SYDNEY GRAMMAR SCHOOL

2011 Annual Examination

FORM II

MATHEMATICS

Monday 7th November 2011

General Instructions
- Writing time — 2 hours
- Write using black or blue pen.
- Calculators are not to be used.
- All necessary working should be shown in every question.
- Start each question on a new page.

Structure of the paper
- Total marks — 130
- All ten questions may be attempted.
- All ten questions are of equal value.

Collection
- Write your name, class and master clearly on each page of your answers and on the tear-off sheet.
- Staple your answers in a single bundle.
- Bundle the tear-off sheet with the question it belongs to.
- Write your name and master on this question paper and submit it with your answers.

2A: BR	2B: SJG	2C: TCW
2D: JMR	2E: BDD	2F: REP
2G: KWM	2H: PKH	2I: FMW

Checklist
- Writing paper required.

Examiner

JMR

QUESTION ONE   (13 marks)   Start a new page.

(a) Simplify the following:

   (i) $3 \times (-5)$

   (ii) $\dfrac{2}{3} + \dfrac{2}{5}$

   (iii) $2 \cdot 05 - 1 \cdot 7$

(b) Find the area of a rectangle that is $8 \cdot 5$ cm long and 4 cm wide.

(c) If $W = Fs$, find W when $F = 11$ and $s = 0 \cdot 24$.

(d)
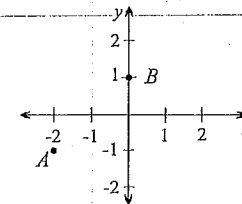
For the diagram above, write down the coordinates of the points A and B.

(e) What is the name of a quadrilateral with only one pair of parallel sides?
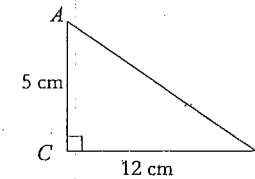
(f) Find 35% of $350.

(g) Simplify:

   (i) $(x^3)^8$

   (ii) $x - 5x - 4x$

(h)

Find the length of AB in the triangle above.

(i) Write $1 \cdot 4$ hours in hours and minutes.

Exam continues next page ...
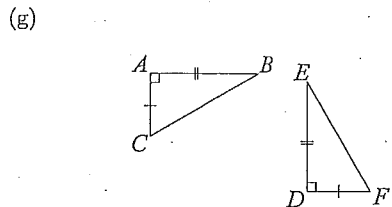
QUESTION TWO   (13 marks)   Start a new page.

(a) Solve the equation $3x - 1 = 17$.

(b) Expand $13x(x - 5)$.

(c) Express $37\frac{1}{2}\%$ as a fraction in lowest terms.

(d) Convert 2560 grams into kilograms.

(e) Find the value of x in each of the diagrams below, giving reasons.
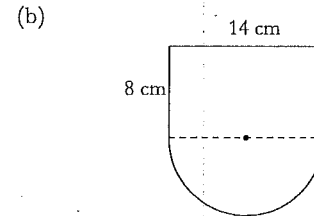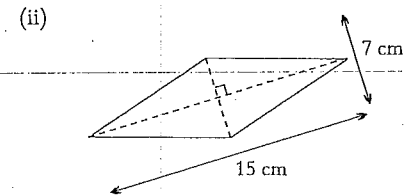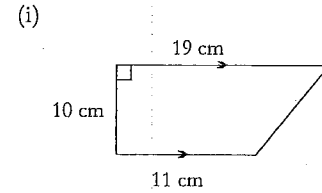
(i)

(ii)

(f) Evaluate $\sqrt{81 + 144}$.

(g)
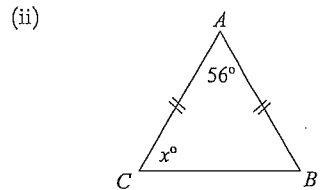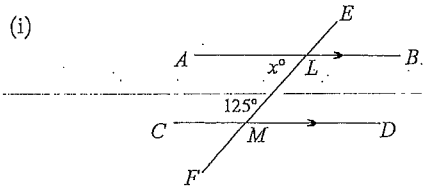
State the congruence test which you would use to show $\triangle ABC \equiv \triangle DEF$. No proof of congruence is required.

