SYDNEY GRAMMAR SCHOOL

2012 Annual Examination

FORM II

MATHEMATICS

Monday 5th November 2012

General Instructions
- Writing time — 2 hours
- Write using black or blue pen.
- Calculators are not to be used.

Total — 130 Marks
- All questions may be attempted.
- All necessary working should be shown.
- Start each question on a new page.

Collection
- Write your name, class and master on each page of your answers.
- Staple your answers in a single bundle.
- Write your name and master on this question paper and submit it with your answers.

2A: KWM	2B: PKH	2C: JMR	2D: LYL
2E: REP	2F: RCF	2G: LRP	2H: FMW
2I: SJE/DS	2J: SO		

Checklist
- Writing paper required.
- Candidature — 185 boys

Examiner
PKH

QUESTION ONE (13 marks) Start a new page.

(a) Simplify the following:

   (i) -7×4

   (ii) $-7 - 11$

   (iii) $3 - 7 \times -2$

(b) Simplify the following:

   (i) $\dfrac{3}{4} \times \dfrac{1}{9}$

   (ii) $\dfrac{2}{5} - \dfrac{3}{10}$

   (iii) $12 \div \dfrac{2}{3}$

(c) Find 18% of $600.

(d) Simplify the following:

   (i) $2x^3 \times x^3$

   (ii) $7x - 5 + 3x - 2$

(e) A right angled triangle has shorter sides of length 6 units and 8 units. What is the length of the hypotenuse?

(f) Expand $-2(x - 2)$.

(g) If $y = 3x^2$, find the value of y when $x = -5$.

QUESTION TWO (13 marks) Start a new page.

(a) Solve the equation $5 + 2x = 11$.

(b) A circle has radius of 10 units. Using $\pi \doteq 3 \cdot 1$, find

   (i) the circle's circumference,

   (ii) the circle's area.

(c) Find 65% as a fraction in simplest form.

(d) Convert $2 \cdot 4\,\text{L}$ to mL.

(e) (i) On a number plane plot the points $A(-1,0)$ and $B(3,3)$. Clearly label the axes and the points.

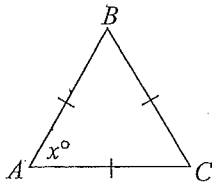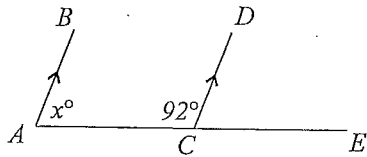(ii) Find the distance AB.

(f)
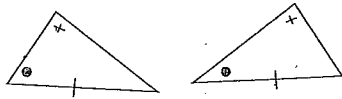
By which test are the triangles above congruent?

(g) Find the value of x in each of the diagrams below, giving reasons.

(i)

(ii)

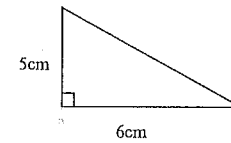**QUESTION THREE** (13 marks) Start a new page.

(a) Expand and simplify the following:

(i) $(a-4)(a+5)$
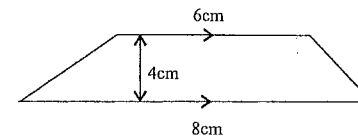
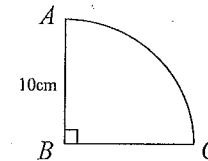(ii) $(2x+3)^2$

(b) Find the areas of the following figures:

(i)

(ii)

(c)

In the diagram above the arc AC is a quarter circle. Using $\pi \doteqdot 3\cdot1$,

(i) find the area of the figure,

(ii) find the perimeter of the figure.

(d) Find the volume of a cylinder with height 6 units and radius 5 units. Leave your answer in terms of π.

(e) How many square centimetres are there in $1\cdot2$ m^2?

QUESTION FOUR   (13 marks)   Start a new page.

(a) Simplify the following ratios:

   (i) $12 : 18$

   (ii) $1 \cdot 5 : 2 \cdot 25$

   (iii) $450 \, \text{mL} : 1 \cdot 8 \, \text{L}$

(b) A boy and a girl have heights in the ratio $6 : 5$. If the girl is 160 cm tall, how tall is the boy?

