SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2014

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instruction

- Reading time – 5 minutes
- Working time – 180 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 11 - 16

Total Marks – 100

Section I **10 Marks**

- Attempt questions 1 - 10
- Allow about 15 minutes for this section

Section II **90 Marks**

- Attempt Questions 11 – 16
- Allow about 2 hour 45 minutes for this section

Examiner: *V. Likourezos*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq 1;\ x \neq 0, \text{ if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x>0$

Section I **Objective response Questions**
Total marks – 10
Attempt Questions 1 – 10

Answer each question on the multiple choice answer sheet provided.

1) Solve $|5x+4| \le 6$

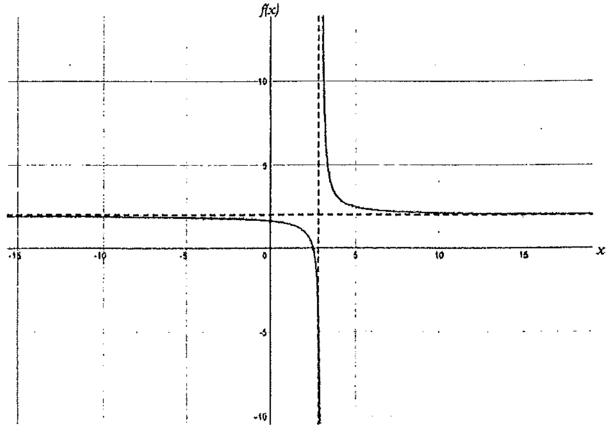
(A) $\frac{-2}{5} \le x \le 2$

(B) $x \ge \frac{2}{5}$ or $x \le -2$

(C) $-2 \le x \le \frac{2}{5}$

(D) $x \ge 2$ or $x \le \frac{-2}{5}$

2) Which function represents the following graph?



(A) $f(x) = \frac{1}{x-3} + 2$

(B) $f(x) = \frac{1}{x-3} - 2$

(C) $f(x) = \frac{1}{x+3} + 2$

(D) $f(x) = \frac{1}{x+3} - 2$

3) Two fair dice are thrown. If the first die thrown is a 3, what is the probability that the sum of the two dice is greater than 7?

(A) $\frac{1}{18}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{1}{12}$

4) Evaluate $\log_{2\sqrt{3}} 144$.

(A) 2

(B) 4

(C) 6

(D) 10

5) When the curve of the equation $y = e^x$ is rotated about the x-axis between $x = -3$ and $x = 3$, the volume of the solid generated is given by:

(A) $2\pi \int_0^3 e^{x^2} dx$

(B) $\pi \int_{-3}^3 e^{2x} dx$

(C) $\pi \int_{-3}^3 e^{x} dx$

(D) $\pi \int_{-3}^3 e^{x^2} dx$

6) The equation $2x^2+6x+4=0$ has roots α and β. The value of $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$ is:

(A) $\frac{-10}{9}$

(B) $\frac{7}{4}$

(C) $\frac{10}{9}$

(D) $\frac{5}{4}$

7) If x,y,z are consecutive terms in a geometric series, $x+y+z=13$, and $xyz=27$, then $x+z=$

(A) 15
(B) 12
(C) 10
(D) 8

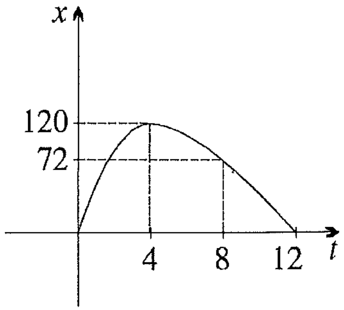
8) If $e^y=\cos x$, $-\frac{\pi}{2}<x<\frac{\pi}{2}$, find $\frac{dy}{dx}$.

(A) $\tan x$
(B) $-\tan x$
(C) $\cot x$
(D) $-\cot x$

9) A parabola has equation $(y+6)=3(x-5)^2$. The co-ordinates of the vertex are:

(A) $(-6,5)$
(B) $(6,-5)$
(C) $(5,-6)$
(D) $(-5,6)$

10) A toy rocket is shot up 120 metres vertically and floats back to the ground, landing in the same place. The accompanying graph shows the height x metres above the ground and t seconds after its launch.
What is the average velocity during the descent?



(A) 10 m/s
(B) -10 m/s
(C) 15 m/s
(D) -15 m/s

End of Section I

Section II **Free response Questions**
Total marks – 90
Attempt Questions 11 – 16

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 11

(a) Evaluate $\dfrac{8.2\times 3.77-(9.32)^2}{\sqrt{5.14}-2}$, correct to 3 significant figures. (1)

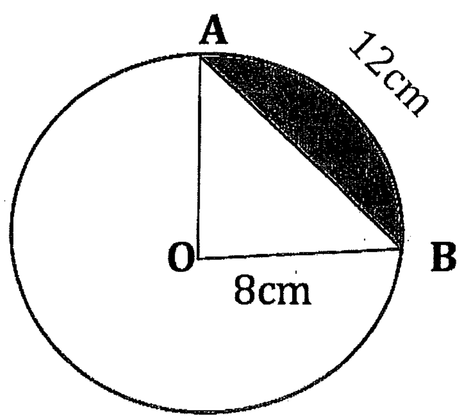
(b) Solve $\dfrac{3x+6}{9}-\dfrac{2-4x}{7}=x$. (1)

(c) $f(x)=\begin{cases}-x-3, & x\le -3\\ x+3, & x>-3\end{cases}$ (1)

Evaluate $f(-4)+f(1)-f(5)$.