(h) Expand:

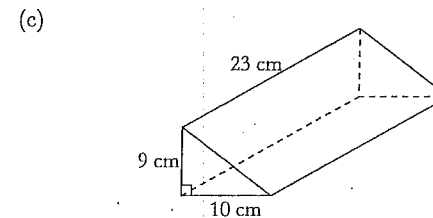(i) $(x + 2)(x - 4)$

(ii) $(a - 3)^2$

QUESTION THREE   (13 marks)   Start a new page.

(a) Find the areas of the quadrilaterals below.

(i)

(ii)

(b)

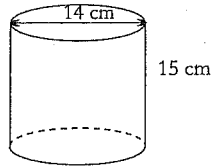In the diagram above, the arc is a semicircle. Using $\pi \doteq \frac{22}{7}$:

(i) find the perimeter of the figure,

(ii) find the area of the figure.

(c)

Find the volume of the triangular prism shown above.

QUESTION THREE (Continued)

(d)

Leaving π in your answer, find the exact volume of the cylinder above.

(e) Convert 2·081 kilolitres into cubic centimetres.

QUESTION FOUR (13 marks)   Start a new page.

(a) Simplify the following ratios.

(i) $35 : 14$

(ii) $\dfrac{3}{8} : \dfrac{1}{16}$

(iii) $2\,\text{cm} : 1\cdot5\,\text{m}$

(b) A school consists of boys and girls in the ratio 7 : 6. If there are 663 pupils in the school, find how many girls there are in the school.

(c) An alloy is made by mixing copper, nickel and lead in the ratio 2 : 3 : 5. How much alloy would be produced if 540 kg of nickel was used?

(d) A coal ship is being filled at the rate of 450 kilograms per minute. Express this rate in tonnes per hour.

(e) Marcus Hellner from Sweden won the 30 km cross-country skiing event at the 2010 Winter Olympics. His winning time was 1 hour and 15 minutes. Calculate his average speed in km/h.

(f) On a scale drawing a rectangular block of land is represented by a rectangle measuring 5 cm by 4 cm. If the scale is 1 : 700, find in metres the actual dimensions of the block of land.

QUESTION FIVE (13 marks)   Start a new page.

(a) Perform the constructions outlined on the tear-off sheet at the end of this examination paper. This sheet should be bundled with the rest of your answers to Question Five.

(b)
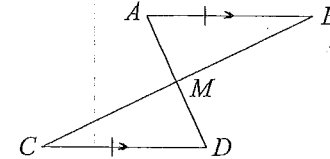
In the diagram above, AB is equal and parallel to CD. Write a formal proof to show that $\triangle ABM \equiv \triangle DCM$.

(c) Decide whether the following statements are true or false. Write TRUE or FALSE as your answer.
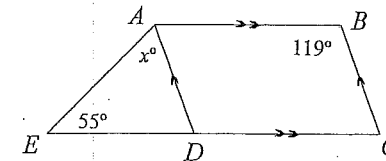
(i) All rectangles have adjacent sides that are equal.

(ii) The area of a rectangle can always be calculated by finding half the product of the lengths of the diagonals.

(iii) The diagonals of a kite are perpendicular.

(iv) All squares are rhombuses.

(d)

Find the value of x in the diagram above. Give reasons for your answer.

QUESTION SIX   (13 marks)   Start a new page.

(a) Write 1·023 as a percentage.

(b) (i) Plot the points $A\,(5,4)$, $B\,(5,8)$ and $C\,(1,4)$ on a number plane and join them to form $\triangle ABC$.
     Use a scale of 1 cm = 1 unit on both the x and y axes.

   (ii) Use Pythagoras' theorem to find the exact length of the interval BC.