(c) A metal bar consists of zinc, silver and copper in the ratio $5 : 1 : 4$. If the metal bar weighs 1200 g, how much copper is in the bar?

(d) A map is drawn to a scale of $1 : 50\,000$. Two towns on the map are 6 cm apart. How far apart are the two towns in kilometres?

(e) A car is travelling at 72 km/hr. What is its speed in m/s?

(f) A rectangle has length 8 cm and width 6 cm. The length is increased in the ratio $5 : 4$ and the width is increased in the ratio $3 : 2$. Find the percentage increase in the area.

QUESTION FIVE   (13 marks)   Start a new page.

(a) Solve the inequation $\dfrac{x}{-2} \leq 3$.

(b) Simplify the following:

   (i) $2x^3 \times 3x^2$

   (ii) $(2x^3)^3$

   (iii) $7x^0$

(c) Which words best complete the following sentences?

   (i) A quadrilateral with both pairs of opposite sides parallel is always a _____.

   (ii) A parallelogram with a pair of adjacent sides equal is always a _____.

(d) Solve the following equations:

   (i) $\dfrac{2x}{3} + 5 = x$

   (ii) $-2(3x - 4) = 6 - 2x$

(e) Simplify:

   (i) $\dfrac{3x}{2} - \dfrac{x}{4}$
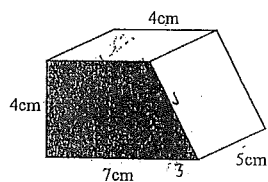
   (ii) $\dfrac{2x - 1}{4} - \dfrac{x - 1}{8}$

QUESTION SIX   (13 marks)   Start a new page.

(a)   (i) Fill in the following table according to the rule $y = 2x + 1$.

x	-1	0	5
y			

(ii) On the same set of axes sketch the lines $y = 2x + 1$ and $x = 5$.

(iii) Write down the co-ordinates of point of intersection of the lines $y = 2x + 1$ and $x = 5$.

(b)

In the diagram above the solid figure is a right prism. The front face is a trapezium. Find:

(i) the volume of the solid,

(ii) the total surface area of the solid.

(c) Express $2 + \dfrac{n+1}{3}$ as a single fraction.

(d) Perform the constructions outlined on the tear-off sheet at the end of this examination paper. This sheet should be bundled with the rest of your answers to Question Six.
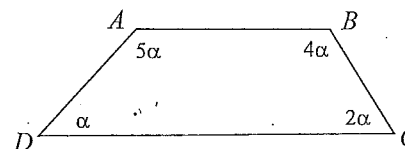
QUESTION SEVEN   (13 marks)   Start a new page.

(a)   (i) Factorise $4y^2 - 2y$.

(ii) Simplify $\dfrac{4y^2 - 2y}{4y}$.

(b) A tennis player has won 36 out of 54 matches. His sponsor says that he must win 60% of his total number of matches to qualify for a bonus. There are 26 matches remaining on the tour. How many more must he win to collect his bonus? You must set up and solve an equation.
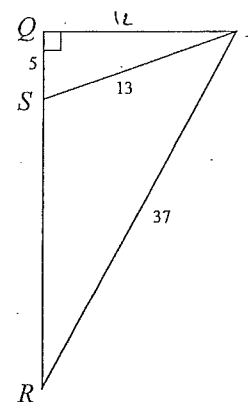
(c)

Prove that the quadrilateral $ABCD$ drawn above is a trapezium.

(d) How long is a train if it is travelling at 45 km/hr and passes a signal in $18\frac{1}{2}$ seconds?

(e)

In the diagram above find:

(i) the length of PQ,

(ii) the perimeter of triangle PQR.

QUESTION EIGHT   (13 marks)   Start a new page.

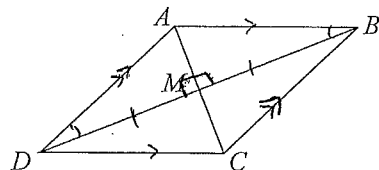(a) Solve the inequation $3 - 5x \leq 12 - 2x$ and graph your solution on a number line.

(b)   (i) For the equation $y = \dfrac{x^2}{2} - 3$, copy and complete the following table of values.

x	-4	-2	0	2	4
y					

   (ii) Using a scale of 1cm for 1 unit on both x and y axes, carefully draw the graph of $y = \dfrac{x^2}{2} - 3$.

