SYDNEY GRAMMAR SCHOOL

2009 Annual Examination

FORM II MATHEMATICS

Wednesday 4th November 2009

General Instructions

- Writing time — 2 hours
- Write using black or blue pen.
- Calculators are not to be used.
- All necessary working should be shown in every question.
- Start each question on a new page.

Structure of the paper

- Total marks — 130
- All ten questions may be attempted.
- All ten questions are of equal value.

Collection

- Write your name, class and master clearly on each page of your answers.
- Staple your answers in a single bundle.

2A: MW	2B: SO	2C: JMR	2D: BR
2E: DNW	2F: MLS	2G: BDD	2H: KWM
2I: SJE	2J: TCW		

Checklist

- Writing paper required.
- Candidature — 192 boys

Examiner

SJE

QUESTION ONE (13 marks) Start a new page.

(a) Calculate:

(i) $-12 + 105$

(ii) $\frac{2}{3} \times \frac{4}{5}$

(iii) $\frac{3}{4} - \frac{7}{8}$

(iv) $0{\cdot}04 \times 1{\cdot}5$

(b) Find 8% of 400.

(c) Simplify:

(i) $48 : 33$

(ii) $a^6 \div a^2$

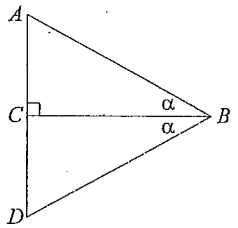
(iii) $2b^2 - 7b + 2b$

(d) Factorise $3x - 12y$.

(e) Simplify $5x^2 \times (-x)$.

(f) Solve $\frac{x}{4} + 3 = -4$.

(g)



State which congruence test would be used to show that $\triangle ABC \equiv \triangle DBC$. Do not prove your result.

Exam continues next page ...

QUESTION TWO (13 marks) Start a new page.

(a) Evaluate:

(i) $16 - 5 \times 7$

(ii) $1{\cdot}3 \div 0{\cdot}2$

(b) Convert $\frac{3}{8}$ to a percentage.

(c) Simplify $\frac{6b}{3b^2}$.

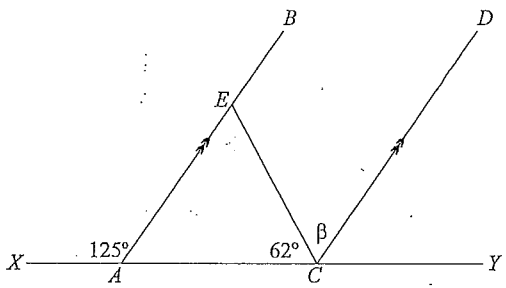
(d) Express $16\frac{1}{2}\%$ as a fraction in simplest form.

(e) Expand and simplify $(x+3)(2x-1)$.

(f) Divide \$63 in the ratio 5 : 2.

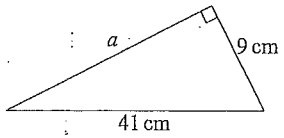
(g) Find the average speed of a car that travels 75 km in $1\frac{1}{2}$ hours.

(h)



Find the value of β in the diagram above, giving a reason.

(i)



In the diagram above find the length of the unknown side a.

Exam continues overleaf ...

QUESTION THREE (13 marks) Start a new page.

(a) Calculate:

(i) $4^3 - 4^2$

(ii) $\frac{1}{7} + \frac{2}{3}$

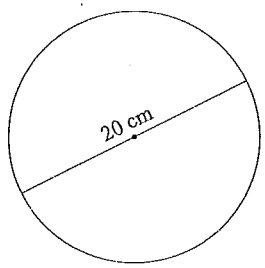
(b) Solve $3x - 2 = 5 + x$.

(c) Express the ratio $\frac{3}{8} : \frac{1}{4}$ in simplest form.

(d) (i) Draw a number plane clearly marking the axes, the origin O and the points $A(1,-2)$ and $B(-2,2)$.

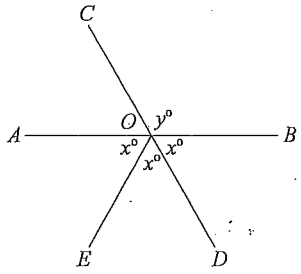
(ii) Use Pythagoras' Theorem to find the length of the interval AB.