   (iii) Indicate on your diagram the point D that will make $ABCD$ a parallelogram (but not a rectangle). Write down the coordinates of this point.

   (iv) Join BD. Label the point of intersection of BD and AC with the letter E. Write down the coordinates of E.

   (v) Given that E is the midpoint of both AC and BD, what property of a parallelogram does this confirm?

(c) A circle marked on a playing field has a circumference of 44 metres. There are 8 rugby players standing on the circumference. Calculate the longest pass between two players that can be made in this circle. Use $\pi \doteq \frac{22}{7}$.

(d)
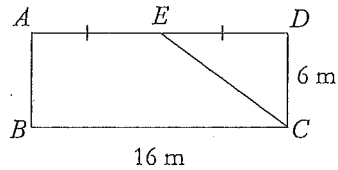
$ABCD$ is a rectangle with $BC = 16\,\text{m}$ and $CD = 6\,\text{m}$. E is the midpoint of AD. Find the length of EC.

QUESTION SEVEN   (13 marks)   Start a new page.

(a) Simplify $\dfrac{14x^2y^3}{7xy} - 2xy^2$.

(b) Expand and simplify $8(2x^2 - 1) - 9(4x^2 - 1)$.

(c) Simplify $\dfrac{5 - 2x}{4} + \dfrac{x + 2}{8}$.

(d) Solve the following equations.

   (i) $2x - 15 = 5x + 9$

   (ii) $\dfrac{3a - 4}{2} - \dfrac{2a + 1}{4} = -5$

   (iii) $\dfrac{3}{7x} - \dfrac{1}{2x} = 3$

QUESTION EIGHT   (13 marks)   Start a new page.

(a) (i) Solve the inequation $5 - \dfrac{2x}{3} < \dfrac{1}{6}$.
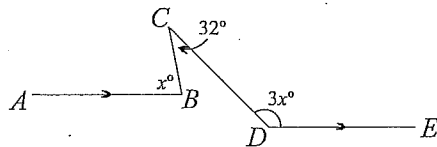
   (ii) Graph your solution on the number line.

(b)
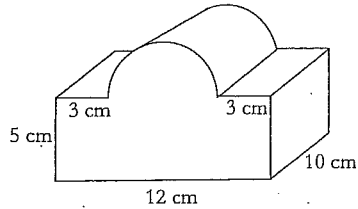
Find the volume of the solid in the diagram above. The cross-section consists of a semicircle on top of a rectangle. Use $\pi \doteqdot 3 \cdot 14$.

(c) (i) For the equation $y = 3 - x^2$, copy and complete the following table of values.

x	-3	-2	-1	0	1	2	3
y							

   (ii) Using a scale of 1cm for 1 unit on both x and y axes, carefully draw the graph of $y = 3 - x^2$.

(d)

In the diagram above $AB \parallel DE$. Copy the diagram. Using a construction line or any other method, form an equation in x and solve it to find the size of $\angle ABC$.

QUESTION NINE   (13 marks)   Start a new page.

(a) Evaluate $\sqrt{\dfrac{-abc^2}{32d^3}}$ if $a = -4$, $b = -1$, $c = \frac{1}{4}$, and $d = -\frac{1}{2}$.

(b) Simplify $\dfrac{(3xy)^2 + 15x^2y^2}{2x \times 3y}$.

(c) In 2010 the teacher : pupil ratio at Sparta Grammar School was 2 : 45. In 2011 the same number of teachers are at the school, but there are 63 extra pupils. The teacher : pupil ratio is now 1 : 24. Let the number of teachers at the School be x.

   (i) In terms of x, how many pupils are at the School in 2011?

   (ii) In terms of x, how many pupils were at the School in 2010?

   (iii) Form an equation and solve it to find the number of teachers at Sparta Grammar School.