(d) A circle with centre O has an arc AB that is $12cm$ in length. $OB = 8cm$.

(i) Find the angle at the centre subtended by the arc AB, leaving your answer in radians. (1)



(ii) Find the area of the shaded segment, correct to 2 significant figures. (2)

(e) Find $\lim\limits_{x\to 5}\dfrac{x-5}{2x^2-9x-5}$. (2)

(f) Find the domain and range of the function $y=\sqrt{9+x^2}$. (2)

(g) Solve for x if $2\log_e x=\log_e 3+\log_e(x+6)$. (2)

(h) Sketch the region defined by $(x+4)^2+(y-2)^2\ge 25$. (2)

(i) Does $\dfrac{16}{27}+\dfrac{8}{9}+\dfrac{4}{3}+\ldots\ldots$ have a limiting sum? (Justify your answer) (1)

End of Question 11

Question 12

(a) Differentiate, with respect to x:

(i) $\log_e 8x^2$ (2)

(ii) $\dfrac{x}{\cos x}$ (2)

(b) Integrate, with respect to x:

(i) $\int \sqrt{e^x}\,dx$ (2)

(ii) $\int_0^{\frac{\pi}{8}} 2\sec^2(2x)\,dx$ (2)

(c) (i) By completing the square, solve for a and b in the following equation: (2)

$$x^2 - 6x + 4 = (x-a)^2 + b$$

(ii) Hence, sketch the graph of $y = x^2 - 6x + 4$. (1)

(d) (i) For the series $11 + 18 + 25 + 32 + \ldots$, which term is equal to 508? (1)

(ii) Find n if $\sum_{k=4}^{n}(6k-7) = 16560$. (3)

End of Question 12

Question 13

(a) Find the equation of the normal to the curve $y = x^2$ at the point $M(6,36)$, leaving your answer in general form. (2)

(b) A and B are the points $\left(-\frac{1}{4}, -\frac{1}{32}\right)$ and $(4,-8)$ respectively. Show that AB is a focal chord of $x^2 = -2y$. (2)

(c) (i) Using the trapezoidal rule, with five function values, estimate $\int_{-2}^{2}\sqrt{4-x^2}\,dx$ (leaving your answer in surd form). (2)

(ii) By considering the curve $y = \sqrt{4-x^2}$, or otherwise, find the exact value of $\int_{-2}^{2}\sqrt{4-x^2}\,dx$. (2)

(iii) Find the percentage error when the trapezoidal approximation is used to estimate $\int_{-2}^{2}\sqrt{4-x^2}\,dx$ (correct to 2 decimal places). (1)

(d) (i) Plot the points $H(2,0)$, $I(5,12)$, $J(8,4)$ and $K(7,0)$ on a number plane and prove that $HIJK$ is a trapezium. (1)

(ii) Show that the equation of HI is $4x - y - 8 = 0$. (1)

(iii) Find the perpendicular distance from K to HI. (1)

(iv) Hence, or otherwise, find the area of the trapezium $HIJK$. (1)

(e) The sides of the triangle ABC are given by the lines:

$l_1 : 2x + y - 11 = 0$;

$l_2 : 2x - y + 3 = 0$; and

$l_3 : x - 2y - 3 = 0$.

(i) If l_1 and l_2 intersect at $(2,7)$, and l_2 and l_3 intersect at $(-3,-3)$, find the third vertex of ΔABC. (1)

(ii) Find the area of the triangle. (1)

End of Question 13

Question 14

(a) Consider the curve $y = x^3(4-x)$.

(i) Find the co-ordinates of the stationary points and determine their nature. (2)

(ii) Find the co-ordinates of any points of inflexion. (2)

(iii) Sketch the curve. (2)

(iv) What is the minimum value of $x^3(4-x)$ in the domain $-2 \le x \le 6$? (1)

(b) The number of litres of water in a spa, t minutes after it has started to drain, is given by the equation $L = 60(14-t)^2$.

(i) Calculate the initial volume of water in the spa (leaving your answer in kilolitres). (1)

(ii) At what rate does the water drain out of the spa after 8 minutes? (2)

(c) (i) Graph $y = 2\sin x$, for $-2\pi \le x \le 2\pi$. (1)

(ii) On your diagram, shade the regions bounded by the curve $y = 2\sin x$, the x-axis, and the lines $x = -\dfrac{3\pi}{2}$ and $x = \dfrac{5\pi}{4}$. (2)

Calculate the total area of these shaded regions (leaving your answer in exact form).

(iii) Solve $2\sin x = 1$, $-2\pi \le x \le 2\pi$. (2)

End of Question 14

Question 15

(a) Consider the equation $x^2 + (k+3)x + 9 = 0$. For what value(s) of k does the equation have:

(i) one root equal to -1? (1)

(ii) distinct real roots? (2)

(b) Solve for x if $2^{2x+2} + 35(2^x) - 9 = 0$. (2)

(c) Following the agreement by the New Zealand government to ban fishing with gillnets in the Maui's dolphins' habitat, the International Whaling Commission scientific committee estimated the dolphin population to be 95 at the start of 2014 and 200 at the end of 2015, with a population growth rate of $\dfrac{dD}{dt} = kD$, for some constant k, where D is the dolphin population t years after the start of 2014.