(e)



Find the area of the circle above. Use $\pi \doteq 3{\cdot}14$.

(f)



In the diagram above AB and CD are straight lines. Find the values of x and y, giving reasons.

Exam continues next page ...

QUESTION FOUR (13 marks) Start a new page.

(a) Simplify $p^2 + p - 2p(1-p)$.

(b) A number is multiplied by 3 and then 11 is added to the result. The answer is 233.

(i) Form an equation involving the unknown number.

(ii) Solve this equation to find the number.

(c) Solve $x + \frac{3}{2} > \frac{1}{2}$ and graph your solution on a number line.

(d) Solve $\frac{m+2}{3} - m = 4$.

(e) (i) Copy and complete the following table for $y = 2x + 3$.

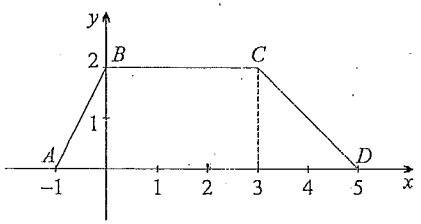
x	-2	0	2
y			

(ii) Plot the points from the table on a number plane and hence sketch the line $y = 2x + 3$.

(iii) On the same number plane draw the line $x = -1$.

(iv) State the coordinates of the point of intersection of the lines $y = 2x + 3$ and $x = -1$.

(f)



Find the area of the trapezium $ABCD$ drawn on the number plane above.

Exam continues overleaf ...

QUESTION FIVE (13 marks) Start a new page.

(a) Factorise fully $6ab^2 - 9a^2$.

(b) Calculate $3\frac{1}{3} \div 3\frac{1}{2}$.

(c) Solve $\frac{3}{x} - 4 = 12$.

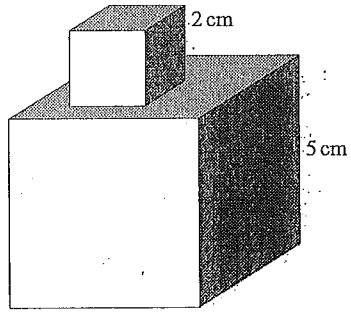
(d) A 1 litre carton of milk is in the shape of a rectangular prism. If the base measures 8 cm by 5 cm what is its height?

(e) A rectangular camping area measures 5 m by 12 m. A tent with rectangular base dimensions of 3 m by 4 m is erected in the camping area.

(i) What area is not covered by the tent?

(ii) What are the dimensions of the largest rectangular groundsheet that can fit in the remaining area?

(f)



The above figure is formed by gluing a cube of side length 2 cm on top of a cube of side length 5 cm. Find the surface area of the figure.

Exam continues next page ...

QUESTION SIX (13 marks) Start a new page.

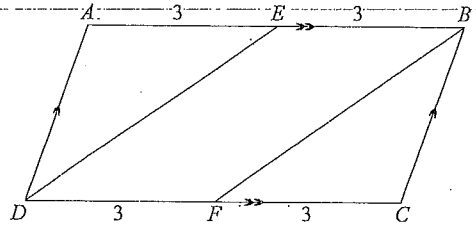
(a) Name all special quadrilaterals from the following list that must have at least one pair of adjacent sides equal.

Kite Parallelogram Rectangle Rhombus Square Trapezium

(b) A wet towel weighs 4·0 kg. It is hung out to dry and when dry it weighs 0·6 kg. Calculate the percentage of the wet towel's weight that was lost.

(c) The dimensions of a rectangular box are in the ratio $2 : 3 : 5$. If the box has a volume of $1920\,\text{cm}^3$, find its dimensions.

(d)



In the diagram above $ABCD$ is a parallelogram. It is known that DE bisects AB and BF bisects DC.

(i) State why $AD = BC$.

(ii) Prove formally that $\triangle ADE \equiv \triangle CBF$.

(iii) Hence state why $\angle ADE = \angle CBF$.