(d)
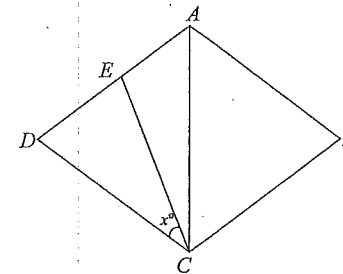
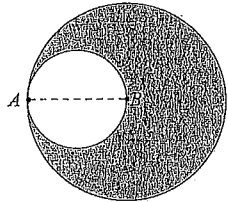In the diagram above, $ABCD$ is a rhombus and CE bisects $\angle ACD$. Let $\angle ECD = x°$.

   (i) Explain why $\angle BCD = 4x°$.

   (ii) Explain why $\angle CED = 3x°$.

QUESTION TEN   (13 marks)   Start a new page.

(a) Expand and simplify $(2x - y)(4x^2 + 2xy + y^2)$.

(b) A can of drink is in the shape of a cylinder with height 15 cm and radius 4 cm. What is the length of the longest straw that will fit inside the can without bending?

(c)

In the diagram above, AB is the diameter of the small circle and B is the centre of the large circle. If the area of the small circle is 11 cm^2, find the exact area of the shaded part of the large circle.
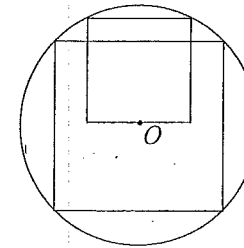
(d)
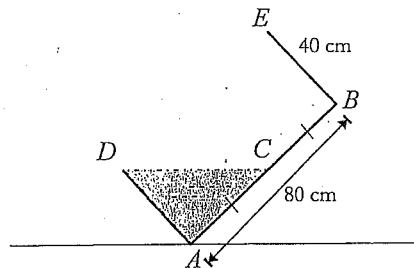
A fish tank is a rectangular prism 80 cm long, 50 cm wide and 40 cm deep. Some of the water is tipped out by tilting the tank along its 50 cm edge as in the diagram above.

The water level reaches C, the midpoint of AB. Find the depth of water in the fish tank once the base AB is returned to the horizontal.

QUESTION TEN (Continued)

(e)

Two squares are drawn inside a circle with centre O. The larger square touches the circle at each of its corners. The smaller square touches the circle at two of its corners and one of its sides passes through the centre of the circle as shown. What is the ratio of the area of the larger square to the area of the smaller square?
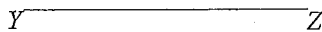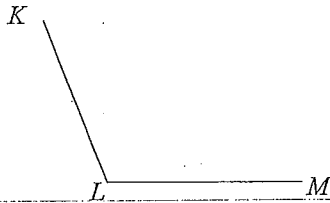
END OF EXAMINATION

NAME: .... CLASS: .... MASTER: ....

DETACH THIS SHEET AND BUNDLE IT WITH THE REST OF QUESTION FIVE.
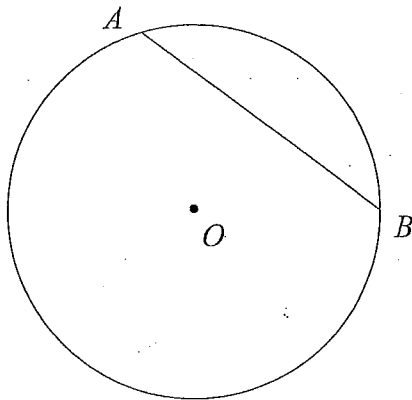
QUESTION FIVE

(a) (i)

Using a compass, ruler and pencil only, copy $\angle KLM$ at Y to form $\angle XYZ$. You must show all construction lines.

(ii)

Construct the perpendicular bisector of the chord AB, clearly showing all construction lines.

FORM II MATHEMATICS ANNUAL 2011
SOLUTIONS

QUESTION ONE

(a) (i) -15 ✓

(ii) $\frac{16}{15}$ or $1\frac{1}{15}$ ✓

(iii) 0.35 ✓

(b) $A = 8.5 \times 4$
$= 34 \text{ cm}^2$ ✓

(c) $W = Fs$
$= 11 \times 0.24$
$= 2.64$ ✓

(d) $A\ (-2,-1)$ ✓
$B\ (0,1)$ ✓

(e) Trapezium ✓

(f) $\frac{35}{100_2} \times \frac{350^7}{1} = 122.5$
Answer $122.50 ✓

(g) (i) x^{24} ✓
(ii) $-8x$ ✓

(h) 13 cm ✓

(i) 1 hour 24 minutes ✓
(Must show correct units in (i)
but no penalty in (b) or (h).)