(i) Show that $D = D_0 e^{kt}$ satisfies the differential equation $\dfrac{dD}{dt} = kD$. (1)

(ii) Find the values of D_0 and k, and hence evaluate the dolphin population at the end of 2018. (2)

(iii) When will the dolphin population reach 2000 (correct to the nearest month)? (1)

(d) A particle moves along a straight line such that its distance, x, measured in metres from a fixed point O, is given by the equation $x = 3 + \cos\left(\frac{1}{2}t\right)$, where t is measured in seconds.

(i) Where is the particle initially? (1)

(ii) What distance has the particle travelled in the first $\frac{2\pi}{3}$ seconds? (1)

(iii) Find the average velocity of the particle in the first $\frac{2\pi}{3}$ seconds. (1)

(iv) Is the particle speeding up or slowing down when $t = \frac{2\pi}{3}$? Justify your answer. (2)

(v) Sketch the displacement, x, as a function of time, from $0 \le t \le 2\pi$. (1)

End of Question 15

Question 16

(a) A roulette wheel has 37 slots, numbered 0 to 36. Slot 0 is green, and the remaining 36 slots alternate between red and black, such that there are 18 red slots and 18 black slots. In a single game, someone must spin the wheel and simultaneously roll a ball around the wheel in the opposite direction. As the wheel slows down, the ball falls into one of the 37 slots. The wheel is carefully balanced so that the ball is equally likely to fall into any of the slots.

(i) In a single game, what is the probability that the ball falls into a black slot? (1)

(ii) Several games are played, one after the other. Assuming that the result of each game is independent of the result of any other game, what is the probability that the first time that the ball falls into a black slot is in the sixth game (correct to 3 decimal places)? (1)

(iii) What is the probability that the ball falls into at least one black slot before the sixth game (correct to 3 decimal places)? (1)

(b) Chloe took out a \$200,000 mortgage on a townhouse on 1 January 2010, and agreed to repay half-yearly instalments, the first being paid on 30 June 2010 and the last on 31 December 2020. She is charged interest at 9% per annum, compounded monthly.

$\$A_n$ represents the amount owing after the n^{th} half-yearly instalment and $\$M$ represents the half-yearly instalment.

(i) Write an expression for the amount owed on 1 July 2010, after Chloe made her first payment. (1)

(ii) How many payments does she make altogether? (1)

(iii) What is the value of $\$M$? (3)

(c) (i) Two cars, represented by points A and B, are travelling due east and due north respectively along two roads represented by two straight lines intersecting at O. At a certain instant, car A is 2 kilometres west of O and car B is 1 kilometre south of O, the former travelling at a constant speed of 1 kilometre per minute and the latter at constant speed V kilometres per minute.

(α) What value of V will cause a collision? (1)

(β) Prove that, in the course of motion, the minimum distance between the cars is $\dfrac{|2V-1|}{\sqrt{(V^2+1)}}$ kilometres. (3)

(ii) Determine the range of values of V which will cause collision if the above problem is made more realistic by representing the cars, not by points A and B, but by rectangles, each of length a kilometres and width b kilometres, with their centres at A and B. (3)

End of Question 16

End of the Examination

STANDARD INTEGRALS

$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n<0$

$\int \frac{1}{x}dx = \ln x,\ x>0$

$\int e^{ax}dx = \frac{1}{a}e^{ax},\ a \neq 0$

$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$

$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$

$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,$

$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$

$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$

$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a},\ a>0,\ -a<x<a$

$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x>a>0$

$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right)$

NOTE: $\ln x = \log_e x,\ x>0$

Student Number: SOLUTIONS

Mathematics
Trial HSC 2014

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

A ~~●~~ B ~~●~~ correct C ○ D ○

Section I: **Multiple choice answer sheet.**

Completely colour the cell representing your answer. Use black pen.

1. (A) (B) ● (D)
2. ● (B) (C) (D)
3. (A) ● (C) (D)
4. (A) ● (C) (D)
5. (A) ● (C) (D)
6. (A) (B) (C) ●
7. (A) (B) ● (D)
8. (A) ● (C) (D)
9. (A) (B) ● (D)
10. (A) (B) (C) ●

QUESTION 11.

(a) $\frac{8.2 \times 3.77 - (9.32)^2}{\sqrt{5.14} - 2} = \boxed{-209}$ (correct to 3 sig. figures)

(b) $\frac{3x+6}{9} - \frac{2-4x}{7} = x$

$7(3x+6) - 9(2-4x) = 63x$

$21x + 42 - 18 + 36x = 63x$

$24 = 6x$

$\boxed{x = 4}$

(c) $f(-4) + f(1) - f(5) = 1 + 4 - 8$

$\boxed{= -3}$

(d) (i) $l = r\theta$

$12 = 8\theta$

$\therefore \boxed{\theta = 1.5^c}$

(ii) $A = \frac{1}{2}r^2(\theta - \sin\theta)$

$= 32(1.5 - \sin 1.5)$

$\boxed{\doteqdot 16\text{ cm}^2}$ (correct to 2 sig. figures)

(e) $\lim_{x\to5} \frac{x-5}{(2x+1)(x-5)} = \lim_{x\to5} \frac{1}{2x+1}$

$\boxed{= \frac{1}{11}}$

(f) $D: x \in R.$

$R: y \geq 3.$

Q11 (CONTD)

(g) $2\ln x = \ln 3 + \ln(x+6)$

$\ln x^2 = \ln 3(x+6)$

$\therefore x^2 = 3x + 18$

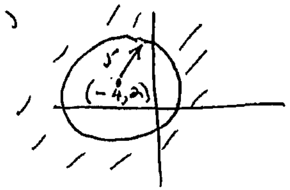
$x^2 - 3x - 18 = 0$

$(x-6)(x+3) = 0$

$x = 6, -3.$ (Clearly $x \neq -3$)

$\therefore \boxed{x = 6.}$

(h)



(i) Given the series $\frac{16}{27} + \frac{8}{9} + \frac{4}{3} + \cdots$

$r = \frac{8}{9} \div \frac{16}{27}$

$= \frac{3}{2}$. $\therefore$ (NO LIMITING SUM $|r| > 1$)

NB we require $|r| < 1$ for a limiting sum. !!