(c) Each diagonal of a square has length 2 units. What is the area of the square?

(d)

   In the diagram above the figure $ABCD$ is a rhombus. The diagonals meet at M.

   (i) Is $ABCD$ a parallelogram?

   (ii) Why does $BM = DM$?

   (iii) By using congruent triangles prove that $\angle DAC = \angle BAC$.

QUESTION NINE   (13 marks)   Start a new page.

(a) Evaluate $0{\cdot}4 \times 0{\cdot}9 \div 0{\cdot}072$.

(b) According to the web site Wolfram Alpha, 790 000 U.S. citizens have Eric as their first name. If this represents $0{\cdot}25\%$ of the population of the U.S.A., what is the population of the U.S.A. to the nearest ten million?

(c) A map is drawn to a scale of $1:10\,000$. On the map, Guppo Forest occupies a rectangular region measuring 10 cm by 100 cm. What is the area of the forest in km^2?

(d)
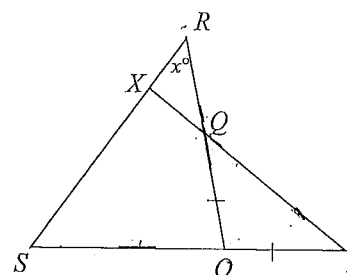
   In the diagram above $OP = OQ$ and $OR = OS$. Prove that PX is perpendicular to RS.

(e) Harry walks at 4 km/hr and runs at 6 km/hr. He saves $3\frac{3}{4}$ minutes by running instead of walking from his home to school. Let d be the distance from home to school in kilometres.
   By forming and solving an equation find the distance in kilometres from his home to his school.

**QUESTION TEN** (13 marks) Start a new page.

(a) Solve $\dfrac{5}{3n} + \dfrac{2}{n} = 6$.

(b) Express 62700 as a product of prime factors in index form.

(c) A parallelogram has diagonals of length 48 units and 14 units. One side of the parallelogram has length 25 units. With the aid of a diagram prove that the parallelogram is a rhombus.

(d) Forty two cubes with 1 cm edges are glued together to form a solid rectangular block. If the perimeter of the base of the block is 18 cm find the height of the block.

(e)
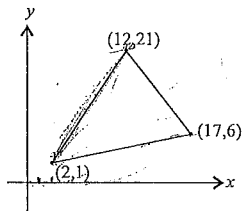
A lattice point is a point (x, y) with x and y both integers. For example $(1, 3)$ is a lattice point but $(2, \dfrac{1}{2})$ is not. In the diagram above how many lattice points lie on the perimeter of the triangle?

(f)
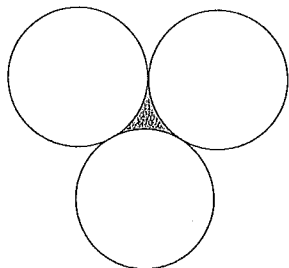
In the diagram above each of the three identical circles touch the other two. The radius of each circle is 6 units.

(i) How many lines of symmetry does the figure above have?

(ii) Find the perimeter of the shaded region.

END OF EXAMINATION

Tear-off pages follow ...

SOLUTIONS - MASTER COPY

a) i) -28 ✓

   ii) $\cancel{77}$ ✗ -18

   iii) $\cancel{42}$ ✗ 17

b) i) $\dfrac{\cancel{3}1}{4} \times \dfrac{1}{\cancel{9}3} = \dfrac{1}{12}$ ✓

   ii) $\dfrac{4}{10} - \dfrac{3}{10} = \dfrac{1}{10}$ ✓

   iii) $\dfrac{\cancel{12}^6}{1} \times \dfrac{3}{\cancel{8}1} = 18$ ✓

c) $\dfrac{18}{\cancel{100}_1} \times \dfrac{\cancel{600}6}{1}$ $\dfrac{6x}{108}$

