SYDNEY GRAMMAR SCHOOL

2010 Annual Examination

# FORM IV MATHEMATICS

Monday 1st November 2010

**General Instructions**
- Writing time — 2 hours
- Write using black or blue pen.
- Board-approved calculators may be used.
- All necessary working should be shown in every question.
- Start each question on a new page.

**Structure of the paper**
- Total marks — 112
- All seven questions may be attempted.
- All seven questions are of equal value.

**Collection**
- Write your name, class and master clearly on each page of your answers.
- Staple your answers in a single bundle.
- Write your name and master on this question paper and submit it with your answers.

| | | | |
|---|---|---|---|
| 4A: MW | 4B: BR | 4C: LYL | 4D: SO |
| 4E: REP | 4F: MK | 4G: BDD | 4H: DS |
| 4I: JMR | 4J: MLS | | |

**Checklist**
- Writing paper required.
- Candidature — 189 boys

Examiner
REP



QUESTION ONE (16 marks) Start a new page.

(a) Find the gradient of the straight line $5y = 2x + 20$.

(b) Use your calculator to find the value of $\sqrt{234 + 877 \times 23{\cdot}3}$ correct to three decimal places.

(c) Expand and simplify:

(i) $(x-6)(x+6)$

(ii) $(2x+3)(x-3)$

(d) Find 32% of 225.

(e) If $W(x) = 1 + 3x^3$ find $W(2)$.

(f) If two twenty-cent coins are tossed what is the probability that both coins will land tails up?

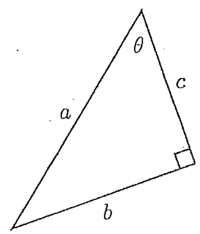
(g) Simplify as a single logarithm to base 5 the expression $\log_5 x - \log_5 y$.

(h) How much interest would you earn if you invested \$7000 at 8% p.a. simple interest for four years? Give your answer correct to the nearest dollar.

(i) Simplify $\left(s^4 t^3\right)^5$.

(j) Solve $(x-4)^2 = 0$.

(k)



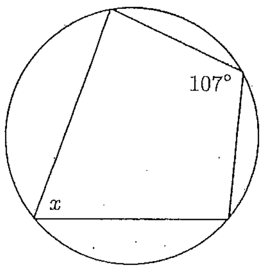

In the diagram above find an expression for $\cos\theta$.

(l) Rewrite the equation $3^7 = 2187$ in logarithmic form.

Exam continues next page ...

QUESTION ONE (Continued)

(m)




Find the size of the angle marked $x$, giving a suitable geometric reason.

(n) Where does the parabola $y = (x+1)(x-3)$ cut the $y$-axis?

Exam continues overleaf ...



QUESTION TWO (16 marks) Start a new page.

(a) Consider the circle $x^2 + y^2 = 144$.

(i) What point on the number plane is the centre of the circle?

(ii) Write down the radius of the circle.

(b) Solve:

(i) $(3x-5)(x+7) = 0$

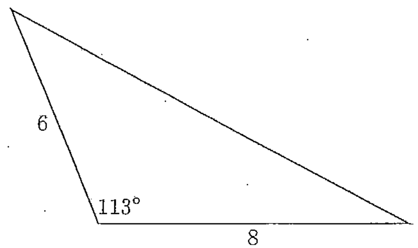
(ii) $x^2 - 3x - 10 = 0$

(iii) $(x-3)^2 = 16$

(c) Solve the following pair of equations simultaneously.

$$y = 1 + 2x$$
$$y = 11 - 3x$$

(d)




Find the area of the triangle above. Give your answer correct to two decimal places.

(e) How much would you expect to pay in interest if you borrowed \$30 000 at 8% per annum compounded annually for 10 years? Give your answer correct to the nearest dollar.

(f) By completing the square express $x^2 - 8x + 19$ in the form $(x-\alpha)^2 + \beta$, where $\alpha$ and $\beta$ are positive integers.

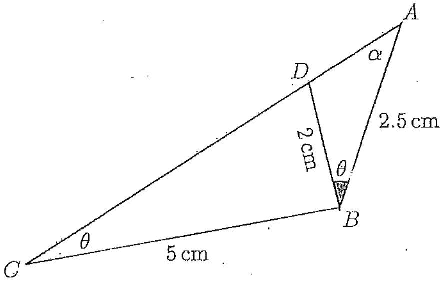
Exam continues next page ...

