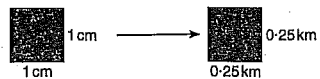# Scales and similarity

Scales and areas on maps

On the map, the lake has area $4\,cm^2$. What is its real area in km^2?
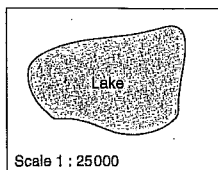
1 cm on the map is 25 000 cm on the ground.
$25\,000\,cm = 250\,m = 0.25\,km$.
So $1\,cm^2$ on the map is $0.25 \times 0.25\,km^2$ on the map $= 0.0625\,km^2$.

Scale 1 : 25000

1cm → 0.25km
1cm   0.25km

Therefore $4\,cm^2$ on the map is really $4 \times 0.0625\,km^2 = 0.25\,km^2$.

Similar objects

Example

These glasses are similar.
The area of the base of the small glass is $30\,cm^2$
and it holds $400\,cm^3$.
Find h and the base area and capacity of the large glass.

The glasses are similar, so the large glass is an
enlargement of the smaller. Looking at the tops,
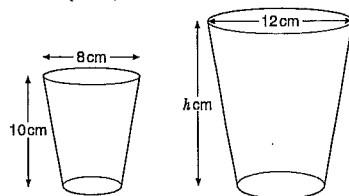we can see that the scale factor is $\frac{12}{8} = 1.5$.
So the length $h\,cm = 1.5 \times 10\,cm = 15\,cm$.

If the length scale factor is 1·5, then the area scale factor is 1.5^2.
So the base area of the large glass is $30 \times 1.5^2\,cm^2 = 67.5\,cm^2$.

If the length scale factor is 1·5, then the volume scale factor is 1.5^3.
So the volume of the large glass is $400 \times 1.5^3\,cm^3 = 1350\,cm^3$.

> Remember:
> Area scale factor is the **square** of the length scale factor.
> Volume scale factor is the **cube** of the length scale factor.

1 The scale on an Ordnance Survey map is 1 : 50000.
   (a) On the map, the distance from Longdon to Upton is 12 cm.
       What is the actual distance in kilometres?
   (b) Upton Meadows have an area of $20\,cm^2$ on the map.
       What is the real area of the Meadows in square kilometres?

2 The Popigai Crater in Siberia is a large circular crater made by
   the impact of an asteroid or comet.
   The diameter of the crater is 100 km.

   A map of Siberia is drawn to a scale of 1 to 8 000 000.
   What is the diameter of the crater on the map in millimetres?   MEG/ULEAC (SMP)

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3 On a clear day, if you stand at Nefyn in North Wales (sea level) and look
   towards Snowdon, another mountain, Garnedd-goch, blocks the view.
   From the map, the distance from Nefyn to Garnedd-goch is 21 km, and
   from Garnedd-goch to Snowdon is 11 km.

   The height of Snowdon is given as 1085 m.
   What is the lowest the height of Garnedd-goch could be?   MEG/ULEAC (SMP)

4 A map of part of the Yorkshire Dales is drawn to a
   scale of 1:50 000.
   (a) On the map the length of Risehill Tunnel is 2·3 cm.
       Calculate the actual length of the tunnel in kilometres.
   (b) Quarry Wood covers an area of $2\,cm^2$ on the map.
       Calculate the actual area of Quarry Wood in hectares.
       (1 hectare $= 10000\,m^2$)   MEG

Scale 1 : 50000
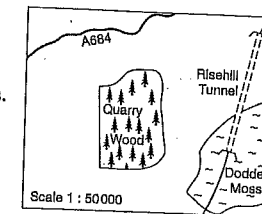
5 In Xian, China, you can buy solid scale models of the famous
   Terracotta Warriors. A model 16 cm tall weighs 270 grams
   and has an armour plate of area $9\,cm^2$.
   (a) Calculate the armour plate area on a similar model of height 24 cm.
   (b) Calculate the weight of the same 24 cm model.   MEG (SMP)

6 A £1 coin has diameter 22 mm.
   In 1989 a £2 coin was issued to commemorate the
   tercentenary of the Bill of Rights. Its diameter is 28 mm.
   Its mass is double that of the £1 coin and it is made
   of the same metal.
   Are they mathematically similar solids
   (apart from the designs)?
   Give reasons and show your working.

