SYDNEY HIGH SCHOOL

MOORE PARK, SURRY HILLS

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

1994

MATHEMATICS

2/3 UNIT

Time allowed — Three hours
(Plus 5 minutes reading time)

Examiner: P. Bigelow

DIRECTIONS TO CANDIDATES

- *ALL* questions may be attempted.
- *ALL* questions are of equal value.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Standard integrals are printed at the back. Approved calculators may be used.
- *Each* section attempted is to be returned in a *separate* bundle, clearly marked Section A (Q1, Q2), Section B (Q3, Q4), Section C (Q5, Q6), Section D (Q7, Q8), or Section E (Q9, Q10). Each bundle must also show your name. Start each question on a new page.
- If required, additional paper may be obtained from the Examination Supervisor upon request.

NOTE: This is a trial paper only and does not necessarily reflect the content or format of the final Higher School Certificate Examination Paper for this subject.

SECTION A (Hand up separately)

QUESTION 1. (Start a new page.)

(a) Solve for x: $|1 + 2x| = 7$

(b) Evaluate correct to one decimal place:

$$\frac{116.4 - 14.9}{5.6 \times 11.8}$$

(c) Factorise completely: (i) $2a^2 - 11a - 6$

(ii) $16 - 2a^3$

(d) Solve the following pair of equations simultaneously:

$$2a = 3 + b$$

$$4a = 5 - b$$

(e) Find the value of $\sqrt{e}$ correct to three significant figures.

(f) Solve $3x^2 = 4x$

QUESTION 2. (Start a new page.)

(a) Express with a rational denominator:

$$\frac{1}{2\sqrt{5} + 3}$$

(b) Evaluate correct to two decimal places:

(i) $\log_3 11$

(ii) $\sin 5.6$

(c) The quadratic equation $14 - 20x + x^2 = 0$ has roots α and β.

Write down the values of

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(d) Make neat sketches of:

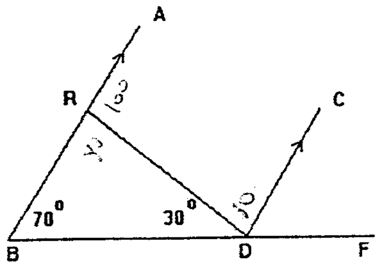
(i) $y = |2 - x|$

(ii) $y = \sqrt{16 - x^2}$

SECTION B (Hand up separately)

QUESTION 3. (Start a new page)

(a) Find ∠RDC given that AB ∥ CD. (Give reasons)



(b) Differentiate with respect to x:

(i) $\sqrt{1 + x^2}$

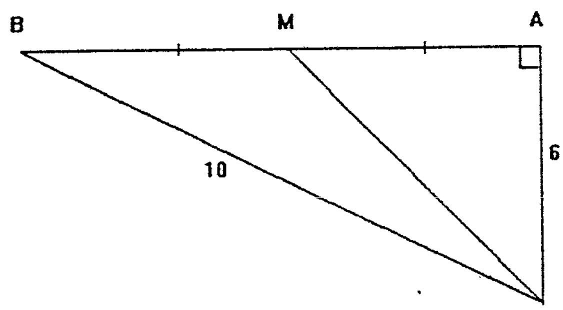
(ii) $\log_e(5 + 4x)$

(iii) $x\cos x$

(iv) $\dfrac{x}{x + 4}$

(c) Find the equation of the normal to the curve $y = e^{-x}$ at the point (0,1).

(d)



M is the mid-point of AB.
BC = 10 and AC = 6.
Find the length of CM, giving reasons.

QUESTION 4. (Start a new page.)

(a) Find primitives of the following:

(i) $\sin\frac{x}{3}$

(ii) $\frac{1}{\sqrt{1 + 3x}}$

(iii) $\frac{1 + 2x}{x^2}$

(b) Solve $\sin 2x = -\frac{\sqrt{3}}{2}$ for $0° \leq x \leq 360°$.

(c) Simplify $\frac{1}{\sec^2 x} + \frac{1}{\operatorname{cosec}^2 x}$

(d) Find the equation of the parabola with vertex (-3, -1) and focus (-3, 5).

SECTION C. (Hand up separately.)