QUESTION THREE (16 marks) Start a new page.

(a) Express $\dfrac{\sqrt{7}}{\sqrt{7}-1}$ in its simplest form with a rational denominator.

(b) Solve $\cos x = -\dfrac{1}{\sqrt{2}}$ for $0° \leq x \leq 360°$.

(c)




In the diagram above $\triangle ADB$ is similar to $\triangle ABC$ with $AB = 2{\cdot}5$ cm, $DB = 2$ cm and $CB = 5$ cm.

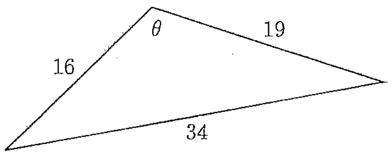
(i) Find the length of $AD$.

(ii) Find the ratio of the area of $\triangle ADB$ to the area of $\triangle ABC$.

(d) Use the remainder theorem to find the remainder when $x^3 - 7x^2 + 5$ is divided by $x - 1$.

(e) Use the quadratic formula to solve $x^2 = 2(3x+1)$. Express your answer in simplest exact form.

(f)




Find the value of $\theta$ correct to the nearest degree.

(g) Find the volume of a pyramid with a square base of side 5 cm and perpendicular height 6 cm.

(h) Solve by completing the square

$$x^2 + 6x - 1 = 0.$$

Give your answers correct to three decimal places.

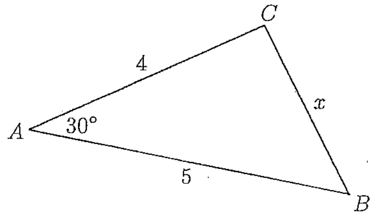
Exam continues overleaf ...



QUESTION FOUR (16 marks) Start a new page.

(a) Find $x$ given that $\sqrt{18} + \sqrt{8} = \sqrt{x}$.

(b)

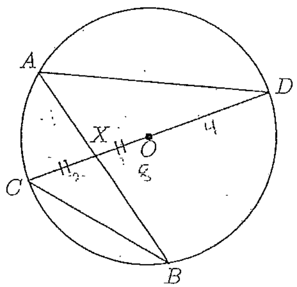



Find $x$ correct to three decimal places.

(c) (i) Draw a neat sketch of the graph of $y = \sin\theta$ for $0° \leq \theta \leq 360°$.

(ii) Hence, or otherwise, solve $\sin\theta = -1$ for $0° \leq \theta \leq 360°$.

(d)



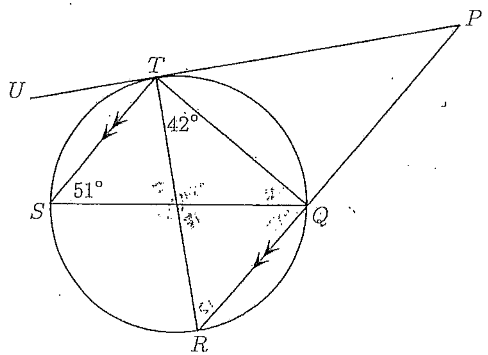

The chord $AB$ and the diameter $CD$ of a circle, centre $O$, intersect at $X$.
$CX = XO$, $CD = 8$ cm and $AX = 3$ cm.
Find, stating all reasons, the length of $XB$.

(e) Find $p$ if $(x-2)$ is a factor of $x^3 + px^2 + x + 6$.

Exam continues next page ...

QUESTION FOUR (Continued)

(f)




In the diagram $PU$ is a tangent touching the circle at $T$. $TS$ is parallel to $PR$.
In each of the following three parts all necessary geometric reasons must be given.

(i) Prove that $\angle STR = 51°$.

(ii) Find $\angle TQS$.

(iii) Find $\angle STU$.

Exam continues overleaf ...



QUESTION FIVE (16 marks) Start a new page.

(a) Use long division to find the polynomial $Q(x)$ if

$$\frac{x^3 + 2x^2 - x + 2}{x + 1} = Q(x) + \frac{4}{x + 1}.$$

(b) Consider the curve $y = 3^x - 2$.

(i) Find where the graph of the curve cuts the $y$-axis.

(ii) Find where the graph of the curve cuts the $x$-axis correct to one decimal place.

(iii) Find the equation of the horizontal asymptote.

(iv) Sketch the curve clearly showing all points of intersection with the axes.

