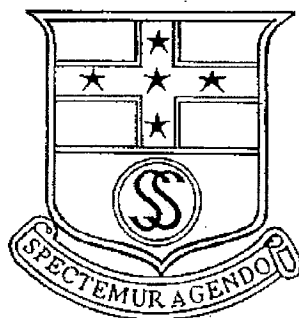


# SOUTH SYDNEY HIGH SCHOOL



Year 12 Assessment Task  
March 2001

# MATHEMATICS

## EXTENSION 1

**Instructions :**

1. All questions may be attempted.
2. Start each question on a new sheet of paper.
3. All necessary working should be shown.
4. Marks may be deducted for poorly arranged or missing working.
5. Approved calculators may be used.

**Time Allowed: 2 periods**

**Question 1 (12 marks)**

**Marks**

(a) Find the primitive function of the following:

**4**

(i)  $\frac{1+x}{x^4}$                       (ii)  $(3x^2 - 1)^2$

(b) Evaluate the following definite integrals

**4**

(i)  $\int_{-1}^1 (x+1)^2 (x-1) dx$

(ii)  $\int_{-2}^0 \frac{1}{\sqrt{9-8x}} dx$

(c) Prove that  $\int_{\frac{1}{2}}^1 \left( x^2 + \frac{6}{x^3} \right) dx = \int_1^2 \left( 6x + \frac{1}{x^4} \right) dx$

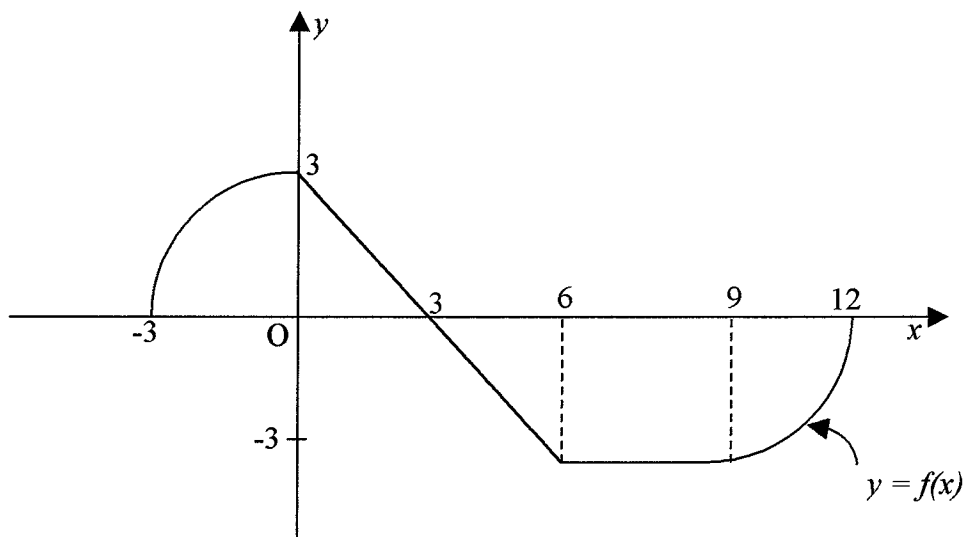
**4**

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**Question 3 (12 marks)**

**Marks**

(a)



The diagram above shows a function  $y = f(x)$  consisting of two quadrants, two triangles and a square.

Find

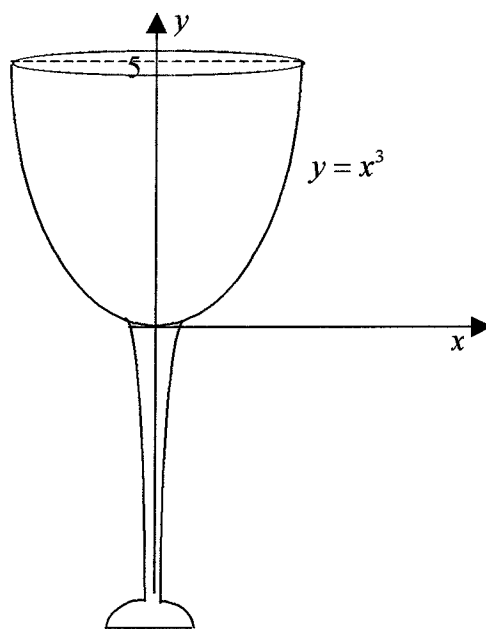
- (i) the exact area enclosed by the function and the  $x$ -axis. 2
- (ii)  $\int_{-3}^{12} f(x) dx$  1
- (b) Find the locus of  $P(x, y)$  which is 2 units away from the line  $3x - 4y + 1 = 0$ . 2
- (c) (i) Find the locus of  $P(x, y)$  whose gradient from the point  $A(-1, 3)$  is twice the gradient from the point  $B(2, 3)$ . 3
- (ii) Explain why the equation of the locus is *not*  $y = 3$ .
- (d)  $P(x, y)$  is a point which moves so that its distance from  $A(-1, 5)$  is always equal to its distance from the line  $y = 1$ . 4
- (i) Show by derivation that the equation of the locus of  $P$  is  $(x+1)^2 = 8y - 24$ .
- (ii) Sketch the locus of  $P$  clearly labeling the vertex, focus and the directrix.