(e) Perform the constructions outlined on the tear-off sheet at the end of this examination paper. This sheeet should be bundled with the rest of your Question Six.

Exam continues overleaf ...

QUESTION SEVEN (13 marks) Start a new page.

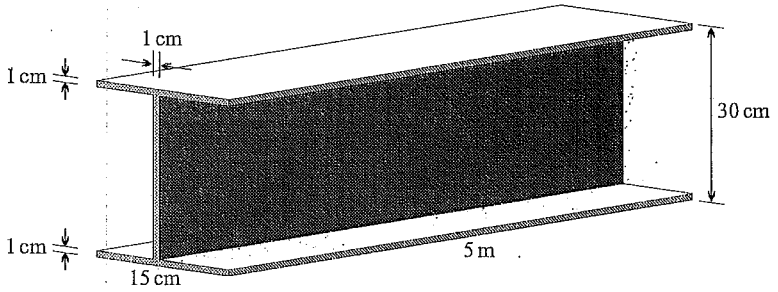
(a) Simplify $\frac{1}{2} \times \frac{1}{3} - \frac{1}{4} \div \frac{1}{5} + \frac{1}{6}$.

(b) Calculate $23\frac{1}{4} - 16\frac{3}{5}$.

(c) Evaluate $\frac{5a}{-2b^2}$, when $a = 3$ and $b = -15$.

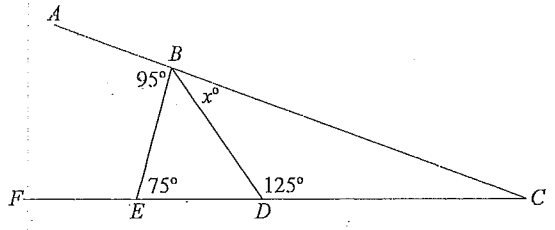
(d) Solve $x + 4{\cdot}505 = 1{\cdot}038$.

(e)



Find the volume of solid steel beam above.

(f)



Find x giving reasons.

Exam continues next page ...

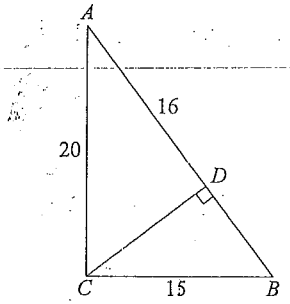
QUESTION EIGHT (13 marks) Start a new page.

(a) Simplify $\dfrac{24xy^5}{(3x^2y)^2}$.

(b) Evaluate $\dfrac{5}{3+\frac{4}{2-\frac{2}{3}}}$.

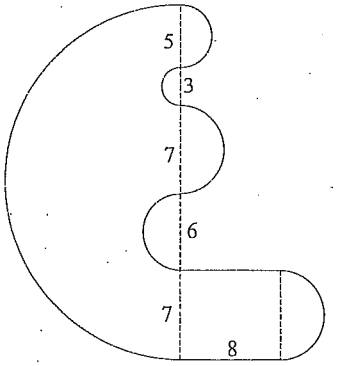
(c) A certain cordial mix is 20% water. How many litres of water must be added to 50 L of this mix to produce a mixture that is $33\frac{1}{3}\%$ water?

(d)



The triangle ABC above looks like it is right-angled at C. Use Pythagoras' Theorem to verify whether it is or not.

(e)



The Go-Kart track above is made of semi-circular turns and two straight sections. All measurements are in metres.

(i) Find the perimeter of the track. Use $\pi \doteqdot \frac{22}{7}$.

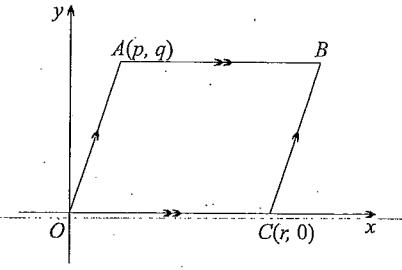
(ii) If the lap record is 20 seconds determine the average speed to the nearest km/h.

Exam continues overleaf ...

QUESTION NINE (13 marks) Start a new page.