    $= \$108$ ✓

d) i) $2x^3 \times x^3 = 2x^6$ ✓

   ii) $7x - 5 + 3x - 2$

      $10x - 7$ ✓

e) $6^2 + 8^2 = 10^2$

    10 units ✓

f) $-2(x-2)$

   $= -2x + 4$ ✓

(11)

g) i) $\cancel{9x}$ $y = 75$ ✓ $-5x - 5 = 25 \times 3 = 75$

$2\pi r$
πr^2

a) $5 + 2x = 11$

   $2x = 6$

   $x = 3$ ✓

b) i) $\dfrac{(6.1)_, 6.2}{10}$ $C = 6.1$ units

    $6\overline{)61.0}$ ✗

Circumference $= 2\pi r$
          $= 2 \times 3.1 \times 10$
          $= 62$ units.

   ii) $\dfrac{3.1y}{100}$ Area $\cancel{2604} = 310$ units^2 ✓

      $3\overline{)100}$

c) $\dfrac{65}{100}$ $\dfrac{130}{200} = \dfrac{13}{20}$ ✓

d) $2.4L = 2400 ml$ ✓

e)

   $3^2 + 4^2 = 5^2$

   $AB = 5$ units
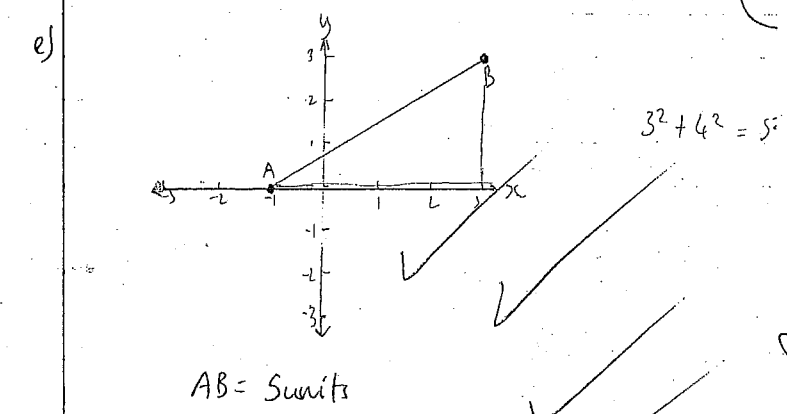
f) AAS

g) $x° = 60°$ ✓ [All $\angle$s of equilateral $\triangle$ are equal

g) ii) $x° = 88°$ [Co-interior $\angle$s (opposite) complementary) $BA \parallel DC$]

Section 3

a i) $(a-4)(a+5)$
$a^2 + 5a - 4a - 20$
$= a^2 + a - 20$ ✓

ii) [diagram crossed out] $x + 4cm$, $x + 4cm$

$2x(2x+3) + 3(2x+3)$
$4x^2 + 6x + 6x + 9$
$= 4x^2 + 12x + 9$ ✓

b i) 5×6 $5 \times 6 \times \frac{1}{2}$
$= 15 cm^2$ ✓✓

ii) $14 \times \frac{1}{2} \times 4$
$= 7 \times 4$
$= 28 cm^2$

c i) $\begin{array}{r} 3.1y \\ 100 \\ \hline 310.0 \end{array}$ $4\overline{)131.0^30}$ 77.5

$A = 77.5 cm^2$

ii) $\begin{array}{r} 6.2x \\ 10 \\ \hline 620 \end{array}$ $4\overline{)16^22.^20}$ 15.5

$15.5 + 20 = 35.5$
$P = 35.5 cm$ ✓✓

d [crossed out] $3.1x$ $\begin{array}{r} 3.1x \\ 2.5 \\ \hline 15.5 \\ 620 \end{array}$ $4\overline{)46^35.0}$ $\frac{77.5x}{6}$ $V = 46.5 units$ ✗✗ ?

(d) Volume of a cylinder
$= \pi r^2 h$
$= \pi (5)^2 \cdot 6$
$= 150\pi \; units^3$

(e) $1m = 100 cm$
$\therefore 1 \times 1 \; m^2 = 100 \times 100 \; cm$
$\therefore 1.2 m^2 = 1.2 \times 10^4 \; cm$

