STUDENT'S NAME ____________________

CLASS (circle one) 12MAE1 Wilson 12MAE4 Paraskevas 12MAE7 Howlett

Sydney Secondary College

QUALITY ■ OPPORTUNITY ■ DIVERSITY

BLACKWATTLE BAY CAMPUS

2014

TRIAL
HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

- **General Instructions**
- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total Marks – 70

Section I Pages 2 – 4

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 5 – 10

60 marks

- Attempt Questions 11 – 14
- Allow about 1hour and 45 minutes for this section

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 What is the domain and range of $y=\frac{1}{2}\cos^{-1}(\frac{x}{2})$?

(A) Domain: $-2\le x\le 2$ Range: $0\le y\le \pi$

(B) Domain: $-1\le x\le 1$ Range: $0\le y\le \pi$

(C) Domain: $-2\le x\le 2$ Range: $0\le y\le \frac{\pi}{2}$

(D) Domain: $-1\le x\le 1$ Range: $0\le y\le \frac{\pi}{2}$

2 When a polynomial $P(x)=x^3+ax+1$ is divided by $(x+2)$ the remainder is 5. What is the value of a?

(A) -6
(B) -3.5
(C) 2
(D) 3

3 Which of the following is an expression for $\int\frac{x}{(2-x^2)^3}dx$?

Use the substitution $u=2-x^2$.

(A) $\frac{1}{2(2-x^2)^3}+C$

(B) $\frac{1}{4(2-x^2)^2}+C$

(C) $\frac{1}{4(2-x^2)^4}+C$

(D) $\frac{1}{8(2-x^2)^4}+C$

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4 What is the exact value of the definite integral $\int_0^1\frac{1}{x^2+1}dx$?

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) π

5 How many arrangements of all of the letters of the word PROBABILITY are possible?

(A) 362 880
(B) 9 979 200
(C) 19 958 400
(D) 39 916 800

6 Find the acute angle between the lines $y=2x$ and $x+y-5=0$. Answer correct to the nearest degree.

(A) 18°
(B) 32°
(C) 45°
(D) 72°

7 What is the exact value of the definite integral $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}(1+\sin^2 x)dx$?

(A) $\frac{\pi}{4}+\frac{1}{2}$

(B) $\frac{\pi}{8}+\frac{1}{8}$

(C) $\frac{\pi}{4}-\frac{\sqrt{3}-2}{4}$

(D) $\frac{\pi}{8}-\frac{\sqrt{3}-2}{8}$

8 What are the coordinates of the point that divides the interval joining the points $A(-1,2)$ and $B(3,5)$ externally in the ratio $3:1$?

(A) $(2.5, 4.25)$

(B) $(2.5, 6.5)$

(C) $(5, 4.25)$

(D) $(5, 6.5)$

9 What is the solution to the inequality $\frac{2x-5}{x-4} \geq x$?

(A) $x \leq -1$ and $4 \leq x \leq 5$

(B) $x \leq -1$ and $4 < x \leq 5$

(C) $x \leq 1$ and $4 \leq x \leq 5$

(D) $x \leq 1$ and $4 < x \leq 5$

10 The velocity of a particle moving in a straight line is given by $v = 2x + 5$, where x metres is the distance from fixed point O and v is the velocity in metres per second. What is the acceleration of the particle when it is 1 metre from O?

(A) $a = 7\ \text{ms}^{-2}$

(B) $a = 12\ \text{ms}^{-2}$

(C) $a = 14\ \text{ms}^{-2}$

(D) $a = 24\ \text{ms}^{-2}$

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Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

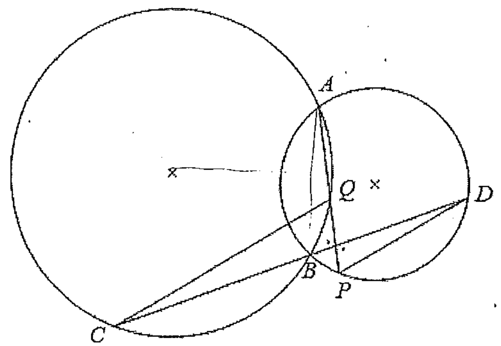
Answer each question in the appropriate writing booklet.

All necessary working should be shown in every question.