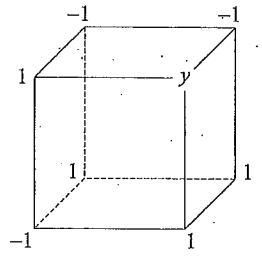
(a) Solve $\dfrac{3k-2}{3} - \dfrac{k+1}{2} = 4+k$.

(b)



In the diagram above $OABC$ is a parallelogram. Write down the coordinates of B in terms of p, q and r.

(c)



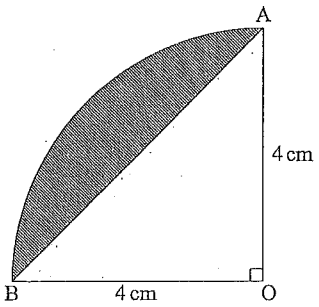
In the diagram above, the cube has integers -1 or 1 at each vertex. However, the number at one vertex is unknown and is labelled y. Each of the six faces has a value formed by multiplying the vertices on that face.

(i) Write down the six expressions that represent the values of the faces.

(ii) Find the sum when all 14 values of those at the vertices and those on the faces are added together.

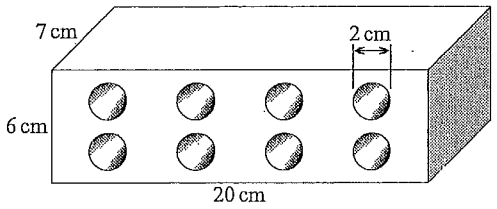
Exam continues next page ...

(d)



A line AB cuts off part of circle of radius 4 cm, as shaded in the diagram above. Find an exact expression for the shaded area.

(e)



A particular type of clay house brick is a rectangular prism with eight cylindrical holes. Its dimensions are given in the diagram above.

(i) Calculate the volume of clay in this brick. Use $\pi \doteqdot \frac{22}{7}$.

(ii) If the density of clay is $2000\,\text{kg/m}^3$, determine the mass of a brick.

Exam continues overleaf ...

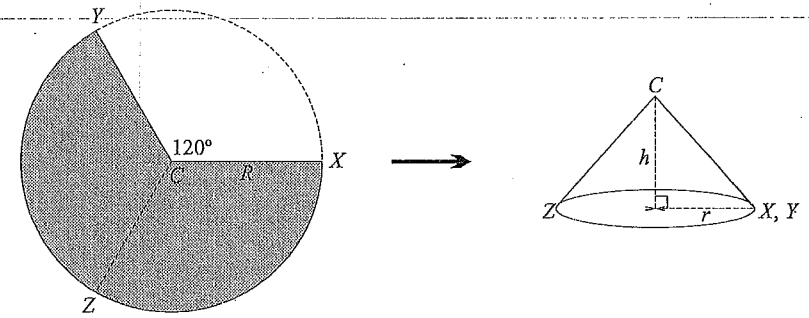
QUESTION TEN (13 marks) Start a new page.

(a) If $\left(t - \frac{1}{t}\right)^2 = 5$, find the value of $t^2 + \frac{1}{t^2}$.

(b) The value of 1 Australian dollar is 0·92 US dollars. How many Australian dollars will 100 US dollars buy? Give your answer to the nearest cent.

(c) The profits from a small company are shared equally between the management team and the staff. The management team of Fred, Daphne and Velma share their half of the profits equally. The staff of Alvin, Simon and Theodore split their half of the profits in the ratio 4 : 3 : 2 respectively. It is known that Fred received \$280 more than Theodore. How much profit did the company make?

(d)



A circular piece of paper with an unknown radius R has a 120°-sector cut out, as in the diagram above. The remaining sector is folded so that X joins Y to make a circular cone.

(i) Find an expression in terms of R for the circumference of the base of the cone.

(ii) Find the radius r of the base of the cone and hence find an exact expression for the height h of the cone. (Both answers will be expressed in terms of R.)

(iii) If the height of the cone is $7\sqrt{5}$ cm, find the radius of the original piece of paper.

(e) Water can be pumped into a tank by two pipes A and B. The tank can also be emptied by a pipe C. Pipe A can fill the empty tank in 4 days working alone and pipe B can fill the empty tank in 6 days working alone. Pipe C empties the full tank in 5 days working alone. In order to fill the empty tank Farmer Hayes opens pipes A and B and closes pipe C.