(c) (i) Copy and complete this table of values for the curve $y = -\dfrac{6}{x}$.

| $x$ | $-6$ | $-3$ | $-2$ | $-1$ | 1 | 2 | 3 | 6 |
|---|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | | |

(ii) Using the values in the table, sketch the graph of $y = -\dfrac{6}{x}$, clearly labelling the asymptotes.

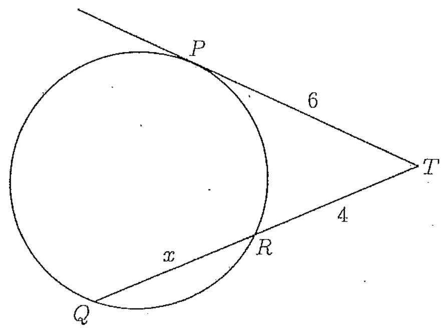
(d) If the remainder when $x^3 - 13x + 14$ is divided by $x - a$ is 2.

(i) Show that $a^3 - 13a + 12 = 0$.

(ii) Hence find all possible values of $a$.

QUESTION SIX (16 marks) Start a new page.

(a)




In the diagram above $TP$ is a tangent, $TP = 6$ cm, $TR = 4$ cm and $RQ = x$ cm. Find the value of $x$ giving a suitable geometric reason.

Exam continues next page ...

QUESTION SIX (Continued)

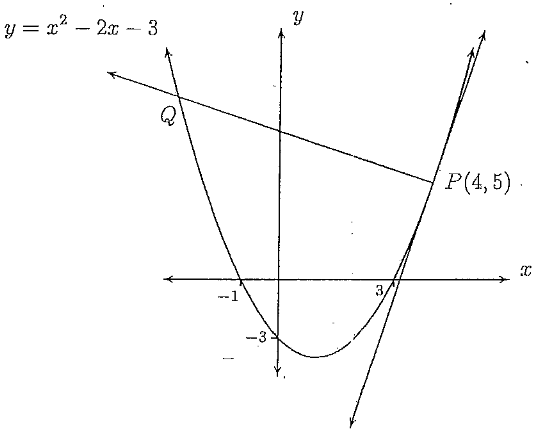
(b) Sketch the graph of the polynomial $y = (x+1)^2(x-2)$. Clearly indicate the points where the graph intersects the axes.

(c) $P(x)$ and $Q(x)$ are polynomials. The degree of $P(x)$ is $n$ and the degree of $Q(x)$ is $m$ where $n > m$.

(i) What is the degree of $P(x) + Q(x)$?

(ii) What is the degree of $P(x) \times Q(x)$?

(d)




The tangent to the parabola $y = x^2 - 2x - 3$ at the point $P(4,5)$ has a gradient of 6. The line through $P$ perpendicular to the tangent cuts the parabola at $Q$.

(i) What is the gradient of the line $PQ$?

(ii) Show that the line $PQ$ has equation $x + 6y - 34 = 0$.

(iii) By solving simultaneously, find the coordinates of $Q$.

(e) In eight hours Jake walks twelve kilometres more than Fiona does in seven hours; and in thirteen hours Fiona walks seven kilometres more than Jake does in nine hours. If Jake walks at $x$ kilometres per hour and Fiona walks at $y$ kilometres per hour, form a pair of simultaneous equations and solve them to find how fast each of Jake and Fiona walk.

(f) If $\frac{1}{6}\log_x(abc) - \frac{1}{2}\log_x(b\sqrt{c}) - \frac{1}{3}\log_x c = 0$, prove that $a^2 = b^4c^5$.

QUESTION SEVEN (16 marks) Start a new page.

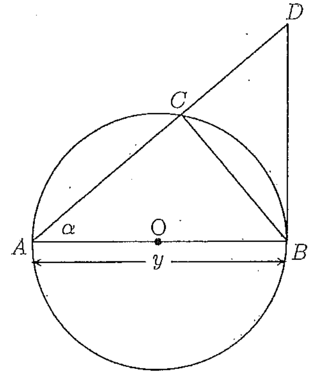
(a) The Mighty Mango, star of the rugby XV is having injury problems. When he is playing the probability that his team will win is $\frac{3}{4}$, but otherwise it is only $\frac{1}{2}$. The probability that he will be fit for the next match is $\frac{1}{3}$. Find the probability that his team will win the match.

Exam continues overleaf ...



QUESTION SEVEN (Continued)

(b)




In the diagram above $O$ is the centre of the circle and $BD$ is a tangent; $AB = y$ and $\angle DAB = \alpha$.