⑩

Section 4

a) i) $12:18$
$2:3$ ✓

ii) $150 : 22\overset{45}{5}$
$30 : 45$
$2 : 3$ ✓

iii) $450 : 18\overset{360}{00}$ $\quad ^{90}$
$90 : 360$
$1ml : 4ml$ ✓

b) $S = \overline{\tfrac{32}{1160}} cm$
The boy is $192 cm$ ✓

c) 1200 $1 unit = 120$
$\begin{array}{r} 120 \times \\ 4 \\ \hline 480 \end{array}$
$Copper = 480g$ ✓✓

d) $1 : 500m \times 6$
$= 3km$ ✓✓

e) $360\overline{)72000}$ 200
$200 m/s$ ✗✓

f) $6 \; [\text{rectangle}] \; 8$

$\begin{array}{r} 10x \\ 9 \\ \hline 90 \end{array}$ $\begin{array}{r} 8x \\ 6 \\ \hline 42 \end{array}$
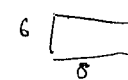
$42 cm^2 \to 90 cm^2$
48% increase

New length is $\frac{5}{4} \times 8 cm = 10 cm$
" width is $\frac{3}{2} \times 6 cm = 9 cm$ New area $= 10 \times 9$
$\%$ increase $= \frac{90}{48} \times 100\% = 187\frac{1}{2}\% - 100\%$
$= 87\frac{1}{2}\%$

⑪

a) $\frac{x}{-2} \le 3$

$x \not{\circledcirc} 6$ ✗

b) i) $2x^3 \times 3x^2$

$6x^5$ ✓

ii) $8x^9$ ✓

$7 \times 1 = 7$

iii) $\cancel{7x^0 = 0 \ne 0}$ ✗

c) i) parallelogram $\cancel{\text{Square}}$

ii) Rhombus ✓

⑧

d) i) $\frac{2x}{3} + 5 = x$

$2x + 15 = 3x$

$15 = x$

$x = \cancel{45} \, 15$ ✓

ii) $-2(3x - 4) = 6 - 2x$

$-6x + 8 = 6 - 2x$

$-6x = 2 - 2x$

$-4x = 2$

$-2x = -1$

$-x = -\frac{1}{2}$

$x = \frac{1}{2}$ ✓

(e) (ii) $\frac{2(2x-1) - (x-1)}{8}$

$= \frac{4x - 2 - x + 1}{8}$

$= \frac{3x - 1}{8}$ ✓

e) i) $\frac{3x}{2} - \frac{x}{4} = \frac{6x}{4} - \frac{x}{4} = \frac{5x}{4}$ ✓

(ii) $\cancel{\frac{3x}{4}} \quad \frac{4x-2}{8} - \frac{x-1}{8}$

a) i) $y = 2x + 1$

x	-1	0	5
y	-1	1	11

$y = 2x + 1$

-4

-5

b) i) $11 \times \frac{1}{2} \times 4$

$\overset{2}{5.5} \times$

$\frac{4}{220}$

$\frac{22}{5} \over 110$ ✓
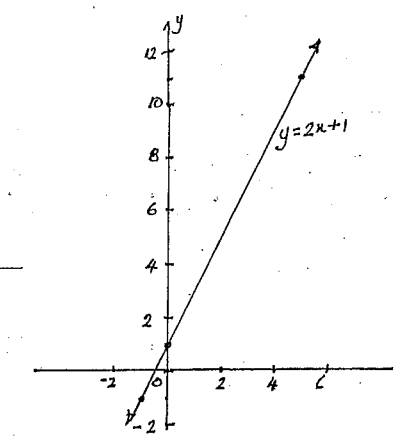
$V = 110 \text{cm}^3$

⑦

(b) (ii) Total surface area $= 2\left(\frac{4}{2}(4+7)\right) + 4\times5 + 7\times4$

$+ 5\times5 + 7\times5$ sq cm
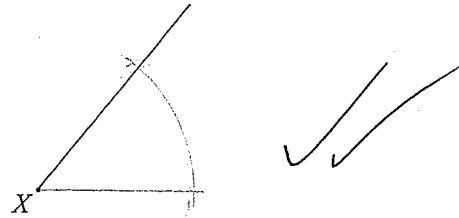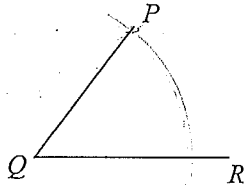
$= 152$ sq cm.

NAME: ....   CLASS: ....   MASTER: ....