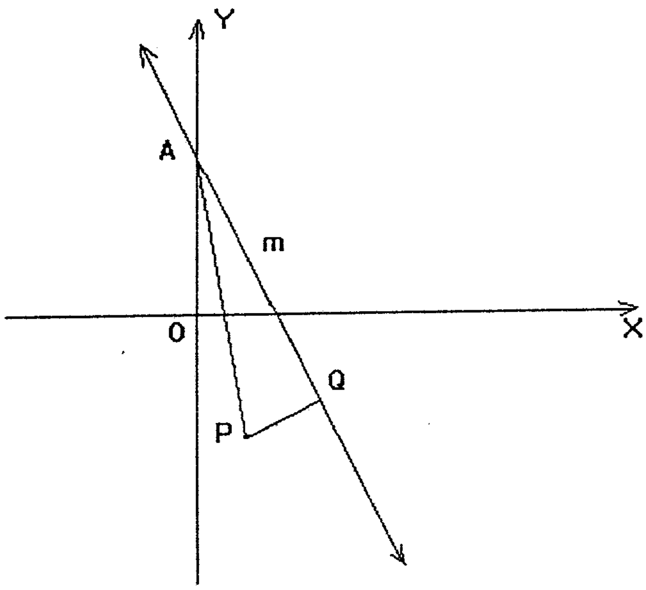
QUESTION 5. (Start a new page.)

(a) The line m has equation $2x + y - 4 = 0$. P is the point (1, -3).

(i) Find the length of PQ, if $PQ \perp m$.

(ii) Find the equation of PQ, and hence the co-ordinates of Q.

(iii) Find the area of $\triangle APQ$.



(b) Write down the quadratic equation with roots $1 + \sqrt{3}$ and $1 - \sqrt{3}$.

(c) The first term of an arithmetic series is 10 and the eighth term is 8.

(i) Find the common difference.

(ii) Find the sum of the first twenty terms.

QUESTION 6. (Start a new page.)

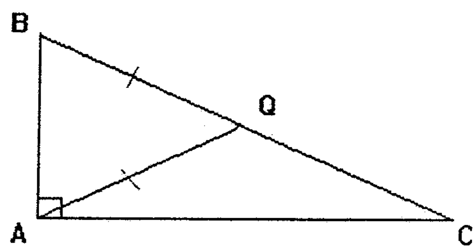
(a) Simplify $\dfrac{\cos(90° - \theta)}{\sin(180° + \theta)}$

(b) Evaluate $\sum_{r=1}^{\infty} 3^{-r}$

(c) Solve $3^x - 3^{x-1} = 54$

(d) $\triangle ABC$ has a rightangle at A.

QB = QA. Prove that QB = QC.



(e) Given that $f(x) = \sqrt{x^2 - 9}$

(i) Is $f(x)$ odd, even, or neither?

(ii) Find the domain and range of $f(x)$.

SECTION D (Hand up separately.)

QUESTION 7. (Start a new page.)

(a) OA and OB are two straight roads intersecting at 37°. A is a house 720m from O, and B is a house 870m from O.

Find the distance between A and B.

(b) For a certain function of x, $f''(x) = -2$, $f'(1) = 0$, and $f(0) = 1$. Find the equation of the function.

(c) Find the area enclosed between $x^2 = 8y$ and the line $2y = x$.