Question 11 (15 marks) **Marks**

(a) What are the roots of the equation $4x^3 - 4x^2 - 29x + 15 = 0$ given that one root is the difference between the other two roots? **3**

(b) Two circles meet at A and B. CBD is a straight line and AQP is a chord of the smaller circle. Prove that CQ is parallel to PD. **2**

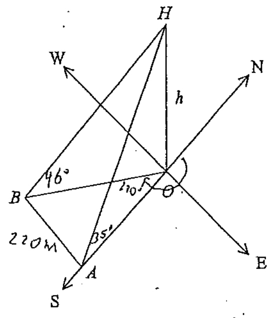


(c) Prove the following identity **2**

$$\frac{\sin\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta}{\cos\theta - \sin\theta} = \tan 2\theta$$

(d) A class consists of 10 boys and 12 girls. How many ways are there of selecting a committee of 3 boys and 2 girls from this class? **2**

(e) Point A is due south of a hill and the angle of elevation from A to the top of the hill is 35°. Another point B is a bearing 200° from the hill and the angle of elevation from B to the top of the hill is 46°. The distance AB is 220 m.



(i) Express OA and OB in terms of h. **2**

(ii) Calculate the height h of the hill correct to three significant figures. **2**

(f) Factorise x^3+3x^2-9x+5 **2**

Question 12 (15 marks) **Marks**

(a) The tangent at the point $P(2ap, ap^2)$ on the parabola $x^2=4ay$ cuts the x-axis at A and the y-axis at B.

(i) Find the coordinates of M, the midpoint of A and B in terms of P. **2**

(ii) Show that the locus of M is a parabola. **1**

(iii) Find the coordinates of the focus of this parabola and the equation of the directrix. **1**

(b) Use the principle of mathematical induction to prove that for all positive integers n: **3**

$$1+2+4+\ldots+2^{n-1}=2^n-1$$

(c) Find the exact value of $\sin\left[\cos^{-1}\frac{2}{3}+\tan^{-1}\left(-\frac{3}{4}\right)\right]$ **2**

(d) Find all the angles θ with $0\le\theta\le 2\pi$ for which $\sin\theta+\cos\theta=1$. **3**

(e) The function $f(x)$ is given by $f(x)=\sin^{-1}x+\cos^{-1}x$, $0\le x\le 1$.

(i) Find $f'(x)$. **1**

(ii) Sketch the graph of $y=f(x)$. **2**

Question 13 (15 marks) **Marks**

(a) (i) Show that the function $f(x) = xe^x - 1$ has a zero between $x = 0$ and $x = 1$. **1**

(ii) Using $x = 0.5$ as the first approximation, use Newton's Method to obtain a second approximation. Answer correct to 2 decimal places. **2**

(b) A golfer hits a golf ball to clear a 6 metres high tree. The tree is 20 metres away on level ground. The golfer uses a golf club that produces an angle of elevation of 40°. Take $g = 10 \text{ ms}^{-2}$

(i) Derive the expressions for the vertical and horizontal components of the displacement of the ball from the point of projection. **3**

(ii) What is the Cartesian equation of the flight path? **2**

(iii) Calculate the speed at which the ball must leave the ground to just clear the tree. Answer correct to one decimal place. **2**

(c) (i) Expand $(1-x)^n$ using the binomial theorem. **1**

(ii) Show that $\int_0^1 (1-x)^n\,dx = {}^nC_0 - \frac{1}{2}{}^nC_1 + \frac{1}{3}{}^nC_2 - \ldots + \frac{(-1)^n}{n+1}{}^nC_n$ **2**

(iii) Show that $\frac{1}{n+1} = \sum_{r=0}^{n} (-1)^r \frac{1}{r+1}{}^nC_r$ **2**

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Question 14 (15 marks) **Marks**

(a) Find $\int \cos^2 2x\,dx$ **2**

(b) A particle moves in a straight line and its position at any time is given by:

$$x = 3\cos 2t + 4\sin 2t$$

(i) Prove that the motion is simple harmonic. **2**

(ii) Calculate the particle's greatest speed. **2**

(c) Water at a temperature of 24°C is placed in a freezer maintained at a temperature of -12°C. After time t minutes the rate of change of temperature T of the water is given by the formula:

$$\frac{dT}{dt} = -k(T+12)$$

where t is the time in minutes and k is a positive constant.