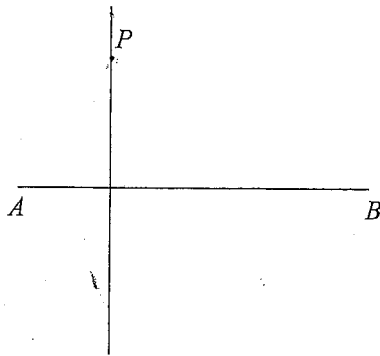DETACH THIS SHEET AND BUNDLE IT WITH THE REST OF QUESTION SIX.

QUESTION SIX

(d) In the following questions leave all construction arcs and use only a ruler and compasses.

  (i) Copy $\angle PQR$ at X to form $\angle ZXY$.

  (ii) From the point P construct an interval perpendicular to AB.

(c) $2 + \dfrac{n+1}{3} = \dfrac{6+n+1}{3}$

$\qquad = \dfrac{7+n}{3}$

(Question 7)

a) i) $4y^2 - 2y$

$\qquad y(2y-2)$ ✓

(ii) $\dfrac{\cancel{2y}(2y-1)}{2\cancel{\times y}} = \dfrac{2y-1}{2}$

(b) Total no. of matches $= 54 + 26$
$\qquad\qquad = 80$

To qualify for a bonus $= \dfrac{6}{10} \times 80$
he must win $\qquad = 48$ matches

Remaining to win $= 48 - 36$ matches
$\qquad\qquad = 12$ matches.

c) $12a = 360°$
$\quad a = 30°$
$\quad \angle ABC + \angle DAB = 180 = 5a + a = 180°$ $(\times 60)$
For a trapezium $\angle ABC + \angle BCD$
$\qquad = 4 \times 30 + 2 \times 30$
$\qquad = 180°$ ∴ Cointerior $\angle s$ are supplementary
$\qquad\qquad AB \parallel DC$

(d) Speed $= \dfrac{Distance}{Time}$

$\dfrac{45 \times 1000 \; m}{60 \times 60 \; s} = \dfrac{D}{18.5}$

∴ $D = \dfrac{25}{2} \times \dfrac{37}{2} = 231.25 \; m$

e) i) $PQ = 12 \; units$ ✓ $[13^2 - 5^2 = 12^2]$

(ii) $QR^2 = 37^2 - 12^2 = 1369 - 144 = 1225$

∴ $QR = 35$

Question 8

a)
$$3 - 5x \leq 12 - 2x$$
$$3 \leq 12 + 3x$$
$$-9 \leq 3x$$
$$3x \geq -9$$
$$x \geq -3$$
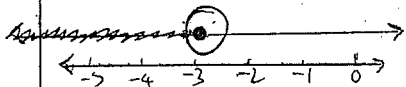
$\frac{2-3}{4}$ -3 $\frac{2}{4}$ -1

X

$y = \frac{x^2}{2} - 3$

$\frac{16}{8} - 3$

$\frac{0}{2}$

$\frac{8-2}{16}$

b)

x	-4	-2	0	2	4
y	5	-1	-3	-1	5

⑦

(c)
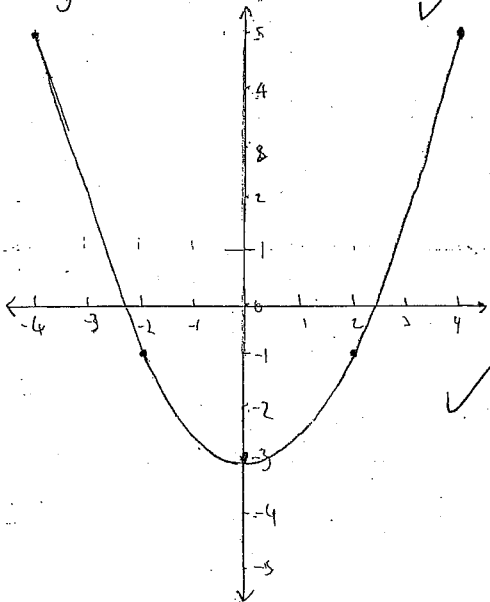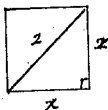
$$\therefore \quad x^2 + x^2 = 2^2$$
$$2x^2 = 4$$
$$x = \sqrt{2}$$

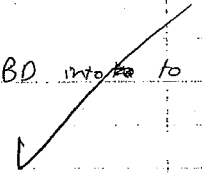$\therefore$ Area of the square
$$= x^2$$
$$= (\sqrt{2})^2 = 2 \text{ sq. units.}$$

d

i) Yes

ii) B̶M̶=̶B̶
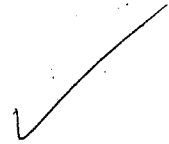
BM = DM because AC bisects B̶D BD into to equal sides.