QUESTION TWO

(a) $3x - 1 = 17$
$3x = 18$
$x = 6$ ✓

(b) $13x(x-5) = 13x^2 - 65x$ ✓

(c) $37\frac{1}{2}\% = \frac{37\frac{1}{2}}{100}$
$= \frac{75}{200}$
$= \frac{3}{8}$ ✓

(d) 2.56 kg ✓

(e) (i) $x° = 55°$ (Co interior angles, $AB \parallel CD$) ✓

(ii) $\angle B = x°$ (Base angles isosceles $\triangle ABC$) ✓
$2x° = 124°$ (Angle sum $\triangle ABC$) ✓
$x = 62$ ✓

(f) $\sqrt{225} = 15$ ✓

(g) SAS ✓

(h)(i) $(x+2)(x-4) = x^2 - 4x + 2x - 8$
$= x^2 - 2x - 8$ ✓

(ii) $(a-3)^2 = a^2 - 6a + 9$ ✓

QUESTION THREE

(a) (i) $A = \frac{10}{2}(11+19)$ ✓

$= 150 \text{ cm}^2$ ✓

(ii) $A = \frac{1}{2} \times 7 \times 15$ ✓

$= 52.5 \text{ cm}^2$ ✓

(b) (i) $P = 2 \times 8 + 14 + \frac{1}{2} \times \frac{22}{7} \times \frac{14}{1}$ ✓

$= 52 \text{ cm}$ ✓

(ii) $A = 8 \times 14 + \frac{1}{2} \times \frac{22}{7} \times \frac{49}{1}$ ✓

$= 112 + 77$

$= 189 \text{ cm}^2$ ✓

(c) $V = \frac{1}{2} \times 10 \times 9 \times 23$ ✓

$= 1035 \text{ cm}^3$ ✓

(d) $V = \pi \times 7^2 \times 15$ ✓

$= 735\pi \text{ cm}^3$ ✓

(e) $2\ 081\ 000 \text{ cm}^3$ ✓

(Penalise lack of or incorrect units once in Qu 3)

QUESTION FOUR

(a) (i) $5:2$ ✓

(ii) $6:1$ ✓

(iii) $1:75$ ✓

(b) Number of girls $= \frac{6}{13} \times \frac{663}{1}$ ✓

$= 306$. ✓

(c) Alloy produced $= 540 \times \frac{10}{3}$ kg ✓

$= 1800 \text{ kg}$ ✓

(d) 450 kg per minute
60×450 kg per hour ✓
27000 kg per hour ✓
27 tonnes per hour. ✓

(e) 30 km in 1 hour 15 minutes
120 km in 5 hours ✓
24 km in 1 hour
Average speed $= 24 \text{ km/h}$ ✓

(f) Actual dimension
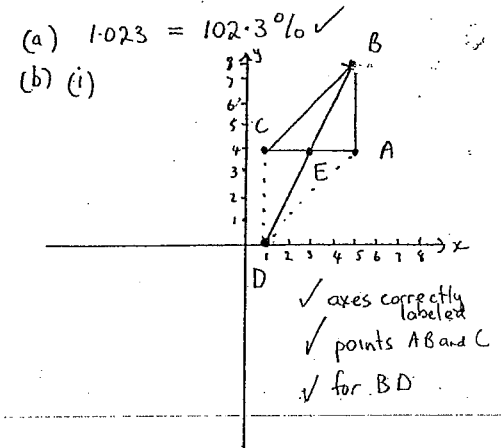5×700 cm by 4×700 cm ✓
or 35 m by 28 m. ✓