(i) Show that $T = Ae^{-kt} - 12$ is a solution of this equation, where A is a constant. **1**

(ii) Find the value of A. **1**

(iii) After 15 minutes the temperature of the water falls to 9°C. Find to the nearest minute the time taken for the water to start freezing. (Freezing point of water is 0°C). **3**

(d) Each rectangular table in a hall has nine seats, five facing the front and four facing the back. In how many ways can 9 people be seated at a table if:

(i) Alex and Bella must sit on the same side. **2**

(ii) Alex and Bella must sit on opposite sides **2**

End of paper

Examination 2014

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Worked solutions and marking guidelines

Section I		
	Solution	**Criteria**
1	Domain: $-1 \le \frac{x}{2} \le 1$ or $-2 \le x \le 2$. Range: $\frac{1}{2} \times 0 \le y \le \frac{1}{2} \times \pi$ or $0 \le y \le \frac{\pi}{2}$	1 Mark: C
2	$P(x) = x^3 + ax + 1$ $P(-2) = (-2)^3 + a \times -2 + 1 = 5$ $-2a = 12$ $a = -6$	1 Mark: A
3	$u = 2 - x^2$ $\frac{du}{dx} = -2x$ $-\frac{1}{2}du = xdx$ $\int \frac{x}{(2-x^2)^3}dx = -\frac{1}{2}\int \frac{1}{u^3}du$ $= -\frac{1}{2} \times -\frac{1}{2}u^{-2} + C$ $= \frac{1}{4(2-x^2)^2} + C$	1 Mark: B
4	$\int_0^1 \frac{1}{x^2+1}dx = \left[\tan^{-1} x\right]_0^1$ $= \frac{\pi}{4} - 0$ $= \frac{\pi}{4}$	1 Mark: A
5	Number of arrangements $= \frac{11!}{2! \times 2!}$ (2 I's and 2 B's) $= 9\ 979\ 200$	1 Mark: B
6	For $y = 2x$ then $m_1 = 2$ For $x + y - 5 = 0$ then $m_2 = -1$ $\tan\theta = \left\lvert \frac{m_1 - m_2}{1 + m_1 m_2} \right\rvert = \left\lvert \frac{2-(-1)}{1+2\times -1} \right\rvert$ $= 3$ $\theta = 71.56505118\ldots$ $\approx 72^\circ$	1 Mark: D

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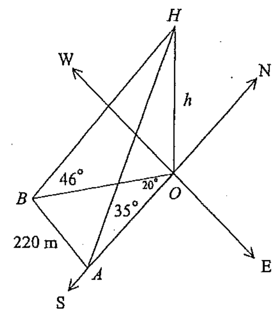
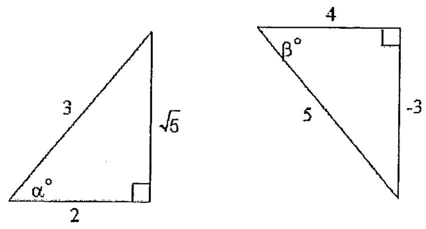
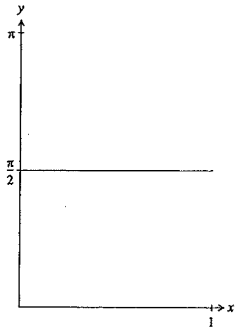
7	$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (1 + \sin^2 x)d = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (1 + \frac{1}{2}(1 - \cos 2x)dx$ $= \left[\frac{3}{2}x - \frac{1}{4}\sin 2x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ $= \left[\left(\frac{\pi}{2} - \frac{1}{4}\sin\frac{2\pi}{3}\right) - \left(\frac{3\pi}{8} - \frac{1}{4}\sin\frac{\pi}{2}\right)\right]$ $= \left[\left(\frac{\pi}{2} - \frac{\sqrt{3}}{8}\right) - \left(\frac{3\pi}{8} - \frac{1}{4}\right)\right]$ $= \frac{\pi}{8} - \frac{\sqrt{3}-2}{8}$	1 Mark: D
8	$A(-1,2)$ and $B(3,5)$ with $3:-1$ $x = \frac{mx_2 + nx_1}{m+n}$ $y = \frac{my_2 + ny_1}{m+n}$ $= \frac{3\times 3 + -1 \times -1}{3 + -1}$ $= \frac{3 \times 5 + -1 \times 2}{3 + -1}$ $= 5$ $= 6.5$ Point is (5,6.5)	1 Mark: D
9	$(x-4)^2 \times \frac{2x-5}{x-4} \ge x \times (x-4)^2 \quad (x \ne 4)$ $(x-4)(2x-5) - x(x-4)^2 \ge 0$ $(x-4)\left[(2x-5) - x(x-4)\right] \ge 0$ $(x-4)(-x^2 + 6x - 5) \ge 0$ $(x-4)(x-5)(1-x) \ge 0$ Critical points are 1,4 and 5 Test values in each region $x \le 1$ and $4 < x \le 5$	1 Mark: D
10	$v = 2x + 5$ $v^2 = 4x^2 + 20x + 25$ $\frac{1}{2}v^2 = 2x^2 + 10x + \frac{25}{2}$ $a = \frac{d}{dx}\left(2x^2 + 10x + \frac{25}{2}\right)$ $= 4x + 10$ When $x = 1$ then $a = 14$	1 Mark: C