(i) Prove that $AC = y\cos\alpha$.

(ii) By using the trigonometric identies $\sin^2\alpha + \cos^2\alpha = 1$ and $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$ prove that $CD = y\sin\alpha\tan\alpha$.

(c) (i) Express $\tan(180° - x)$ in terms of $\tan x$.

(ii) Suppose $A$, $B$, $C$ are the angles of a triangle and $\tan A = 1$ and $\tan B = 2$.

($\alpha$) Use the formula $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$ to prove that $\tan C = 3$.

($\beta$) If $a$ $b$, $c$ are the corresponding sides of the triangle, prove that

$$\frac{a}{\sqrt{5}} = \frac{b}{2\sqrt{2}} = \frac{c}{3}.$$

(d) (i) Expand $\left(y - \dfrac{1}{y}\right)^3$.

(ii) If $\left(y - \dfrac{1}{y}\right)^2 = 3$ and $y - \dfrac{1}{y} > 0$, without finding the value of $y$ find the value of:

($\alpha$) $y^3 - \dfrac{1}{y^3}$

($\beta$) $y^4 + \dfrac{1}{y^4}$

END OF EXAMINATION

16 marks per question

Do not penalise omission of units

1. (a) $\frac{2}{5}$ [√]

(b) 143·764 [√]

(c) (i) $x^2 - 36$ [√]

(ii) $2x^2 - 3x - 9$ [√]

(d) 72 [√]

(e) $W(2) = 25$ [√]

(f) $\frac{1}{4}$ [√]

(g) $\log_5 \frac{x}{y}$ [√]

(h) $I = 7000 \times 0{\cdot}08 \times 4$

So $I = \$2240$ [√]

(i) $s^{20}t^{15}$ [√]

(j) $x = 4$ [√]

(k) $\cos\theta = \frac{c}{a}$ [√]

(l) $\log_3 2187 = 7$ [√]

(m) $x = 73°$ [√] (Opp. ∠s cyclic quad. supplementary) [√]

(n) $-3$ or $(0, -3)$ [√]

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2. (a) (i) Centre is $(0, 0)$ [√]

(ii) Radius $= 12$ [√]

(b) (i) $x = \frac{5}{3}$ or $-7$ [√]

(ii) $x^2 - 3x - 10 = 0$

So $(x-5)(x+2) = 0$ [√]

So $x = -2$ or $5$ [√]

(iii) $(x-3)^2 = 16$

So $x - 3 = \pm 4$ [√]

So $x = 3 \pm 4$

So $x = 7$ or $-1$ [√]

(c) $y = 1 + 2x$

$y = 11 - 3x$

So $1 + 2x = 11 - 3x$ [√]

So $5x = 10$

So $x = 2$

So $y = 5$ [√]

(d) $A = \frac{1}{2} \times 6 \times 8 \times \sin 113°$ [√]

So $A = 22{\cdot}09$ units$^2$(2 d.p.) [√]

(e) Amt. owed $= 30\,000(1 + 0{\cdot}08)^{10}$ [√]

$= 30\,000 \times 1{\cdot}08^{10}$

$= 64767{\cdot}75$ [√]

So interest $= \$34768$(nearest \$) [√]

(f) $x^2 - 8x + 19$

$= x^2 - 8x + 16 + 3$ [√]

$= (x-4)^2 + 3$ [√]

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3. (a) $\frac{\sqrt{7}}{\sqrt{7}-1} = \frac{\sqrt{7}}{\sqrt{7}-1} \times \frac{\sqrt{7}+1}{\sqrt{7}+1}$ [√]

$= \frac{7+\sqrt{7}}{6}$ [√]

(b) $\cos x = -\frac{1}{\sqrt{2}}$

So $x = 180° \pm 45°$ [√]

So $x = 135°$ or $225°$ [√]

(c) (i) $\triangle ADB \mathrel{|||} \triangle ABC$

(Corr. sides proportional)

So $\frac{AD}{2{\cdot}5} = \frac{2}{5}$

So $AD = 1$ cm [√]

(ii) Ratio of lengths is 2 : 5

So ratio of areas is 4 : 25 [√]

(d) $R = 1^3 - 7 \times 1^2 + 5$ [√]

So $R = -1$ [√]

(e) $x^2 = 2(3x+1)$

So $x^2 - 6x - 2 = 0$

So $x = \frac{6 \pm \sqrt{36+8}}{2}$ [√]