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MEG/ULEAC (SMP)

7 Sudsy shampoo comes in plastic bottles.
   Two different sizes are available, and the two bottles are similar.
   The height of the large size is 1·3 times the height of the small size.
   The small size costs £1·40 and the large size costs £2·40.

   Which size gives you more for your money?
   Explain clearly how you worked it out.

8
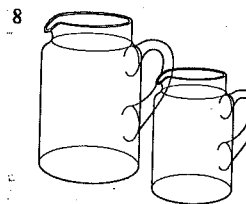
At dinner in a restaurant in France, we had on our table
a 50 cl jug for wine and a 25 cl jug for milk.
The jugs were mathematically similar in shape.
The smaller jug was 15 cm high.
What was the height of the larger jug?   MEG/ULEAC (SMP)

Scales and similarity (page 60)

1 (a) 1 cm on the map is really 50 000 cm
   = 500 m = 0·5 km on the ground.
   So 12 cm is really 6 km.

  (b) 1 cm stands for 0·5 km, so
   1 cm² stands for 0·5 × 0·5 km² = 0·25 km².
   So 20 cm² stands for 20 × 0·25 km² = 5 km².

2 1 mm on the map = 8 000 000 mm on the ground
   = 800 000 cm = 8000 m = 8 km.
   So 100 km on the ground = 100 ÷ 8 mm
   = 12·5 mm on the map.
   Alternatively convert 100 km to mm and then
   divide by 8 000 000.

3 *First you need a sketch, supposing that Nefyn (N) and*
   the tops of Snowdon (S) and Garnedd-goch (G) are
   in a straight line.
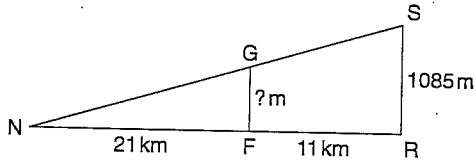
   SRN is an enlargement of GFN, with scale factor
   $\frac{32}{21} = 1\cdot523\ldots$
   So GF = SR ÷ 1·523… = 712·03…m.
   So Garnedd-goch must be at least 720 m high.
   Note that we have rounded up here (because
   Garnedd-goch must be higher than GF), but it does not
   really matter much! An answer of 710 m would be
   quite acceptable.

4 (a) 1 cm on the map is 50 000 cm on the ground
   = 500 m = 0·5 km.

   So 2·3 cm on the map = 2·3 × 0·5 km on
   the ground = 1·15 km
   = 1·2 km (to 2 s.f.).

  (b) 1 cm represents 50 000 cm = 500 m, so
   1 cm² represents 500 × 500 m² = 250 000 m²
   = 250 000 ÷ 10 000 hectares = 25 hectares.
   So 2 cm² represents 2 × 25 hectares
   = 50 hectares.

5 (a) The length scale factor from the small to the
   large model is $\frac{24}{16} = 1\cdot5$.
   So the area scale factor is 1·5².
   So the armour area = 9 × 1·5² cm²
   $\qquad\qquad\qquad\qquad = 20\cdot25\,cm^2$
   $\qquad\qquad\qquad\qquad = 20\,cm^2$ (to 2 s.f.)

  (b) Length scale factor = 1·5, so volume scale
   factor = 1·5³ = 3·375.
   So weight (which is proportional to volume)
   = 270 × 3·375 grams
   = 911·25 grams
   = 910 grams (to 2 s.f.).

6 If the coins are similar, then the length scale
   factor = $\frac{28}{22} = 1\cdot2727\ldots$
   So the volume factor will be
   (1·2727…)³ = 2·061…
   In actual fact we are told the weight (volume)
   factor is 2. The difference between 2 and 2·061…
   is probably within the tolerances we are given in
   the question, so they probably are similar.

7 The volume factor is 1·3³ = 2·197.
   If you could buy them, 2·197 of the small bottles
   would cost 2·197 × £1·40 = £3·0758 = £3·08
   (to nearest penny).
   This would cost more than the £2·40 cost of the
   large size, so the large size gives you more for
   your money.

8 The volume scale factor is $\frac{50}{25} = 2$.
   So the length scale factor = $\sqrt[3]{2} = 1\cdot2599\ldots$
   So the large jug is 15 × 1·2599…cm high
   = 18·898…cm = 19 cm (to 2 s.f.).

More help or practice

Similarity ► Book Y1 pages 81 to 85

Effects of enlargement on length, area and volume
► Book Y4 pages 9 to 19