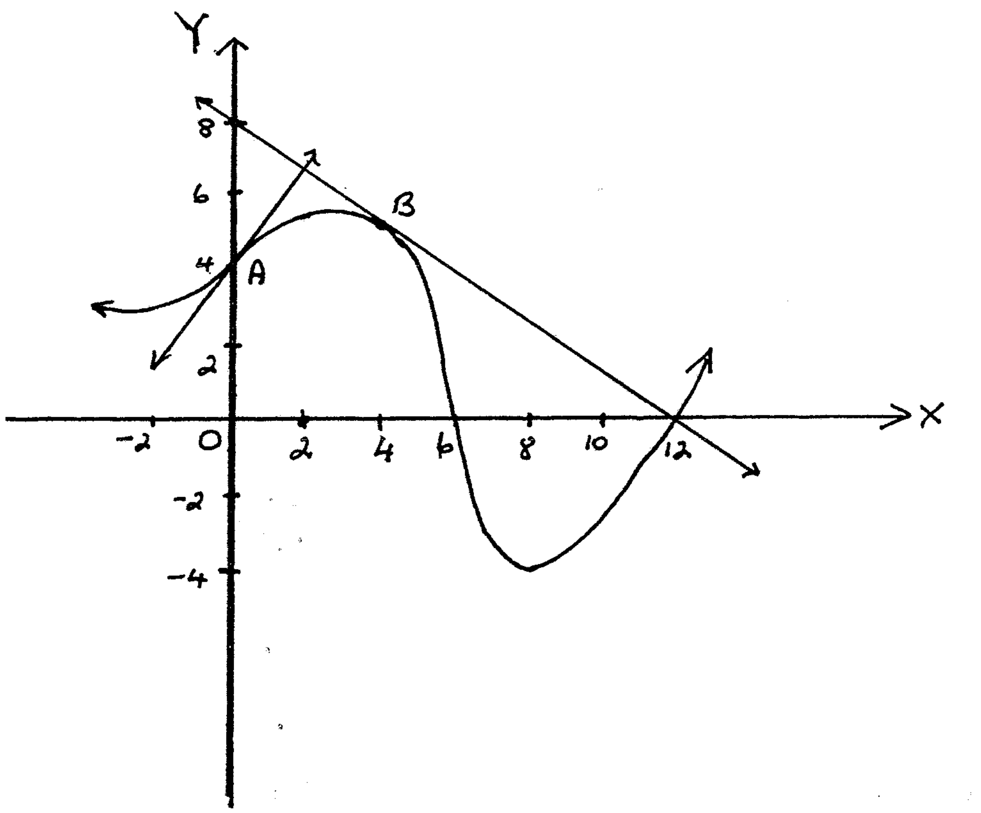
(d) Does the origin lie inside or outside the circle $x^2 + y^2 - 8x + 6y + 9 = 0$? Justify your answer.

QUESTION 8. (Start a new page.)

(a) The area bounded by the curve $y = 1 - e^{-x}$, the x-axis, and the ordinates $x = 0$ and $x = 1$ is rotated about the x-axis.
Prove that the volume of the generated solid, in cubic units, is

$$\frac{\pi}{2}(4e^{-1} - e^{-2} - 1)$$

(b)



The above is a graph of the function $y = f(x)$. Tangents are drawn at A(0,4) and B(4,5). Use the graph to evaluate:

(i) $f(6)$ (ii) $f'(4)$ (iii) $f'(8)$ (iv) $f''(0)$

(c) Calculate in degrees and minutes the angle subtended at the centre of a circle of radius 2.31 cm by an arc of length 5.29 cm.

SECTION E (Hand up separately.)

QUESTION 9. (Start a new page.)

(a) Two dice are rolled. The score for the roll is given by the difference between the numbers on the uppermost faces (e.g. if the numbers are 2 and 6, the score is 4).

Find the probability that the score will be

(i) 0

(ii) at least 3.

(b) Sketch the curve $y = -3\cos 2x$ for $0 \leq x \leq 2\pi$.

(c) Consider the curve $y = x^3 - 12x$.

(i) Determine all stationary points, and their natures.

(ii) Find any points of inflexion.

(iii) Sketch the curve for $-4 \leq x \leq 5$.

(iv) What is the maximum value of y in this domain?

(d) Consider the curve $y = \dfrac{1}{4x + 1}$.

Calculate the area enclosed between the curve, the x-axis, and the ordinates $x = 1$ and $x = 5$, by Simpson's Rule using five function values (correct to 2 significant figures).

QUESTION 10. (Start a new page.)

(a) A loan of \$20,000 is to be repaid by equal annual instalments. Compound interest, calculated yearly, is 8% p.a. If the annual instalment is \$P, then:

A_1 = \$(20,000 x 1.08 - P) is the amount owing at the end of one year.

A_2 = \$(20,000 x $(1.08)^2$ - P(1 + 1.08)) is the amount owing at the end of two years.

(i) Write an expression for A_n, the amount owing at the end of n years.

(ii) If the loan is exactly repaid at the end of n years, write an expression for the annual instalment \$P.

(iii) Calculate the annual instalment, \$P, when $n = 25$.

(b) ABCDE is a pentagon of fixed perimeter P cm. Its shape is such that ABE is an equilateral triangle and BCDE is a rectangle. If the length AB is x cm.

(i) Show that the length BC is $\dfrac{P - 3x}{2}$ cm.

(ii) Show that the area of the pentagon is given by

$$A = \tfrac{1}{4}[2Px - (6 - \sqrt{3})x^2]$$

(iii) Find the value of $\dfrac{P}{x}$ for which the area of the pentagon is a maximum.