So $x = \frac{6 \pm \sqrt{44}}{2}$

So $x = 3 \pm \sqrt{11}$ [√]

(f) $\cos\theta = \frac{16^2 + 19^2 - 34^2}{2 \times 16 \times 19}$ [√]

So $\theta = 152°$ (nearest °) [√]

(g) $V = \frac{1}{3} \times 5^2 \times 6$ [√]

So $V = 50\text{ cm}^3$ [√]

(h) $x^2 + 6x - 1 = 0$

So $x^2 + 6x + 9 = 10$

So $(x+3)^2 = 10$ [√]

So $x + 3 = \pm\sqrt{10}$

So $x = -3 \pm \sqrt{10}$

So $x = 0{\cdot}162$ or $-6{\cdot}162$ [√]

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4. (a) $\sqrt{18} + \sqrt{8} = \sqrt{x}$

So $3\sqrt{2} + 2\sqrt{2} = \sqrt{x}$ [√]

So $5\sqrt{2} = \sqrt{50} = \sqrt{x}$

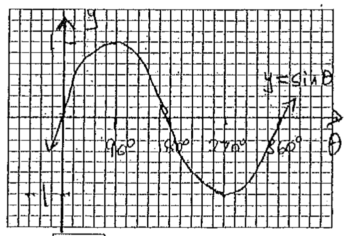
So $x = 50$ [√]

(b) $x^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \times \cos 30°$ [√]

So $x^2 \approx 6{\cdot}35898\ldots$

So $x = 2{\cdot}522$ units (3 d.p.) $x > 0$ [√]

(c) (i)




[√√]

(ii) $\theta = 270°$ [√]

(d) $CX = 2$ (radii & $X$ bisects $CO$ )

Similarly, $XD = 6$ [√]

Now $AX \times XB = CX \times XD$

(int. chords theorem) [√]

So $3XB = 2 \times 6$

So $XB = 4$ cm [√]

(e) $x - 2$ is a factor, so

$2^3 + 2^2 \times p + 2 + 6 = 0$ [√]

So $8 + 4p + 8 = 0$

So $p = -4$ [√]

(f) (i) $\angle SQR = 51°$ (alt. ∠s, ∥ lines) [√]

$\angle STR = 51°$ (∠s on same arc) [√]

(ii) $\angle TQS = 36°$ (∠ sum $\triangle TSQ$) [√]

(iii) $\angle STU = 36°$ (∠ in alt. seg. theorem) [√]

5. (a)

$$
\begin{array}{r|l}
 & x^2 + x - 2 \\
\hline
x+1 & x^3 + 2x^2 - x + 2 \\
 & x^3 + x^2 \\
\hline
 & \quad x^2 - x \\
 & \quad x^2 + x \\
\hline
 & \qquad -2x + 2 \\
 & \qquad -2x - 2 \\
\hline
 & \qquad\qquad 4
\end{array}
$$

√

So $Q(x) = x^2 + x - 2$ √

(b) (i) $-1$ or $(0, -1)$ √

(ii) Cuts $x$-axis where $3^x - 2 = 0$

So $3^x = 2$ √

So $x \log 3 = \log 2$ (Base of log not necessary)

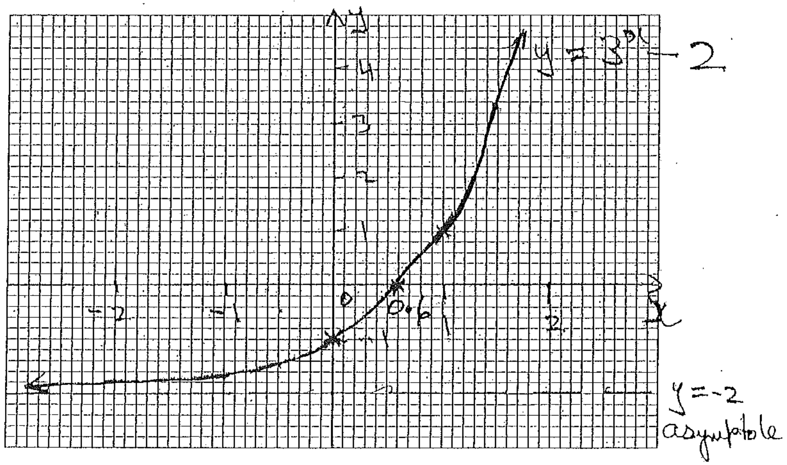
So $x = \dfrac{\log 2}{\log 3}$

So $x \approx 0{\cdot}6$

So the curve cuts the $x$-axis at $0{\cdot}6$ or $(0{\cdot}6, 0)$ (1 d.p.) √

(iii) Horizontal asymptote is $y = -2$ √

(iv)