Section II		
11(a)	Let the roots be α, β and $\alpha - \beta$. $4x^3 - 4x^2 - 29x + 15 = 0$ $\alpha + \beta + (\alpha - \beta) = -\frac{b}{a} = -\frac{-4}{4} = 1$ $2\alpha = 1$ or $\alpha = \frac{1}{2}$ $\alpha\beta(\alpha - \beta) = -\frac{d}{a}$ $\frac{1}{2}\beta(\frac{1}{2} - \beta) = -\frac{15}{4}$ $\beta(\frac{1}{2} - \beta) = -\frac{15}{2}$ $2\beta^2 - \beta - 15 = 0$ $(2\beta + 5)(\beta - 3) = 0$ $\beta = -\frac{5}{2}$ or $\beta = 3$ Roots are $x = -\frac{5}{2}$, $x = \frac{1}{2}$ and $x = 3$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Finds the sum or product of the roots.
11(b)	Join A to B $\angle BCQ = \angle BAQ$ (Angle in the same segment standing on the same arc are equal) $\angle BAP = \angle BDP$ (Angle in the same segment standing on the same arc are equal) $\angle BCQ = \angle BDP$ ($\angle BAQ$ and $\angle BAP$ are the same angle) $\therefore CQ \parallel PD$ (alternate angles are equal).	2 Marks: Correct answer. 1 Marks: Makes some progress towards the solution.
11(c)	$\text{LHS} = \frac{\sin\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta}{\cos\theta - \sin\theta}$ $= \frac{\sin\theta(\cos\theta - \sin\theta) + \sin\theta(\cos\theta + \sin\theta)}{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}$ $= \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$ $= \frac{\sin 2\theta}{\cos 2\theta}$ $= \tan 2\theta = \text{RHS}$	2 Marks: Correct answer. 1 Marks: Uses a relevant trigonometric identity
11(d)	Number of ways $= {}^{10}C_3 \times {}^{12}C_2$ $= 120 \times 66$ $= 7920$ Class can be selected in 7920 ways.	2 Marks: Correct answer. 1 Marks: Shows some understanding.

11(e) (i)	 In ΔHOA $\tan 35^\circ = \frac{h}{OA}$ $OA = \frac{h}{\tan 35^\circ}$ In ΔHOB $\tan 46^\circ = \frac{h}{OB}$ $OB = \frac{h}{\tan 46^\circ}$	2 Marks: Correct answer. 1 Mark: One correct expression or shows some understanding of the problem.
11(e) (ii)	$AB^2 = OA^2 + OB^2 - 2 \times OA \times OB \times \cos 20^\circ$ $220^2 = \left(\frac{h}{\tan 35^\circ}\right)^2 + \left(\frac{h}{\tan 46^\circ}\right)^2 - 2 \times \frac{h}{\tan 35^\circ} \times \frac{h}{\tan 46^\circ} \times \cos 20^\circ$ $= h^2\left(\frac{1}{\tan^2 35^\circ} + \frac{1}{\tan^2 46^\circ} - 2 \times \frac{\cos 20^\circ}{\tan 35^\circ \times \tan 46^\circ}\right)$ $h^2 = 220^2 \div \left(\frac{1}{\tan^2 35^\circ} + \frac{1}{\tan^2 46^\circ} - 2 \times \frac{\cos 20^\circ}{\tan 35^\circ \times \tan 46^\circ}\right)$ $= 127296.7453...$ $h = 356.7866944... \approx 357$ m	2 Marks: Correct answer. 1 Mark: Uses the cosine rule with at least one correct value.
11(f)	Factors of 5 are $\{\pm 1, \pm 5\}$ $P(1) = 1^3 + 3 \times 1^2 - 9 \times 1 + 5 = 0$ Therefore $(x-1)$ is a factor of $x^3 + 3x^2 - 9x + 5$ $x - 1 \overline{) x^3 + 3x^2 - 9x + 5}$ quotient $x^2 + 4x - 5$ $\underline{x^3 - x^2}$ $4x^2 - 9x$ $\underline{4x^2 - 4x}$ $-5x + 5$ $\underline{-5x + 5}$ $P(x) = x^3 + 3x^2 - 9x + 5 = (x-1)(x^2 + 4x - 5)$ $= (x-1)(x-1)(x+5) = (x-1)^2(x+5)$	2 Marks: Correct answer. 1 Mark: Finds one factor or shows some understanding.