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**Question 4 (12 marks)**

**Marks**

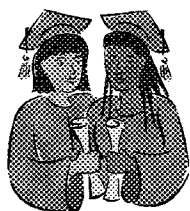
- (a) The curves  $y^2 = 4x$  and  $y = 2x^2$  meet at the origin  $O$  and the point  $A$  forming a loop. **5**
- (i) Show that the coordinates of  $A$  is  $(1, 2)$ .
- (ii) Show that the line  $OA$  divides the loop into two equal parts.
- (b) The bowl of a wine glass is formed by rotating that part of the curve  $y = x^3$ , for the values of  $y$  from 0 to 5, through one complete revolution about the  $y$ -axis. **5**



Show that the amount of wine in the glass when the depth is  $y$  is  $\frac{3}{5}\pi\sqrt[3]{y^5}$  cm<sup>3</sup>.

Find the capacity of the glass if it is 5 cm deep (to the nearest cu. cm).

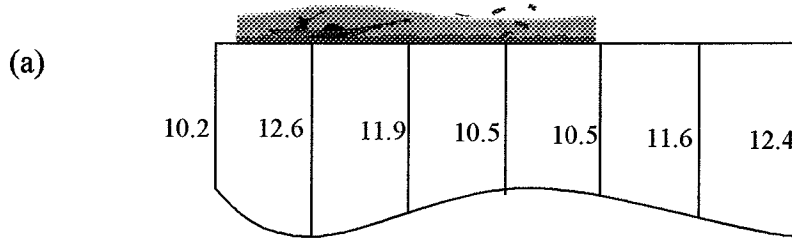
- (c) The latus rectum of a parabola measures 12 units and its extremities are  $(-6, 3)$  and  $(6, 3)$ , find the equation of this parabola. **2**



**End of Assessment task 2**

**Question 2 (12 marks)**

**Marks**



**6**

The diagram above shows the cross-section of a river bed taken by firing sonar waves at equal intervals into the water. The width of the river bed is 240 metres long and the different depths ( $h$ ), in metres, are as shown. Copy and complete the table of values below into your Writing Booklet.

$x$	0	40					240
$h$	10.2						12.4

- (i) Using ( $\alpha$ ) Trapezoidal rule and ( $\beta$ ) Simpson's rule, find the approximate cross-sectional area of the river bed (to 1 decimal place).
- (ii) Use the answer in ( $\beta$ ) for this part. If the river current is flowing at the rate of 1.5 metres per second, how much water would flow through in 1 minute? (Express the answer in Megalitres, you may assume  $1ML = 10^6$  cubic metres).
- (b) (i) Sketch the graph of  $y = \sqrt{x+2}$  and shade in the areas enclosed by the curve, the  $x$  and  $y$  axes and the line  $y = 3$ . **4**
- (ii) Show that the shaded area is  $\left(\frac{8\sqrt{2}}{3} + 3\right)$  units<sup>2</sup>.
- (c) If  $\frac{dy}{dx} = 3\sqrt{x}$ , find the equation of the curve  $y = f(x)$  if the curve passes through the point (4, 15). **2**

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(1) (a) (i)  $-\frac{1}{3x^3} - \frac{1}{2x^2} + c$

(ii)  $\frac{9x^5}{5} + 2x^3 + x + c$

(b) (i)  $-1\frac{1}{3}$

(ii)  $\frac{1}{2}$

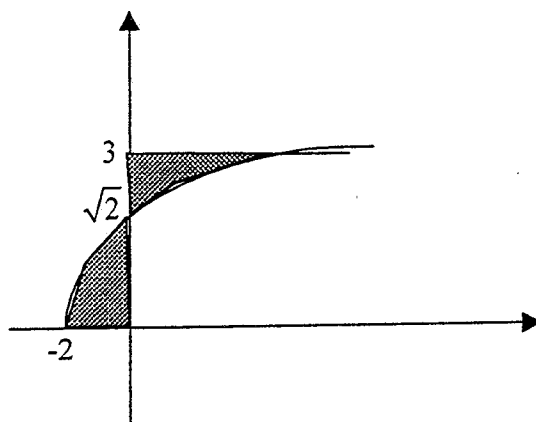
(c) Show that LHS = RHS =  $9\frac{7}{24}$ .

(2)

40	80	120	160	200
12.6	11.9	10.5	10.5	11.6

(a) (i)  $(\alpha) \overset{2750.4}{2736.0} (\beta) \overset{2768.5}{2749.3}$   
 (ii)  $0.2474 \text{ ML}$   
 $0.249165 \text{ ML}$

(b) (i)



(ii) Proof

(c)  $y = 2\sqrt{x^3} - 1$

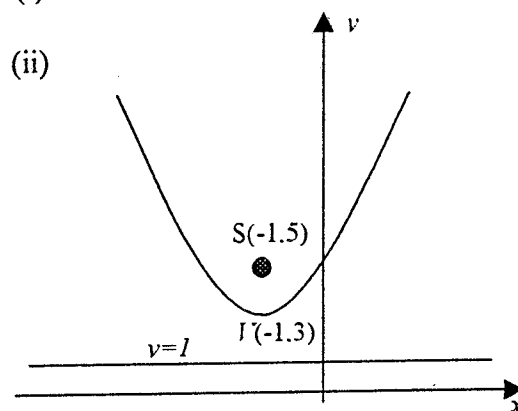
(3) (a) (i)  $18 + \frac{9\pi}{2}$  (ii)  $-9$

(b)  $3x - 4y - 9 = 0$  or  $3x - 4y + 11 = 0$

(c) (i)  $x = -4; y \neq 3$

(ii)  $y = 3$  has no gradient.

(d) (i) Proof



(4) (a) (i) Proof (ii) Proof

(b) Proof;  $28 \text{ cm}^3$

(c)  $x^2 = 12y$