(i) What fraction of the tank would be full after 2 days?

(ii) Suppose when he opened pipes A and B, he accidently left pipe C open. What percentage of the tank would be full after the two days?

END OF EXAMINATION

Tear-off pages follow ...

NAME: ..... CLASS: ..... MASTER: .....

DETACH THIS SHEET AND BUNDLE IT WITH THE REST OF QUESTION SIX.

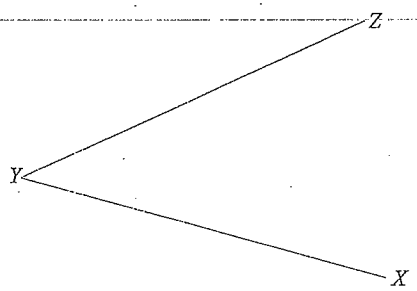
QUESTION SIX

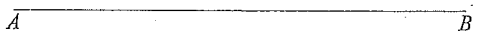
(e) In the following constructions leave all construction arcs visible.

(i) Copy $\angle XYZ$ at A below to form $\angle BAP$.

(ii) Construct $\angle ABQ = 60°$.

(iii) Let AP and BQ intersect at C. Draw $\triangle ABC$.





Form II 2009 Annual Exam - Solutions

Question 1

a) (i) $-12 + 105 = 93$ ✓

(ii) $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$ ✓

(iii) $\frac{3}{4} - \frac{7}{8} = -\frac{1}{8}$ ✓

(iv) $0.04 \times 1.5 = 0.06$ ✓

b) $8\% \times 400 = 32$ ✓

c) (i) $48 : 33 = 16 : 11$ ✓

(ii) $a^6 \div a^2 = a^4$ ✓

(iii) $2b^2 - 7b + 2b$

$= 2b^2 - 5b$ ✓

d) $3x - 12y = 3(x - 4y)$ ✓

e) $5x^2 \times (-x) = -5x^3$ ✓

f) $\frac{x}{4} + 3 = -4$

$\frac{x}{4} = -7$ ✓

$x = -28$ ✓

g) AAS ✓

(13)

Question 2

a) (i) $16 - 5 \times 7 = 16 - 35$

$= -19$ ✓

(ii) $1.3 \div 0.2 = 6.5$ ✓

b) $\frac{3}{8} = 37.5\%$ ✓

c) $\frac{66}{36^2} = \frac{2}{6}$ ✓

d) $16\frac{1}{2}\% = \frac{33}{200}$ ✓

e) $(x+3)(2x-1) = 2x^2 + 6x - x - 3$ ✓

$= 2x^2 + 5x - 3$ ✓

f) $5 : 2$

$\$45 : \$18$ ✓

g) $75\text{km}/1\frac{1}{2}\text{hours} = 50\text{km/h}$ ✓

h) $\beta + 62° = 125°$ (corresponding angles, $AB \parallel CD$) ✓

$\beta = 63°$ (adjacent angles) ✓

i) $a^2 = 41^2 - 9^2$ ✓

$= 1681 - 81$

$= 1600$

$\therefore a = 40$ cm ✓

(13)

Question 3

a) (i) $4^3 - 4^2 = 64 - 16$

$= 48$ ✓

(ii) $\frac{1}{7} + \frac{2}{3} = \frac{3}{21} + \frac{14}{21}$

$= \frac{17}{21}$ ✓

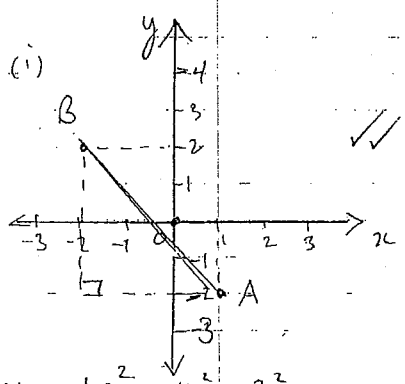
b) $3x - 2 = 5 + x$

$2x = 7$ ✓

$x = \frac{7}{2}$ ✓

c) $\frac{3}{8} : \frac{1}{4} = 3 : 2$ ✓

d) (i)