Question 12

(a) (i) $\frac{d}{dx}\ln 8x^2 = \frac{1}{8x^2}\frac{d}{dx}(8x^2)$ **2**

$= \frac{16x}{8x^2}$

$= \frac{2}{x}$

(ii) $\frac{d}{dx}\frac{x}{\cos x} = \frac{\cos x.1 - x.(-\sin x)}{\cos^2 x}$ **2**

$= \frac{\cos x + x\sin x}{\cos^2 x}$

$= \sec x + x\sec x\tan x$

(b) (i) $\int \sqrt{e^x}\,dx = \int e^{\frac{1}{2}x}\,dx$ **2**

$= 2e^{\frac{1}{2}x} + C$

$= 2\sqrt{e^x} + C$

(ii) $\int_0^{\frac{\pi}{8}} 2\sec^2(2x)\,dx = 2\left[\frac{1}{2}\tan(2x)\right]_0^{\frac{\pi}{8}}$ **2**

$= [\tan 2x]_0^{\frac{\pi}{8}}$

$= 1$

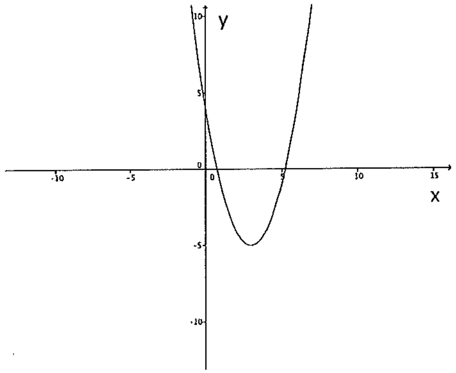
(c) (i) $x^2 - 6x + 4 \equiv x^2 - 6x + 9 - 9 + 4$ **2**

$= (x-3)^2 - 5$

Thus $a = 3$, and $b = -5$.

(ii) Curve crosses the x-axis when **1**

$$x=\frac{6\pm\sqrt{36-16}}{2}$$
$$=\frac{6\pm\sqrt{20}}{2}$$
$$=3\pm\sqrt{5}$$



(d) (i) This is an Arithmetic Series, $d=7$, $a=11$. **1**

$$u_n=a+(n-1)d$$
$$508=11+7(n-1)$$
$$497=7n-7$$
$$504=7n$$
$$n=72$$

508 is the 72nd term.

(ii) $\sum_{k=4}^{n}(6k-7)=16560$ **3**

$u_1=-1,\ u_2=5,\ u_3=11$, the sum of which is 15.

Thus $\sum_{k=4}^{n}(6k-7)=\sum_{k=1}^{n}(6k-7)-15=16560$

Therefore $\sum_{k=1}^{n}(6k-7)=16575$

This is an Arithmetic Series with $a=-1$ and $d=6$.

$$S_n=\tfrac{n}{2}(2a+(n-1)d)$$

$$16575=\tfrac{n}{2}(-2+6(n-1))$$
$$33150=n(6n-8)$$
$$0=3n^2-4n-16575$$
$$n=\frac{4\pm\sqrt{16+12\times16575}}{6}$$
$$=\frac{4\pm446}{6}$$
$$=75\ \text{ since } n>0\,.$$

Hence, $n=75$.

Question 13

a) $y = x^2$ at $M(6, 36)$

$y' = 2x$

$\therefore m_T = 2(6)$ $\qquad m_N \perp m_T$:

$= 12$

$\therefore m_N = -\frac{1}{12}$ [1]

$y - y_1 = m_N(x - x_1)$

$y - 36 = -\frac{1}{12}(x - 6)$

$12y - 432 = -x + 6$

$x + 12y - 438 = 0$ [1]

b)

method 1:

Let S be the focus: $\qquad A\left(-\frac{1}{4}, -\frac{1}{32}\right), B(4, -8)$

$\therefore 4a = -2$

$a = -\frac{1}{2}$ $\qquad \therefore S\left(0, -\frac{1}{2}\right)$

$m_{AS} = \dfrac{-\frac{1}{2} - \left(-\frac{1}{32}\right)}{0 - \left(-\frac{1}{4}\right)}$ $\qquad m_{SB} = \dfrac{-8 - \left(-\frac{1}{2}\right)}{4 - 0}$

$= -\frac{15}{8}$ [1] $\qquad = -\frac{15}{8}$

$\therefore m_{AS} = m_{SB}$ [1]

$\therefore$ AB is a focal chord of $x^2 = -2y$.

method 2:

$y - y_1 = m(x - x_1)$

$y - (-8) = \left(\dfrac{-1/32 - (-8)}{-1/4 - 4}\right)(x - 4)$

$y + 8 = -\frac{15}{8}(x - 4)$

$8y + 64 = -15x + 60$

$15x + 8y + 4 = 0$ [1]

Checking: $S\left(0, -\frac{1}{2}\right)$

$LHS = 15(0) + 8\left(-\frac{1}{2}\right) + 4$

$= 0$

$= RHS$ [1]

$\therefore$ AB is a focal chord of $x^2 = -2y$

c) i)

x	-2	-1	0	1	2
$\sqrt{4-x^2}$	0	$\sqrt{3}$	2	$\sqrt{3}$	0

$\therefore A \doteqdot \frac{1}{2}\left[0 + 0 + 2(\sqrt{3} + 2 + \sqrt{3})\right]$ [1]

$\doteqdot \frac{1}{2}\left[2(2\sqrt{3} + 2)\right]$

$\doteqdot 2\sqrt{3} + 2$ [1]

ii) $y = \sqrt{4 - x^2}$ (upper semi-circle with $r = 2$)

$\therefore A = \frac{1}{2} \times 2^2 \times \pi$

$= 2\pi$ [2]

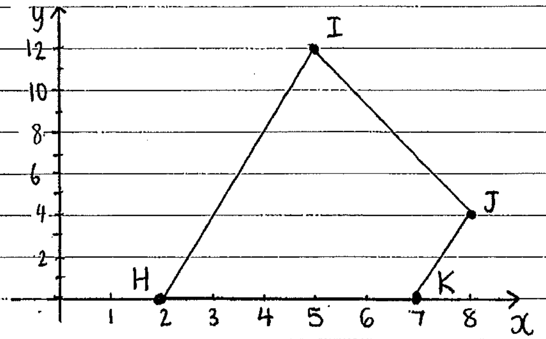
iii) Percentage error

$$= \frac{(2\pi - (2\sqrt{3}+2))}{2\pi} \times 100$$

$= 13.03612\ldots\ \%$

$= 13.04\%$ (2 d.p.) [1]

d) i)



H(2,0)
I(5,12)
J(8,4)
K(7,0)

$m_{HI} = \frac{12-0}{5-2} = \frac{12}{3} = 4$

$m_{JK} = \frac{4-0}{8-7} = \frac{4}{1} = 4$

$\therefore$ HIJK is a trapezium as $m_{HI} = m_{JK}$ [1]

ii) $m_{HI} = 4$ (from i)

$y - y_1 = m(x - x_1)$

$y - 0 = 4(x-2)$

$y = 4x - 8$

$\underline{4x - y - 8 = 0}$ [1]

iii) $d = \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$

$= \frac{|4(7) + (-1)(0) + (-8)|}{\sqrt{4^2 + (-1)^2}}$

$= \frac{20}{\sqrt{17}}$

$= \frac{20\sqrt{17}}{17}$ [1]

iv) method 1:

$d_{HI} = \sqrt{(5-2)^2 + (12-0)^2}$

$= \sqrt{153}$

$= 3\sqrt{17}$

$d_{JK} = \sqrt{(8-7)^2 + (4-0)^2}$

$= \sqrt{17}$

$\therefore$ Area $= \frac{1}{2} \times \frac{20\sqrt{17}}{17} \times (3\sqrt{17} + \sqrt{17})$

$= \frac{1}{2} \times \frac{20\sqrt{17}}{17} \times 4\sqrt{17}$

$= 40\ u^2$ [1]

method 2:

Area $= 12 \times (8-2) - \left[\frac{1}{2} \times 12 \times 3 + \frac{1}{2} \times 1 \times 4 + \frac{1}{2} \times 3 \times 8\right]$

$= 40\ u^2$ [1]

e) i) l_1 and l_3 intersect at:

$$2x + y - 11 = 0 \quad (1)$$
$$x - 2y - 3 = 0 \quad (2)$$

From (1): $y = 11 - 2x \quad (3)$
Sub (3) into (2):

$$x - 2(11 - 2x) - 3 = 0$$
$$x - 22 + 4x - 3 = 0$$
$$5x = 25$$
$$\underline{x = 5}$$

Sub $x = 5$ into (3):

$$y = 11 - 2(5)$$
$$= 11 - 10$$
$$\underline{y = 1}$$

$\therefore$ l_1 and l_3 intersect at $(5, 1)$ [1]

ii) l_1 and l_3 are perpendicular as $m_1 = -2$ and $m_3 = \frac{1}{2}$

$$m_1 \times m_3 = -2 \times \frac{1}{2}$$
$$= -1$$

$d_1 = \sqrt{(5-2)^2 + (1-7)^2} = \sqrt{45} = 3\sqrt{5}$

$d_2 = \sqrt{(5-(-3))^2 + (1-(-3))^2} = \sqrt{80} = 4\sqrt{5}$

$\therefore$ Area $= \frac{1}{2} \times 3\sqrt{5} \times 4\sqrt{5}$
$= 30 \text{ u}^2$ [1]

2U Trial Solutions SBHS 2014 Question 14.

(a) (i)
$$y = x^3(4 - x)$$
$$y = 4x^3 - x^4$$
$$y' = 12x^2 - 4x^3$$
$$y'' = 24x - 12x^2.$$

Stat pts exist when $y' = 0$

$$12x^2 - 4x^3 = 0$$
$(\div 4)$ $3x^2 - x^3 = 0$
$$x^2(3 - x) = 0$$
$x = 0$ and $x = 3$
$y = 0$ and $y = 27$
$(0,0)$ and $(3,27)$.