QUESTION FIVE

(a) see tear-off sheet

(b) In $\triangle ABM$ and $\triangle DCM$
$\angle AMB = \angle DMC$ (Vertically opposite) ✓
$\angle MAB = \angle MDC$ (Alternate angles, AB||CD) ✓
$AB = DC$ (Given) ✓
So $\triangle ABM \equiv \triangle DCM$ (AAS) ✓

(c) (i) FALSE ✓
(ii) FALSE ✓
(iii) TRUE ✓
(iv) TRUE ✓

(d) $\angle ADC = 119°$ (Opposite angles of parallelogram) ✓
$x° = 64°$ (Exterior angle $\triangle AED$) ✓

QUESTION SIX

(a) $1.023 = 102.3\%$ ✓

(b) (i)

✓ axes correctly labeled
✓ points AB and C
✓ for BD

(ii) $BC^2 = 4^2 + 4^2$ ✓
$= 32$
$BC = \sqrt{32}$ ✓

(iii) D $(1,0)$ ✓

(iv) E $(3,4)$ ✓

(v) The diagonals of a parallelogram bisect each other ✓

(c)

$C = 44 m$

Longest pass will be the diameter ✓
$C = \pi D$
$D = \frac{C}{\pi}$
$= 44 \div \frac{22}{7}$
$= 44 \times \frac{7}{22}$
$= 14 \text{ metres}$ ✓

(d) $EC^2 = ED^2 + DC^2$
$= 8^2 + 6^2$ ✓
$= 100$
$EC = \sqrt{100}$
$= 10 \text{ metres}$ ✓

QUESTION SEVEN

(a) $\dfrac{14x^2y^2}{7xy} - 2xy^2 = 2xy^2 - 2xy^2$ ✓

$= 0$ ✓

(b) $8(2x^2-1) - 9(4x^2-1) = 16x^2 - 8 - 36x^2 + 9$ ✓

$= -20x^2 + 1$ ✓

(c) $\dfrac{5-2x}{4} + \dfrac{x+2}{8} = \dfrac{2(5-2x) + (6x+2)}{8}$ ✓

$= \dfrac{10 - 4x + x + 2}{8}$

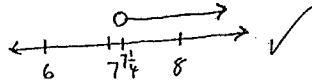$= \dfrac{12 - 3x}{8}$ ✓

(d) (i) $2x - 15 = 5x + 9$

$-15 = 3x + 9$ ✓

$-24 = 3x$ ✓

$x = -8$ ✓

(ii) $\dfrac{3a-4}{2} - \dfrac{2a+1}{4} = -5$

$2(3a-4) - (2a+1) = -20$ ✓

$6a - 8 - 2a - 1 = -20$ ✓

$4a - 9 = -20$

$4a = -11$ ✓

$a = -\dfrac{11}{4}$ ✓

(iii) $\dfrac{3}{7x} - \dfrac{1}{2x} = 3$

$6 - 7 = 42x$ ✓

$-1 = 42x$

$x = -\dfrac{1}{42}$ ✓

QUESTION EIGHT

(a) (i) $5 - \dfrac{2x}{3} < \dfrac{1}{6}$ ✓
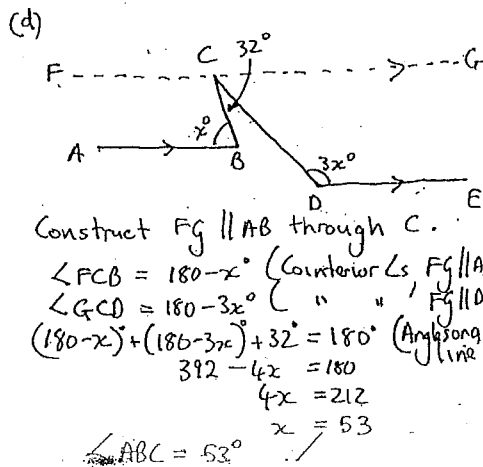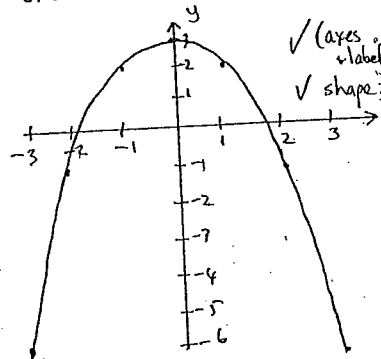
$30 - 4x < 1$