√√

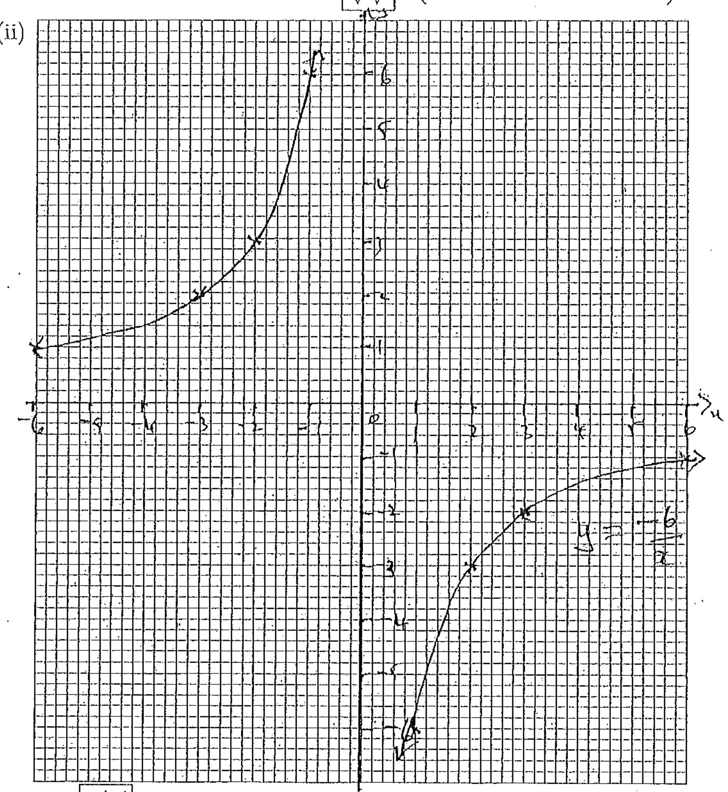
Deduct one for axial intercepts not labelled.
Shape should be clear.

(c) (i)

| $x$ | $-6$ | $-3$ | $-2$ | $-1$ | $1$ | $2$ | $3$ | $6$ |
|---|---|---|---|---|---|---|---|---|
| $y$ | $1$ | $2$ | $3$ | $6$ | $-6$ | $-3$ | $-2$ | $-1$ |

√√ (Deduct 1 for each error.)

(ii)



√√

One for reasonable shape and one for the graph approaching the asymptotes correctly.

(d) (i) By the remainder theorem:

$$a^3 - 13a + 14 = 2$$

So $a^3 - 13a + 12 = 0$ $\boxed{\surd}$

(ii) By inspection $a = 1$ is a solution. $\boxed{\surd}$

Either by further inspection or long division:

$(a-1)(a^2 + a - 12) = 0$ $\boxed{\surd}$ for this or the long division below.

$$
\begin{array}{r|rrrr}
 & a^2 & +\ a & -\ 12 & \\
a - 1 & a^3 & & -\ 13a & +\ 12 \\
 & a^3 & -\ a^2 & & \\
\hline
 & & a^2 & -\ 13a & \\
 & & a^2 & -\ a & \\
\hline
 & & & -12a & +\ 12 \\
 & & & -12a & +\ 12 \\
\hline
 & & & & 0
\end{array}
$$