1/15

When $x = 0$ $y'' = 0$ (1)
inconclusive for a max/min stat pt
When $x = 3$, $y'' = 72 - 108 < 0$ (1)
$(3, 27)$ is a MAX stat. point.

(ii) Inflexions occur when $y'' = 0$ and a sign change exists.

$$y'' = 24x - 12x^2$$
$$24x - 12x^2 = 0$$
$(\div 12)$ $2x - x^2 = 0$
$$x(2 - x) = 0$$
$x = 0$ and $x = 2$.
$y = 0$ and $y = 16$.
$(0,0)$ and $(2,16)$

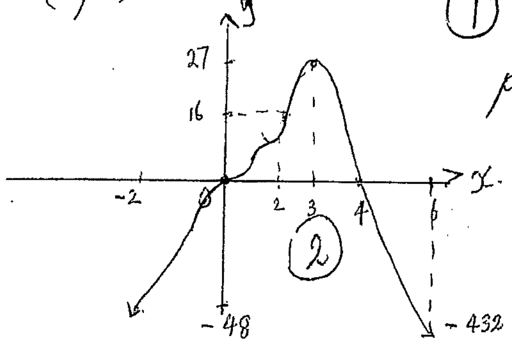
At $x = 0 - \varepsilon$ $(-.1)$ $y'' < 0$
$x = 0 + \varepsilon$ $(.1)$ $y'' > 0$ } sign change

(1) $(0,0)$ is a point of inflexion.

At $x = 2 - \varepsilon$ (1.9) $y'' > 0$
$x = 2 + \varepsilon$ (2.1) $y'' < 0$ } sign change

(1) $(2,16)$ is a point of inflexion

(iii)



(iv) minimum value is -432 (when $x = 6$) (1)

(14) (b) $L = 60(14-t)^2$.

(i) when $t=0$, $L = 60\times 14^2$

$= 11760\ L$

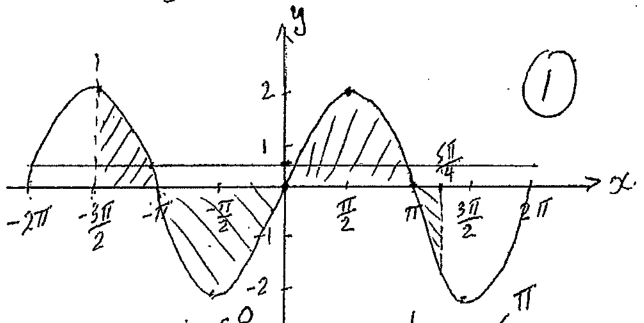
$= 11{\cdot}76\ kL$. (1)

(ii) $\frac{dL}{dt} = 60\times 2(14-t)^1 \times -1$

$= -120(14-t)$

when $t=8$, $\frac{dL}{dt} = -120\times 6 = -720\ L/min$ (2)

(c)



(1)

$$A = \int_{-\frac{3\pi}{2}}^{-\pi} 2\sin x\, dx + \left|\int_{-\pi}^{0} 2\sin x\, dx\right| + \int_{0}^{\pi} 2\sin x\, dx + \left|\int_{\pi}^{\frac{5\pi}{4}} 2\sin x d\right|$$

$$= 2\int_0^{\pi} 2\sin x\, dx$$

$$A = -2\cos x\Big]_{-\frac{3\pi}{2}}^{-\pi} - 4\cos x\Big]_0^{\pi} + \left|-2\cos x\Big]_{\pi}^{\frac{5\pi}{4}}\right|$$

$$= -2\left(\cos(-\pi) - \cos\left(-\tfrac{3\pi}{2}\right)\right) - 4\left(\cos\pi - \cos 0\right) + \left|-2\left(\cos\tfrac{5\pi}{4} - \cos\pi\right)\right|$$

$$= -2(-1-0) - 4(-1-1) + \left|-2\left(-\tfrac{1}{\sqrt{2}} - -1\right)\right| = 2+8+\left|\tfrac{2}{\sqrt{2}} - 2\right|$$

$$A = 2+8+2\left|1-\tfrac{1}{\sqrt{2}}\right|$$

$$= 2+8+2\left|\tfrac{\sqrt{2}-1}{\sqrt{2}}\right|$$

$$= 2+8+\cancel{2}\left|\tfrac{(\sqrt{2}-1)\cdot\sqrt{2}}{\cancel{2}}\right|$$

$$= 10+\left|2-\sqrt{2}\right|\ u^2$$

$= \cancel{12-\sqrt{2}}\ \cancel{u^2}$ ✓ (2)

(iii) $2\sin x = 1$

$\sin x = \frac{1}{2}$ see diagram.

4 answers $30^\circ = \frac{\pi}{6}$ ✓

$180^\circ - 30^\circ = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ ✓

$-180^\circ - 30^\circ = -\pi - \frac{\pi}{6} = -\frac{7\pi}{6}$ ✓ (2)

$-360^\circ + 30^\circ = -2\pi + \frac{\pi}{6} = -\frac{11\pi}{6}$ ✓

an alternative to 14(c) is

$$\int_{-\frac{3\pi}{2}}^{-\pi} 2\sin x\, dx + \left|\int_{\pi}^{\frac{5\pi}{4}} 2\sin x\, dx\right|$$

$= 2 + \left|2-\sqrt{2}\right|\ u^2$ (2)

$4-\sqrt{2}\ u^2$.