✓✓

(ii) $AB^2 = 4^2 + 3^2$

$AB^2 = 25$

$AB = 5$ ✓

e) Area $= \pi\left(\frac{20}{2}\right)^2$ ✓

$= 3.14 \times 100$

$= 314\ \text{cm}^2$ ✓

f) $3x° = 180°$ (straight angle) ✓

$x° = 60°$ ✓

$y° = 2x°$ (vertically opposite angles)

$y = 120°$ ✓

(13)

Question 4

a) $p^2 + p - 2p(1-p)$

$= p^2 + p - 2p + 2p^2$ ✓

$= 3p^2 - p$ ✓

b) Let the number be x

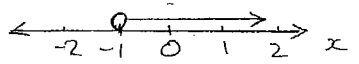
$3x + 11 = 233$ ✓

$3x = 222$

$x = 74$ ✓

c) $x + \frac{3}{2} > \frac{1}{2}$

$x > -1$ ✓



✓

d) $\frac{m+2}{3} - m = 4$

$m + 2 - 3m = 12$ ✓

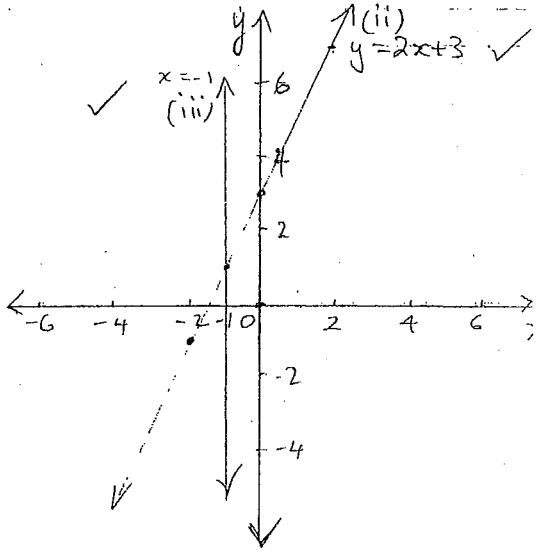
$-2m = 10$

$m = -5$ ✓

e) (i)

x	-2	0	2
y	-1	3	7

✓



✓ ✓

(iv) $(-1, 1)$ ✓

f) Area $= \frac{1}{2}(2)[6+3]$

$= 9\ \text{units}^2$ ✓

(13)

Question 5

a) $6ab^2 - 9a^2 = 3a(2b^2 - 3a)$ ✓

b) $3\frac{1}{3} \div 3\frac{1}{2} = \frac{10}{3} \div \frac{7}{2}$ ✓

$= \frac{10}{3} \times \frac{2}{7}$

$= \frac{20}{21}$ ✓

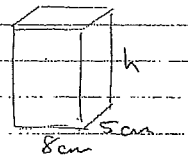
c) $\frac{3}{x} - 4 = 12$ ✓

$\frac{3}{x} = 16$

$\frac{x}{3} = \frac{1}{16}$

$x = \frac{3}{16}$ ✓

d)



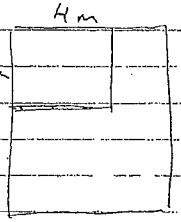
$V = 1$ Litre $= 1000\ cm^3$

let h be height

$(h \times 5 \times 8)_{cm^3} = 1000\ cm^3$ ✓

$h = 25$ cm ✓

e)



Remaining area $= (5 \times 12) - (3 \times 4)\ m^2$ ✓

$= 48\ m^2$ ✓

f) $9m \times 5m$ ✓

g) $SA = 6 \times 5^2 + 4 \times 2^2$ ✓

$= 150 + 16$

$= 166\ cm^2$ ✓✓

(13)

Question 6

(a) Kite Rhombus Square ✓

(b) $\frac{\text{Weight lost}}{\text{original Weight}} \times 100\% = \frac{3.4}{4} \times 100\%$ ✓

