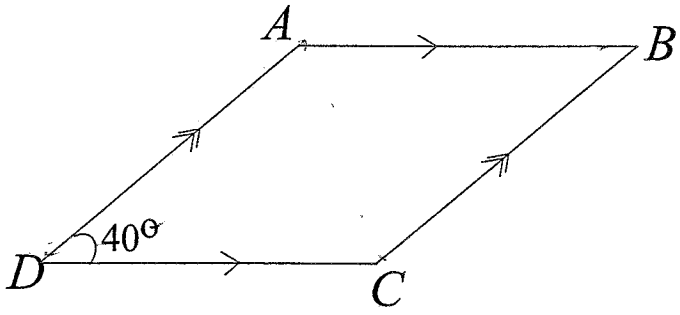
(a) The points $A(-1,-2)$, $B(2,4)$ and $C(4,8)$ are collinear. Write down the size of the angle $\angle ABC$.

(b)



Find the value of x in the diagram above. Give reasons.

(c)

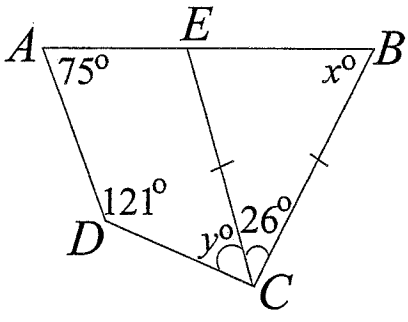


In the diagram above, $ABCD$ is a parallelogram and $\angle ADC = 40°$.

(i) Find the size of $\angle ABC$. Give a reason.

(ii) If the diagonals AC and BD are perpendicular, what can you say about the lengths of sides AD and AB? Give reasons.

(d)



In the diagram above, $BC = EC$. Find the values of x and y. Give reasons.

(e) Detach the tear-off page at the end of this examination paper. Then use ruler and compasses to carry out the constructions described on that page. Staple the page with the rest of question four.

Exam continues overleaf ...

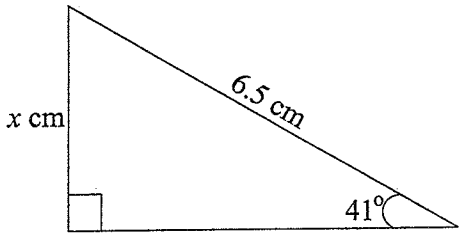
QUESTION FIVE Start a new page.

(a) Simplify:

(i) $\dfrac{2a-3}{6a-9}$

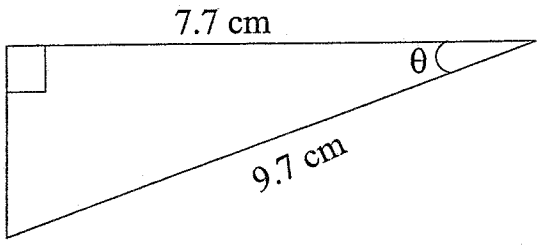
(ii) $\dfrac{x^2-9}{x^2+9x+18}$

(b)



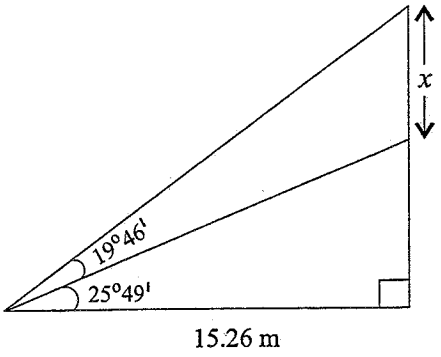
Find the value of x in the triangle above. Give your answer correct to one decimal place.

(c)



Find the size of the angle θ in the triangle above. Give your answer correct to the nearest minute.

(d)



Find the value of x in the diagram above. Give your answer correct to two decimal places.

Exam continues next page ...

QUESTION SIX Start a new page.

(a) Given the points $A(3,5)$ and $B(-1,2)$, find the distance between A and B.

(b) State the equation of the line that has a y-intercept of -2 and a gradient of 5.

(c) Which (if either) of the points $A(3,-1)$ and $B(-3,-11)$ lies on the line $y = 2x - 5$? You must show your working clearly.

(d) Find the equation of the line that passes through $A(2,3)$ and is parallel to the x-axis.

(e) A line passes through the points $P(1,3)$ and $Q(-2,5)$.

(i) Find the gradient of the line.

(ii) Find the equation of the line.

QUESTION SEVEN Start a new page.