$-4x < -29$ ✓

$x > \dfrac{29}{4}$ ✓

(ii)

 ✓

(b) $V = $ Cross-sectional area $\times 10$ ✓

$= (5 \times 12 + \frac{1}{2} \times 3.1 \times 3^2) \times 10$ ✓

$= (60 + 13.95) \times 10$

$= 739.5$ cm³ ✓

(c) (i)

x	-3	-2	-1	0	1	2	3
y	-6	-1	2	3	2	-1	-6

(-1 for 1 or 2 errors)
(No marks for 3 or more errors)

✓ (axes + label)
✓ shape

(d)

Construct FG ∥ AB through C.

∠FCB = $180 - x°$ (Co-interior ∠s, FG∥AB)

∠GCD = $180 - 3x°$ (" " , FG∥DE)

$(180-x) + (180-3x) + 32 = 180°$ (Angles on line)

$392 - 4x = 180$

$4x = 212$

$x = 53$

∠ABC = 53°

QUESTION NINE

(a) $\sqrt{\dfrac{-abc^2}{32d^3}} = \sqrt{\dfrac{-(4) \times (-1) \times \frac{1}{16}}{32 \times -\frac{1}{8}}}$

$= \sqrt{\dfrac{-\frac{1}{4}}{-4}}$ ✓

$= \sqrt{\dfrac{1}{16}}$

$= \dfrac{1}{4}$ ✓

(b) $\dfrac{(3xy)^2 + 15x^2y^2}{2x \times 3y} = \dfrac{9x^2y^2 + 15x^2y^2}{6xy}$ ✓
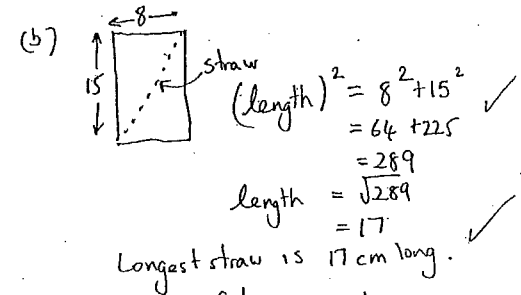
$= \dfrac{24x^2y^2}{6xy}$

$= 4xy$ ✓

(c) (i) $24x$ ✓

(ii) $\dfrac{45x}{2}$ ✓

(iii) $24x = \dfrac{45x}{2} + 63$ ✓

$48x = 45x + 126$

$3x = 126$

$x = 42$ ✓

There are 42 teachers at Sparta Grammar School.

(d) (i) ∠ECA $= x°$ (CE bisects ∠ACD)

∴ ∠ACD $= 2x°$ (Sum of adjacent angles)

∠ACB $= 2x°$ (Diagonal of rhombus bisects the vertex) ✓

So ∠BCD $= 4x°$ (Sum of adjacent angles) ✓

(ii) ∠ECA $= x°$ } as above
∠ACD $= 2x°$ }

So ∠ECB $= 3x°$ (Sum of adjacent angles)

and ∠CED $= 3x°$ (Alternate angles) AD∥BC

QUESTION TEN

(a) $(2x-y)(4x^2 + 2xy + y^2) = 8x^3 + 4x^2y + 2xy^2 - 4x^2y$
$\qquad - 2xy^2 - y^3$ ✓

$= 8x^3 - y^3$ ✓

(b)

straw

$(\text{length})^2 = 8^2 + 15^2$ ✓

$= 64 + 225$

$= 289$

length $= \sqrt{289}$

$= 17$ ✓

Longest straw is 17 cm long.