(iii) In $\triangle DAM$, $\triangle BAM$

DH = MB (proves above in (ii))

$\angle DMA = \angle BMA = 90°$ (Property of a rhombus)

AM is common

$\therefore \triangle DAM \equiv \triangle BAM$ (S.A.S)

$\therefore \angle DAC = \angle BAC$ (matching $\angle$s congruent $\triangle$s)

Question Nine

(a) $0.4 \times 0.9 \div 0.072$

$= \dfrac{0.360}{0.072} = \dfrac{\overset{5}{\cancel{360}}}{\underset{2}{\cancel{72}}} = 5$
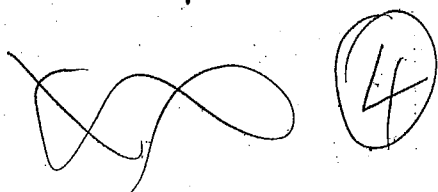
(b) $\dfrac{0.25}{100} \, x = 790\,000$

$\therefore x = \dfrac{7.9 \times 10^5 \times 10^2 \times 10^2}{0.25}$

$= 7.9 \times 10^7 \times 4$

$= 31.6 \times 10^7$

$= 32 \times 10^7$ (to nearest 10 million)
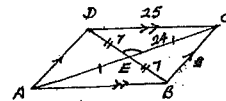

10km

Guppo Forest $\overset{is}{\approx}$ 10km²

d) $\angle OQP = \angle OPQ = 45°$ (~~base ∠s~~ isosceles △)
$\angle QOP$

$\angle RSP = x°$ [isosceles △ base ∠s]

(e) Using Speed $= \dfrac{\text{Distance}}{\text{Time}}$

$t_w = \dfrac{d}{4}$, $t_r = \dfrac{d}{6}$

But $t_w \bar{\div} 3\frac{3}{4} = t_r$

$\therefore \dfrac{d}{4} - \dfrac{15}{4} = \dfrac{d}{6}$

$\therefore 6d - 4d = 90$

$\therefore 2d = 90$

$d = 45\text{km}$

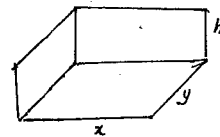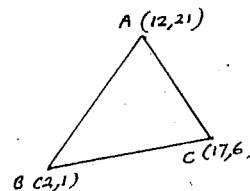
(c)

In △ DEC
$25^2 = 7^2 + 24^2$
$625 = 49 + 576$ which is true

$\therefore \angle DEC = 90°$ (Diagonals of a rhombus bisect at 90°)

(d)

$2x + 2y = 18$
$x + y = 9 \Rightarrow$ By trial + error
$x = 7 , y = 2$

$x.y.h = 42$
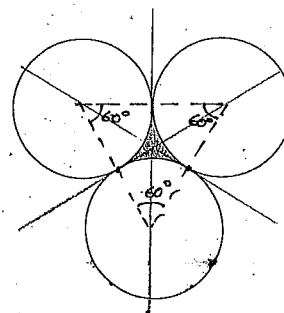$\therefore h = \dfrac{42}{x.y} = \dfrac{42}{7.2}$

$= 3.$

(e)

A (12,21)
B (2,1)   C (17,6)

For BC $y - 1 = \frac{1}{3}(x-2)$
$\underset{5,8,11,14,}{\uparrow}$ (4 lattice points)

For AC $y - 6 = -3(x-17)$
$\underset{13,14,15,16,17}{\uparrow}$ (4 "   ")

For AB $y - 1 = 2(x-1)$
$\underset{3,4,5,\ldots,11}{\uparrow}$ (9 "   ")

Total lattice points = 20.

(f)

(i) 3 axes of symmetry

(ii) Perimeter of the shaded region
$= 3 \times \left(\dfrac{60}{360} \times 2\pi \times 6\right)$
$= 6\pi$ units.