2014 TRIAL Q15 MATHS(2U)

(15) a) (i) $x=-1$ is a root of

$$x^2+(k+3)x+9=0$$

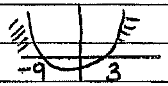
sub $x=-1$

$$(-1)^2-k-3+9=0$$
$$-k+7=0$$
$$\underline{\underline{k=7}}$$

(ii) For distinct real roots $\Delta>0$

$$\Delta = b^2-4ac$$
$$=(k+3)^2-4(1)(9)$$

$$k^2+6k+9-36$$
$$=k^2+6k-27$$
$$=(k-3)(k+9)$$
$$>0$$



$\underline{\underline{k<-9}}$ or $\underline{\underline{k>3}}$

2014 TRIAL 2U. Q15

(15) b) $2^{2x+2}+35(2^x)-9=0$

$$2^{2x}\times 2^2+35(2^x)-9=0$$

$$4\times 2^{2x}+35(2^x)-9=0$$

Let $u=2^x$

$$4u^2+35u-9=0$$
$$4u^2+36u-u-9=0$$
$$4u(u+9)-(u+9)=0$$

$$(4u-1)(u+9)=0$$

Either

$4u=1$ or $u=-9$

$u=\frac{1}{4}$

sub $u=2^x$

$2^x=2^{-2}$ $\qquad$ $2^x=-9$

$x=-2$ // $\qquad$ $x=$ NO SLTN

2014 THSC 2U Q15

15 c

i) $\frac{dD}{dt} = KD$

$D = D_0e^{Kt}$ is a sltn.

$LHS = \frac{dD}{dt} = \frac{d}{dt}\left(D_0e^{Kt}\right)$

$= K D_0 e^{Kt}$

$= K D$

$= RHS$.

(ii) a) $D = D_0e^{Kt}$ $D=95$ $t=0$

$95 = D_0e^0$ $\therefore$ $D_0 = 95$

b) $D=200$ $t=2$

$D = D_0e^{Kt}$

$200 = 95e^{2K}$

$e^{2K} = \frac{200}{95}$

$2K = \ln\left(\frac{200}{95}\right)$

$K = \frac{1}{2}\ln\left(\frac{200}{95}\right)$

$= 0.372$ (3dp)

c)

Pop. at End of 2018 when $t=5$

$D = 95e^{5K}$

$= 610.927$

$\doteq 610$ or 611 DOLPHINS.

15 2014 THSC 2U Q15.

c iv)

$D = 2000$ $t = ?$

$2000 = 95e^{Kt}$

$\frac{2000}{95} = e^{Kt}$

$Kt = \ln\left(\frac{2000}{95}\right)$

$t = \frac{1}{K}\ln\left(\frac{2000}{95}\right)$ $\left(K = \frac{1}{2}\ln\frac{40}{19}\right)$

$\doteq$ ~~8~~8.186 yrs

$= 98$ mths.

2014 + ~~8~~8.186 yrs = 2022.186 yrs

= 2022 + 2.23 mnths.

$\boxed{\text{MARCH 2022}}$

2014 THSC 2U Q15

15 d)

i) Initially, $t=0$ $\quad x = 3 + \cos(0/2)$

$= 3+1$

$= 4$ m.

the particle is 4 m to the right of the origin O

ii) At $t = 2\pi/3$

$x = 3 + \cos(2\pi/3 \div 2)$

$= 3 + \cos(\frac{\pi}{3})$

$= 3 + \frac{1}{2}$

$= 3\frac{1}{2}$ m.

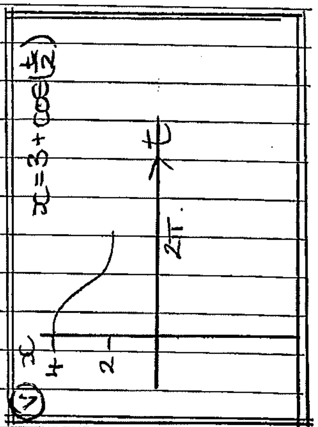
The particle has travelled $\frac{1}{2}$ m

iii) Ave vel $= \dfrac{x_1 - x_0}{t_1 - t_0}$

$= \dfrac{4 - 3.5}{0 - 2\pi/3}$

$= \frac{1}{2} \div -\frac{2\pi}{3}$

Ave Vel $= \dfrac{-3}{4\pi}$ ms^{-1}



iv) $x = 3 + \cos(\frac{t}{2})$

$\dot{x} = v = -\frac{1}{2}\sin(\frac{t}{2})$

$\ddot{x} = a = -\frac{1}{4}\cos(\frac{t}{2})$

At $t = 2\pi/3$ $\quad v = -\frac{1}{2}\sin(\pi/3) = -\frac{1}{2}\cdot\frac{\sqrt{3}}{2} = \dfrac{-\sqrt{3}}{4}$ ms^{-1}.

$a = -\frac{1}{4}\cos(\pi/3) = -\frac{1}{4}(\frac{1}{2})$

$= -\frac{1}{8}$ ms^{-2}

As $v<0$ and $a<0$

speeding up in the negative direction.