(c) Let radius of larger circle $= r$.

So $\pi \left(\dfrac{r}{2}\right)^2 = 11$

$\pi r^2 = 44$

$r = \sqrt{\dfrac{44}{\pi}}$ ✓

Area of larger circle $= \pi \times \left(\sqrt{\dfrac{44}{\pi}}\right)^2$

$= 44$ cm²

So, Area of shaded region $= 44 - 11$ ✓
$= 33$ cm²

(d) At tilting point

$V = \frac{1}{2} \times 40 \times 40 \times 50$ ✓

$= 40\,000$ cm³

Once horizontal again

$V = 80 \times 50 \times$ depth ✓

depth $= \dfrac{40000}{80 \times 50}$

$= 10$ cm ✓

(e)

Let AB $= x$, So BO $= \dfrac{x}{2}$

In △ABO $r^2 = x^2 + \left(\dfrac{x}{2}\right)^2$

$r^2 = \dfrac{5x^2}{4}$

$x^2 = \dfrac{4r^2}{5}$

$x = \dfrac{2r}{\sqrt{5}}$ PTO
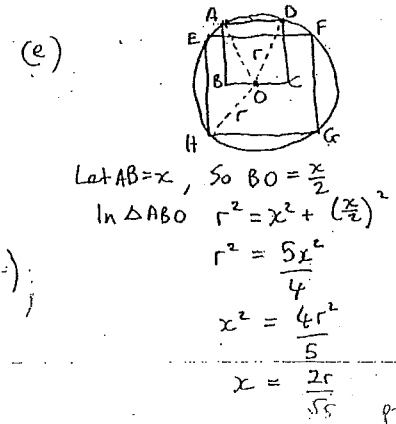
Area of small square $= x^2$

$$= \frac{4r^2}{5}$$

Area of large square $= \frac{1}{2} \times 2r \times 2r$

$$= 2r^2$$

Ratio of large square to small square
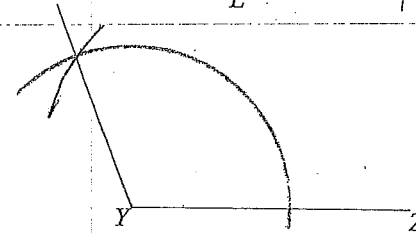
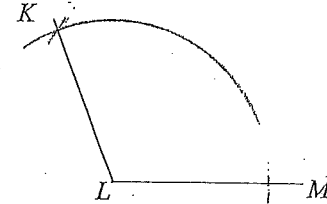$$2r^2 : \frac{4r^2}{5}$$

$$= 2 : \frac{4}{5}$$

$$= 5 : 2 \quad /// \checkmark$$

NAME: .... CLASS: .... MASTER: ....

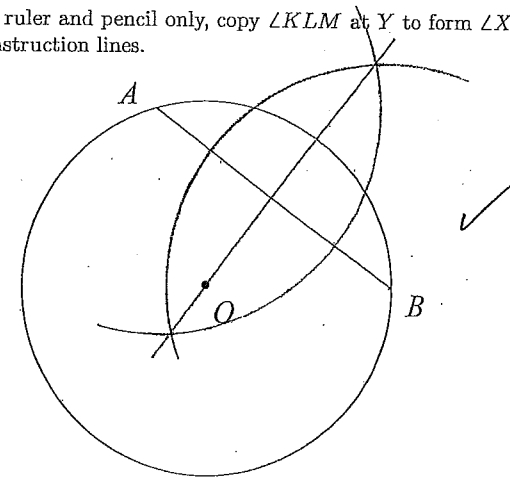DETACH THIS SHEET AND BUNDLE IT WITH THE REST OF QUESTION FIVE.

QUESTION FIVE

(a) (i)

Using a compass, ruler and pencil only, copy $\angle KLM$ at Y to form $\angle XYZ$. You must show all construction lines.

(ii)

Construct the perpendicular bisector of the chord AB, clearly showing all construction lines.