So $(a-1)(a+4)(a-3) = 0$

So $a = -4$, 1 or 3 $\boxed{\surd}$

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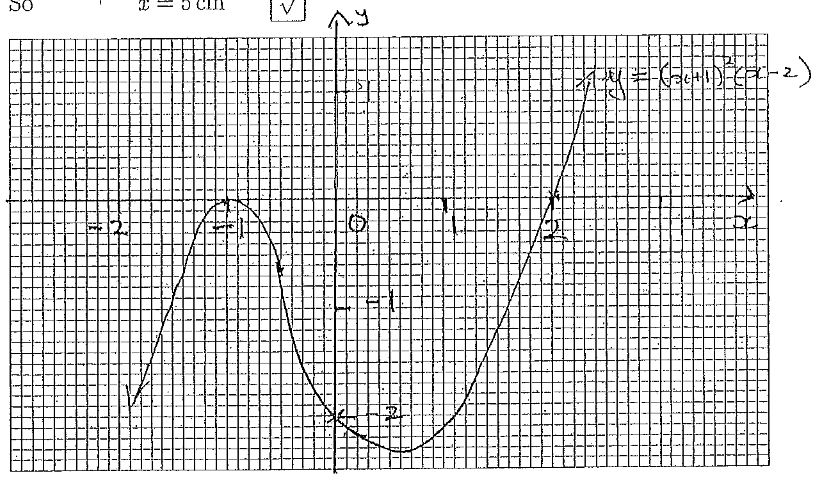
6. (a) $QT \times RT = PT^2$ (tangent/secant theorem) $\boxed{\surd}$

So $(4 + x) \times 4 = 6^2$

So $4 + x = 9$

So $x = 5$ cm $\boxed{\surd}$

(b)




correct roots $\surd$ | axial intercepts $\surd$ | shape $\surd$

(c) (i) The degree of $P(x) + Q(x) = n$ $\boxed{\surd}$

(ii) The degree of $P(x) \times Q(x) = n + m$ $\boxed{\surd}$

(d) (i) Gradient of tangent $= 6$.

Hence the gradient of the line $\perp$ is $-\frac{1}{6}$ $\boxed{\surd}$

(ii) So $PQ$ is $y - 5 = -\frac{1}{6}(x - 4)$

So $6y - 30 = -x + 4$

So $x + 6y - 34 = 0$, as required. $\boxed{\surd}$

(iii) Solve simultaneously:

$$x + 6y - 34 = 0$$
$$x^2 - 2x - 3 = y$$

So $x + 6x^2 - 12x - 18 - 34 = 0$

So $6x^2 - 11x - 52 = 0$ $\boxed{\surd}$

So $(6x + 13)(x - 4) = 0$

So $x = -\frac{13}{6}$ or $x = 4$ $\boxed{\surd}$

So at $Q$, $x = -\frac{13}{6}$

and $Q$, $y = \left(34 + \frac{13}{6}\right) \div 6 = \frac{37}{6}$

So $Q$ is $\left(-\frac{13}{6}, \frac{37}{6}\right)$ or $\left(-2\frac{1}{6}, 6\frac{1}{6}\right)$ $\boxed{\surd}$

(e) The first piece of data yields:

$8x = 7y + 12$

The second piece of data yields:

$13y = 9x + 7$

So $8x - 7y = 12 \quad \boxed{1}$

$9x - 13y = -7 \quad \boxed{2} \quad \boxed{\checkmark}$

$\boxed{1} \times 9 : 72x - 63y = 108 \quad \boxed{3}$

$\boxed{2} \times 8 : 72x - 104y = -56 \quad \boxed{4}$

$\boxed{3} - \boxed{4} : 41y = 164$

So $y = 4$

So $x = \frac{1}{8}(12 + 7 \times 4)$

So $x = 5$

So Jake walks at 5 km/h and Fiona walks at 4 km/h $\boxed{\checkmark}$

(f) $\frac{1}{6}\log_x(abc) - \frac{1}{2}\log_x(b\sqrt{c}) - \frac{1}{3}\log_x c = 0$

So $\log_x(abc)^{\frac{1}{6}} = \log_x(b\sqrt{c})^{\frac{1}{2}} + \frac{1}{3}\log_x c^{\frac{1}{3}}$

So $\log_x(abc)^{\frac{1}{6}} = \log_x\left((bc^{\frac{1}{2}})^{\frac{1}{2}}c^{\frac{1}{3}}\right) \quad \boxed{\checkmark}$

So $a^{\frac{1}{6}}b^{\frac{1}{6}}c^{\frac{1}{6}} = b^{\frac{1}{2}}c^{\frac{1}{4}}c^{\frac{1}{3}}$

So $a^{\frac{1}{6}} = b^{\frac{1}{3}}c^{\frac{5}{12}}$

So $a^2 = b^4c^5$, as required. $\boxed{\checkmark}$

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7. (a) The probability the team wins is two mutually exclusive events.