Question 16:

a) (i) $P(\text{black}) = \dfrac{18}{37}$

(ii) $P(\bar{B}\bar{B}\bar{B}\bar{B}\bar{B}B) = \left(\dfrac{19}{37}\right)^5\left(\dfrac{18}{37}\right)$

$= 0.017$

(iii) $1 - \left(\dfrac{19}{37}\right)^5$

$= 0.964$

b) (i) $A_1 = 200000(1.0075)^6 - M$

(ii) 22 repayments

(iii) $A_2 = 200\,000(1.0075)^{12} - M(1 + 1.0075^6)$

1 $\quad A_3 = 200\,000(1.0075)^{18} - M(1 + 1.0075^6 + 1.0075^{12})$

$\vdots$

$A_{22} = 200000(1.0075)^{132} - M(1 + 1.0075^6 + \dots + 1.0075^{126})$

$0 = 200000(1.0075)^{132} - M\left(\dfrac{1.0075^{132} - 1}{1.0075^6 - 1}\right)$

$M\left(\dfrac{1.0075^{132} - 1}{1.0075^6 - 1}\right) = 200000(1.0075)^{132}$

1 $\quad M = \dfrac{200\,000(1.0075)^{132}}{\left(\dfrac{1.0075^{132} - 1}{1.0075^6 - 1}\right)}$

1 $\quad \boxed{\therefore M = \$14624.79}$

Question 16 :

c) (i) α) $V_A = 1$ km/min $\quad V_B = V$

$T_A = 2$ min $\quad \boxed{\therefore V = \tfrac{1}{2} \text{ km/min}}$ (1)

β). $V_A = 1 \quad V_B = V$

$x_A = t - 2 \quad x_B = Vt - 1$

$D = \sqrt{(t-2)^2 + (Vt-1)^2}$ (1)

$D^2 = (t-2)^2 + (Vt-1)^2$

$\frac{dD^2}{dt} = 2(t-2) + 2V(Vt-1)$

$2(t-2) + 2V(Vt-1) = 0$

$2t - 4 + 2V^2t - 2V = 0$

$t - 2 + V^2t - V = 0$

$(1+V^2)t = V+2$

$$\boxed{t = \frac{V+2}{1+V^2}}$$

$\frac{d^2D^2}{dt^2} = 2 + 2V^2$

> 0 (1)

$\therefore t = \frac{V+2}{1+V^2}$ is a minimum

$$D^2 = \left(\frac{V+2}{1+V^2} - 2\right)^2 + \left(\frac{V^2+2V}{1+V^2} - 1\right)^2$$

$$= \left(\frac{V+2-2-2V^2}{1+V^2}\right)^2 + \left(\frac{V^2+2V-1-V^2}{1+V^2}\right)^2$$

$$= \left(\frac{V-2V^2}{1+V^2}\right)^2 + \left(\frac{2V-1}{1+V^2}\right)^2$$

$$= \frac{V^2(1-2V)^2 + (2V-1)^2}{(1+V^2)^2}$$

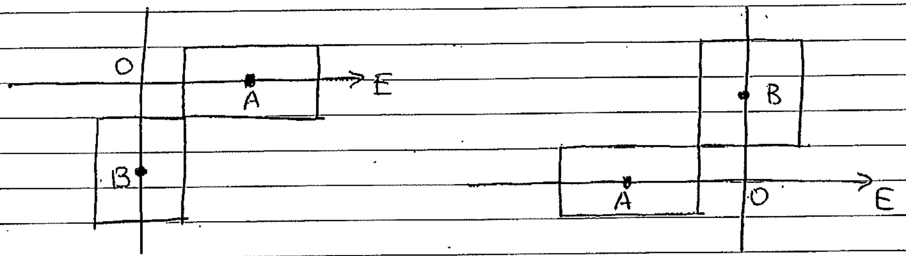
$$D^2 = \frac{(2V-1)^2(V^2+1)}{(1+V^2)^2}$$ (1)

$$D = \frac{\sqrt{(2V-1)^2}}{\sqrt{1+V^2}}$$

$$\therefore D = \frac{|2V-1|}{\sqrt{1+V^2}}$$

Question 16:

c). (ii) Collisions occur between 2 extremes:



Vminimum Vmaximum

Vminimum: Collision occurs when A is $\frac{a+b}{2}$ E of O and when B is $\frac{a+b}{2}$ S of O.

In time t, A travels,

$$2+\frac{a+b}{2}=\frac{4+a+b}{2} \text{ km}$$

and B travels,

$$1-\frac{a+b}{2}=\frac{2-a-b}{2} \text{ km}$$

$$\therefore \frac{\frac{4+a+b}{2}}{1}=\frac{\frac{2-a-b}{2}}{V_{min}} \qquad \left(\frac{d}{V}=t\right) \qquad 1$$

$$\therefore V_{min}=\frac{2-a-b}{4+a+b}$$

Vmaximum: Collision occurs when A is $\frac{a+b}{2}$ W of O and when B is $\frac{a+b}{2}$ N of O.

In time t, A travels.

$$2-\frac{a+b}{2}=\frac{4-a-b}{2} \text{ km}$$

and B travels

$$1+\frac{a+b}{2}=\frac{2+a+b}{2} \text{ km} \qquad 1$$

$$\therefore \frac{\frac{4-a-b}{2}}{1}=\frac{\frac{2+a+b}{2}}{V_{max}}$$

$$\therefore V_{max}=\frac{2+a+b}{4-a-b}$$

The range of values of V,

$$\boxed{\therefore \frac{2-a-b}{4+a+b}\le V\le\frac{2+a+b}{4-a-b}} \qquad 1$$