$= 85\%$ ✓

(c) Let the dimensions be $2x, 3x, 5x$ ✓

So $2x \times 3x \times 5x = 1920\ cm^3$

$30x^3 = 1920\ cm^3$

$x^3 = 64\ cm^3$

$x = 4$ cm ✓

∴ Dimensions are $8 \times 12 \times 20$ cm ✓

(d)

(i) Opposite sides of a parallelogram are equal ✓

(ii) In $\triangle ADE$ and $\triangle CBF$

$AD = BC$ (opposite sides of parallelogram)

$\angle DAE = \angle BCF$ (opposite angles of a parallelogram) ✓

$AE = CF$ (given)

∴ $\triangle ADE \equiv \triangle CBF$ (SAS) ✓

(iii) matching angles of congruent triangles ✓

(e) see tear-off sheet ✓✓

(13)

Question 7

(a) $\frac{1}{2} \times \frac{1}{3} - \frac{1}{4} \div \frac{1}{5} + \frac{1}{6}$

$= \frac{1}{6} - \left(\frac{1}{4} \times \frac{5}{1}\right) + \frac{1}{6}$ ✓

$= \frac{2}{6} - \frac{5}{4}$

$= \frac{4 - 15}{12}$

$= \frac{-11}{12}$ ✓

(b) $23\frac{1}{4} - 16\frac{3}{5}$

$= 23 + \frac{1}{4} - 16 - \frac{3}{5}$

$= 22 + \frac{5}{4} - 16 - \frac{3}{5}$ ✓

$= 6 + \frac{25 - 12}{20}$

$= 6\frac{13}{20}$ ✓

(c) $\frac{5a}{-2b^2} = \frac{5(3)}{-2(-15)^2}$

$= \frac{15}{-450}$ ✓

$= -\frac{1}{30}$ ✓

(d) $x + 4.505 = 1.038$

$3.467 = -x$

$x = -3.467$ ✓

(e) $V = Ah$

$= (1 \times 15 + 1 \times 15 + 1 \times 28) \times 500\ cm^3$ ✓✓

$= 58 \times 500\ cm^3$

$= 29000\ cm^3$ ✓

(f) $\angle EBD + 75° = 125°$ (exterior angle of $\triangle BED$)

$\angle EBD = 50°$

$x + 50° + 95° = 180°$ (straight angle)

$x = 180° - 95° - 50°$

$= 35°$ ✓

(13)

Question 8

(a) $\frac{24xy^5}{(3x^2y)^2} = \frac{24xy^5}{9x^4y^2}$ ✓

$= \frac{8y^3}{3x^3}$ ✓

(b) $\frac{5}{3 + \frac{4}{2 - \frac{3}{3}}} = \frac{5}{3 + \frac{4}{\frac{4}{3}}}$

$= \frac{5}{6}$ ✓

(c) let the amount (in L) of water to be added be x

$\therefore \frac{10 + x}{50 + x} = \frac{1}{3}$ ✓

$30 + 3x = 50 + x$

$2x = 20$

$x = 10$ ✓

∴ 10 L of water must be added

(d) $CO^2 = 20^2 - 16^2$

$= 144$

$\therefore CO = 12$ ✓

$DB^2 = 15^2 - 12^2$

$= 81$

$\therefore DB = 9$ ✓

Now $AB = 16 + 9$

$= 25$

Question 8 (cont.)

d) (cont.)

If $\triangle ABC$ is right-angled Pythagoras must hold

i.e. $AB^2 = AC^2 + CB^2$

$LHS = 25^2$

$= 625$

$RHS = 20^2 + 15^2$

$= 400 + 225$

$= 625$

$\therefore = LHS$ ✓

$\therefore \triangle ABC$ is right-angled ✓

e) Perimeter $= \frac{\pi}{2}(5+3+7+6+7)\pi$ ✓

i) $+ \frac{1}{2}(5+3+7+6+7)\pi + 16$

$= 28\pi + 16$

$\doteq 88 + 16$

$\doteq 104$ m ✓

ii) Average Speed $= \frac{104\text{ m}}{20\text{ s}}$

$= \frac{0.104\text{ km}}{\frac{20}{3600}\text{ h}}$ ✓

$= 18.72$ km/h

$\doteq 19$ km ✓

(13)