So $P(\text{team wins}) = P(\text{Win with Mango}) + P(\text{Win without Mango}) \quad \boxed{\checkmark}$

$$P(\text{team wins}) = \tfrac{3}{4} \times \tfrac{1}{3} + \tfrac{1}{2} \times \tfrac{2}{3}$$
$$= \tfrac{1}{4} + \tfrac{1}{3}$$

So $P(\text{team wins}) = \frac{7}{12} \quad \boxed{\checkmark}$

(b) (i) $\angle ACB = 90^\circ$ ($\angle$ in a semi-circle $= 90^\circ$)

So $\dfrac{AC}{y} = \cos\alpha$

So $AC = y\cos\alpha$, as required. $\boxed{\checkmark}$

(ii) $\angle ABD = 90^\circ$ (tangent meets a radius)

So $\dfrac{y}{AD} = \cos\alpha$

So $AD = \dfrac{y}{\cos\alpha}$

But $CD = AD - AC$

So $CD = \dfrac{y}{\cos\alpha} - y\cos\alpha \quad \boxed{\checkmark}$

$$= y\left(\frac{1}{\cos\alpha} - \cos\alpha\right)$$
$$= y\left(\frac{1 - \cos^2\alpha}{\cos\alpha}\right)$$
$$= y\left(\frac{\sin^2\alpha}{\cos\alpha}\right), \text{ as } \sin^2\alpha + \cos^2\alpha = 1.$$
$$= y\sin\alpha\frac{\sin\alpha}{\cos\alpha}$$

So $CD = y\sin\alpha\tan\alpha$, as $\dfrac{\sin\alpha}{\cos\alpha} = \tan\alpha$. $\boxed{\checkmark}$

(c) (i) $\tan(180^\circ - x) = -\tan x \quad \boxed{\checkmark}$

(ii) ($\alpha$) $C = 180^\circ - (A + B)$ ($\angle$ sum $\triangle ABC = 180^\circ$)

So $\tan C = \tan(180^\circ - (A + B))$

$$= -\tan(A + B) \quad \boxed{\checkmark}$$
$$= -\frac{\tan A + \tan B}{1 - \tan A\tan B}$$
$$= -\frac{1 + 2}{1 - 1 \times 2}$$

So $\tan C = 3 \quad \boxed{\checkmark}$

($\beta$) If $\tan\theta = k$ then $\sin\theta = \dfrac{k}{\sqrt{1+k^2}}$ so:

$$\boxed{\surd\surd} \quad \begin{cases} \sin A = \dfrac{1}{\sqrt{2}} \\ \sin B = \dfrac{2}{\sqrt{5}} \\ \sin C = \dfrac{3}{\sqrt{10}} \end{cases}$$

All three ratios must be found.

Hence by the sine rule:

$$\frac{a}{\frac{1}{\sqrt{2}}} = \frac{b}{\frac{2}{\sqrt{5}}} = \frac{c}{\frac{3}{\sqrt{10}}}$$

So $$\frac{a}{\sqrt{5}} = \frac{b}{2\sqrt{2}} = \frac{c}{3} \quad \boxed{\surd}$$

(d) (i) $\left(y - \dfrac{1}{y}\right)^3 = y^3 - 3y + \dfrac{3}{y} - \dfrac{1}{y^3}$ $\boxed{\surd}$

(ii) From (i) $y^3 - \dfrac{1}{y^3} = \left(y - \dfrac{1}{y}\right)^3 + 3\left(y - \dfrac{1}{y}\right)$ $\boxed{\surd}$

But $y - \dfrac{1}{y} = \sqrt{3}$, as $y - \dfrac{1}{y} > 0$

So $y^3 - \dfrac{1}{y^3} = \left(\sqrt{3}\right)^3 + 3\sqrt{3}$

So $y^3 - \dfrac{1}{y^3} = 6\sqrt{3}$ $\boxed{\surd}$

(iii) Now $\left(y - \dfrac{1}{y}\right)^2 = 3$

So $y^2 - 2 + \dfrac{1}{y^2} = 3$

So $y^2 + \dfrac{1}{y^2} = 5$ $\boxed{\surd}$

So $\left(y^2 + \dfrac{1}{y^2}\right)^2 = 25$

So $y^4 + 2 + \dfrac{1}{y^4} = 25$

So $y^4 + \dfrac{1}{y^4} = 23$ $\boxed{\surd}$

If $y$ is found explicitly $\left(y = \frac{1}{2}(\sqrt{3} + \sqrt{7})\ \&\ \frac{1}{y} = \frac{1}{2}(\sqrt{7} - \sqrt{3})\right)$ then one mark out of four may be awarded for parts (ii) and (iii).

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