THIS IS THE END OF THE PAPER

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \; n \neq -1; \; x \neq 0, \; if \; n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0.$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \; a \neq 0.$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \; a \neq 0.$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \; a \neq 0.$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \; a \neq 0.$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \; a \neq 0.$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \; a \neq 0.$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \; a > 0, \; -a < x < a.$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln \left[x + \sqrt{x^2 - a^2} \right], \; |x| > |a|.$$

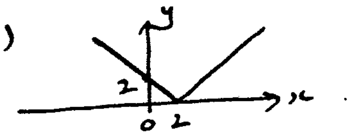
$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln \left[x + \sqrt{x^2 + a^2} \right].$$

NOTE: $\ln x = \log_e x, \; x > 0.$

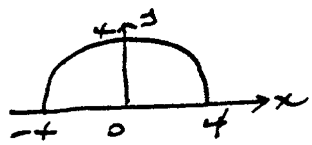
Sydney Boys High 2U Trial 1994 Answers.

(1) (a) $x = 3, -4$ (b) 1·5 (c) (i) $(2a+1)(a-6)$ (ii) $2(2-a)(4+2a+a^2)$
(d) $a = \frac{4}{3}$, $b = -\frac{1}{3}$ (e) 1·65 (f) $x = 0, \frac{4}{3}$

(2) (a) $\frac{2\sqrt{5}-3}{11}$ (b) (i) 2·18 (ii) −0·63 (c) (i) 20 (ii) 14
(d) (i)



(ii)



(3) (a) $\angle RDC = 80°$ (b) (i) $\frac{x}{\sqrt{1+x^2}}$ (ii) $\frac{4}{4x+5}$ (iii) $\cos x - x\sin x$
(iv) $\frac{4}{(x+4)^2}$ (c) $y = x+1$ (d) $CM = 2\sqrt{13}$

(4) (a) (i) $-3\cos\frac{x}{3} + c$ (ii) $\frac{2}{3}\sqrt{1+3x} + c$ (iii) $-\frac{1}{x} + 2\ln x + c$
(b) $x = 120°, 150°, 300°, 330°$. (c) 1 (d) $(x+3)^2 + (y-5)^2 = (y+7)^2$ or
$x^2 + 6x - 15 = 24y$.

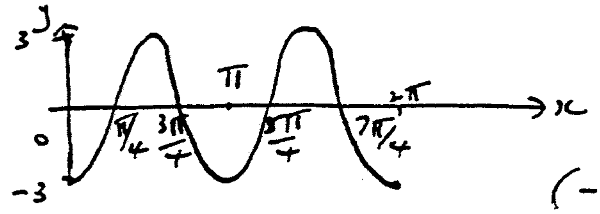
(5) (a) (i) $PQ = \sqrt{5}$ (ii) $x - 2y - 7 = 0$, Q is $(3, -2)$ (iii) $\frac{15}{2}$ units²
(b) $(x-1-\sqrt{3})(x-1+\sqrt{3}) = 0$ or $x^2 - 2x - 2 = 0$ (c) (i) $-\frac{2}{7}$ (ii) $\frac{1020}{7}$

(6) (a) −1 (b) ½ (c) $x = 4$ (d) (i) Even (ii) $x \leq -3$ or $x \geq 3$, $f(x) \geq$

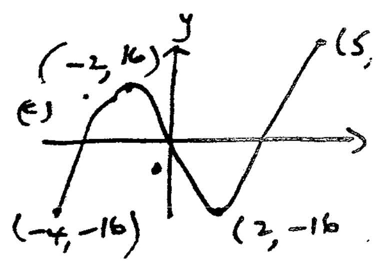
(7) (a) $AB \doteqdot 524$ (b) $f(x) = -x^2 + 2x + 1$ (c) $\frac{4}{3}$ (d) outside

(8) (b) (i) 0 (ii) $-\frac{2}{3}$ (iii) 0 (iv) 0 (c) 131°13′

(9) (a) (i) $\frac{1}{6}$ (ii) $\frac{1}{3}$ (b)



(c) $(2, -16)$ MIN. $(-2, 16)$ MAX $(0, 0)$ P. of Infl. (e)



(c) max value is 65
(d) 0·36

(10) (a) (i) $A_n = 20000 \times (1\cdot08)^n - P(1 + 1\cdot08 + 1\cdot08^2 + \dots + \text{to } n \text{ terms})$
(ii) $P = \frac{20000 \times (1\cdot08)^n \times 0\cdot08}{(1\cdot08)^n - 1}$. (iii) when $n = 25$, $P = 1873\cdot5$
(b) (ii) $\frac{P}{x} = 6 - \sqrt{3}$