Question 9

(a) $\frac{3k-2}{3} - \frac{k+1}{2} = 4+k$

$2(3k-2) - 3(k+1) = 6(4+k)$ ✓

$6k - 4 - 3k - 3 = 24 + 6k$ ✓

$-3k = 31$

$k = -\frac{31}{3}$ ✓

b) $B(p+r, q)$ ✓

(c) (i) $y, -1, -y, 1, 1, -y$ ✓✓

(ii) $-1 - 1 + y + 1 + 1 + 1 + 1 - 1$

$+ y - 1 - y + 1 + 1 - y$

$= 2$ ✓

(d) Area $= \frac{1}{4}\pi 4^2 - \frac{1}{2}(4 \times 4)$ ✓

$= 4\pi - 8$ cm^2 ✓

(e) (i) Volume $= (20 \times 6 \times 7)$ ✓

$- 8(\pi 1^2 \times 7)$

$= 840 - 8 \times 22 \times 7$

$= 664$ cm^3 ✓

(ii) Density $= 2000$ kg/m^3

$= \frac{2000000\text{ g}}{1000000\text{ cm}^3}$

$= 2$ g/cm^3 ✓

$\therefore$ Mass is 1328 g ✓

or 1.328 kg

(13)

Question 10

(a) $\left(t - \frac{1}{t}\right)^2 = t^2 - 2t\frac{1}{t} + \frac{1}{t^2}$

$= t^2 + \frac{1}{t^2} - 2$

$= 5$

$\therefore t^2 + \frac{1}{t^2} = 7$ ✓

(b) 1 AUD $= 0.92$ USD

$\frac{100}{0.92}$ AUD $= 100$ USD ✓

$\therefore \$100$ USD $\doteq \$108.70$ AUD ✓

(c) let the profits be x

F : D : V : A : S : T

$\frac{2x}{6} : \frac{2x}{6} : \frac{x}{6} : \frac{4x}{18} : \frac{3x}{18} : \frac{2x}{18}$ ✓

Now $\frac{2x}{6} = \frac{2x}{18} + \$280$

$\frac{3x - 2x}{18} = \$280$

$x = \$5040$ ✓

$\therefore$ The profit was \$5040

(d) (i) Circumference of base $= \frac{2}{3} 2\pi R$

$= \frac{4}{3}\pi R$ ✓

(ii) $2\pi r = \frac{4}{3}\pi R$

$\therefore r = \frac{2}{3}R$ ✓

$h^2 = R^2 - \left(\frac{2}{3}R\right)^2$

$= R^2 - \frac{4}{9}R^2$

$h = \frac{\sqrt{5}}{3}R$ ✓

Question 10 (cont.)

d) (iii) (cont.) $7\sqrt{5} = \frac{\sqrt{5}}{3} R$

$R = 21$ ✓

e) (i) Pipe A: 1 tank in 4 days

$\therefore \frac{1}{4}$ tank in 1 day ✓

Pipe B $\frac{1}{6}$ tank in 1 day

A and B $\frac{1}{4} + \frac{1}{6}$ tank in 1 day

$= \frac{5}{12}$ tank in 1 day

After 2 days:

Tank is $2 \times \frac{5}{12}$ full $= \frac{5}{6}$ full ✓

(ii) $\frac{5}{12} - \frac{1}{5}$ tank in 1 day

$= \frac{13}{60}$ tank in 1 day ✓

$\therefore$ after 2 days $\frac{26}{60}$ full $= \frac{13}{30} \times 100\%$ full

$= 43\frac{1}{3}\%$ full ✓

(13)

NAME: ..... CLASS: ..... MASTER: .....

DETACH THIS SHEET AND BUNDLE IT WITH THE REST OF QUESTION SIX.

QUESTION SIX

(e) In the following constructions leave all construction arcs visible.

(i) Copy $\angle XYZ$ at A below to form $\angle BAP$.

(ii) Construct $\angle ABQ = 60°$.

(iii) Let AP and BQ intersect at C. Show $\triangle ABC$.