BOYS		GIRLS
Leaf	Stem	Leaf
9 8	1	
8 7 4 4 4 1 0 0	2	0 3 6 7 8 8 9
3 2 0	3	0 1 1 3 8
9	4	0 1

The back-to-back stem-and-leaf display above shows the number of seconds taken by each member of a class of primary school children to swim 25 metres.

(a) How many pupils are there in the class?

(b) Which score may be considered an outlier?

(c) What is the range for the whole class?

(d) For the boys' scores:

(i) What is the mode?

(ii) What is the mean? Give your answer correct to the nearest second.

(iii) What is the standard deviation? Give your answer correct to one decimal place.

(e) For the girls' scores:

(i) What is the median?

(ii) What is the upper quartile?

(iii) What is the lower quartile?

(iv) What is the inter-quartile range?

Exam continues overleaf ...

QUESTION EIGHT Start a new page.

(a) A climber standing on the highest point of the Sydney Harbour Bridge is 134 metres above sea level. He spots a kayaker and the angle of depression from the climber to the kayaker is $32°15'$.

(i) Draw a diagram to show the information given. Show the angle of depression clearly on your diagram.

(ii) Calculate the distance that the kayaker needs to paddle before he is directly under the climber. Give your answer correct to the nearest metre.

(b) Solve $\dfrac{4}{2x-3} = \dfrac{5}{x+1}$.

(c) Factorise completely:

(i) $2y^2 + 2y - 12$

(ii) $8x^2 - 2x - 15$

(d) Expand and simplify $(2\sqrt{7} - \sqrt{3})^2$.

QUESTION NINE Start a new page.

(a) Simplify $\dfrac{3}{x-2} - \dfrac{2}{x+2} + \dfrac{4}{x^2-4}$.

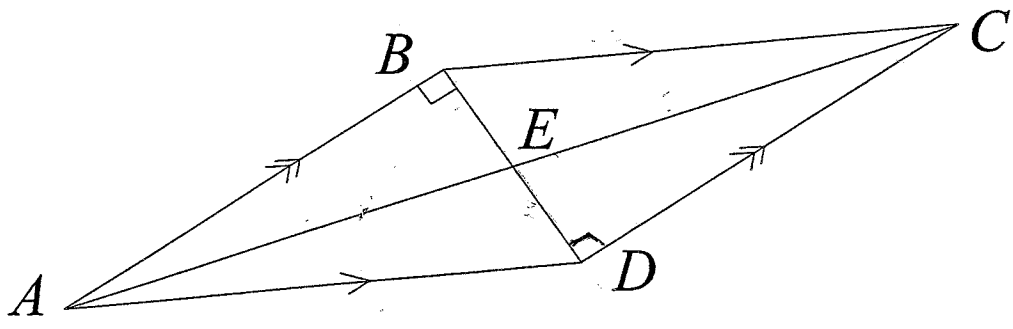
(b) The time t hours taken to complete the painting of a courtyard varies inversely as the number N of painters to do the work. Six painters can complete the job in 11 hours.

(i) Find a formula relating t and N.

(ii) How long would it take 8 painters to complete the job?

(c) The weekly wages of five carpenters and three apprentices amount to \$7190 and the wages of three carpenters and five apprentices amount to \$5850. Use simultaneous equations to find the weekly wages of a carpenter and of an apprentice.

(d)



In the parallelogram $ABCD$ drawn above, the diagonals AC and BD intersect at E and $\angle ABD = 90°$. Prove that

$$AD^2 = AE^2 + 3ED^2.$$

Exam continues next page ...

QUESTION TEN Start a new page.

(a) The points $A(-2a, 0)$, $B(2a, 0)$ and $C(a, 2a)$ are the vertices of a triangle.

(i) Find the equation of the perpendicular bisector of AB.

(ii) Show that the equation of the perpendicular bisector of BC is $2x - 4y + a = 0$.

(iii) Find the coordinates of the point S where the perpendiculars meet.

(iv) Show that the point S is equidistant from the vertices A, B and C.

(v) Show that $\angle CSB = 2 \times \angle CAB$.

(b) For any positive integer n, define

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1.$$

Use this definition to simplify

$$\frac{(n+1)!\ (n-1)!}{(n+2)!\ n!}.$$

(c) (i) Show that every multiple of 4 can be expressed as the difference of the squares of two integers.

(ii) Show that 30 cannot be expressed as the difference of the squares of two integers.

END OF EXAMINATION

Tear-off pages follow ...