12(a) (i)	To find the gradient of the tangent $y=\frac{1}{4a}x^2$ and $\frac{dy}{dx}=\frac{1}{2a}x$ At $P(2ap,ap^2)$ $\frac{dy}{dx}=\frac{1}{2a}\times 2ap=p$ Equation of the tangent at $P(2ap,ap^2)$ $y-y_1=m(x-x_1)$ $y-ap^2=p(x-2ap)$ $y=px-ap^2$ x-intercept ($y=0$) then $x=ap$. Hence $A(ap,0)$ y-intercept ($x=0$) then $y=-ap^2$. Hence $B(0,-ap^2)$ Midpoint of A and B. $M\left(\frac{ap+0}{2},\frac{0+-ap^2}{2}\right)=M\left(\frac{ap}{2},\frac{-ap2}{2}\right)$	2 Marks: Correct answer. 1 Mark: Finds the gradient of the tangent or the coordinates of A and B.
12(a) (ii)	To find the locus of M eliminate p from coordinates of M Now $x=\frac{ap}{2}$ (1) and $y=\frac{-ap^2}{2}$ (2) From (1) $p=\frac{2x}{a}$ and sub into eqn (2) $y=\frac{-a(\frac{2x}{a})^2}{2}=\frac{-a}{2}\times\frac{4x^2}{a^2}=-\frac{2x^2}{a}$ or $x^2=-\frac{1}{2}ay$ (parabola)	1 Mark: Correct answer.
12(a) (iii)	$x^2=-\frac{1}{2}ay=4\times\left(-\frac{1}{8}a\right)\times y$ Focus is $\left(0,-\frac{1}{8}a\right)$ and equation of the directrix $y=\frac{1}{8}a$	1 Mark: Correct answer.
12(b)	Step 1: To prove the statement true for $n=1$ LHS $=1$ RHS $=2^1-1=1$ Result is true for $n=1$ Step 2: Assume the result true for $n=k$ $1+2+4+...+2^{k-1}=2^k-1$ To prove the result is true for $n=k+1$ $1+2+4+...+2^{k-1}+2^k=2^{k+1}-1$	3 Marks: Correct answer. 2 Marks: Proves the result true for $n=1$ and attempts to use the result of $n=k$ to prove the result for $n=k+1$.

	LHS $=1+2+4+...+2^{k-1}+2^k$ $=2^k-1+2^k$ $=2\times 2^k-1$ $=2^{k+1}-1$ $=$ RHS Result is true for $n=k+1$ if true for $n=k$ Step 3: Result true by principle of mathematical induction.	1 Mark: Proves the result true for $n=1$.
12(c)	Let $\alpha=\cos^{-1}\frac{2}{3}$ and $\beta=\tan^{-1}\left(-\frac{3}{4}\right)$  $\sin\left[\cos^{-1}\frac{2}{3}+\tan^{-1}\left(-\frac{3}{4}\right)\right]=\sin(\alpha+\beta)$ $=\sin\alpha\cos\beta+\cos\alpha\sin\beta$ $=\frac{\sqrt{5}}{3}\times\frac{4}{5}+\frac{2}{3}\times-\frac{3}{5}=\frac{4\sqrt{5}-6}{15}$	2 Marks: Correct answer. 1 Mark: Sets up the two triangles or shows some understanding of the problem.
12(d)	Let $\sin\theta+\cos\theta=R\sin(\theta+\alpha)$ $=R\sin\theta\cos\alpha+R\cos\theta\sin\alpha$ $\therefore R\cos\alpha=1$ and $R\sin\alpha=1$ $R^2(\cos^2\alpha+\sin^2\alpha)=2$ and $\tan\alpha=1$ or $\alpha=\frac{\pi}{4}$ $R=\sqrt{2}$ $\sin\theta+\cos\theta=\sqrt{2}\sin(\theta+\frac{\pi}{4})=1$ $\sin(\theta+\frac{\pi}{4})=\frac{1}{\sqrt{2}}$ $\theta+\frac{\pi}{4}=\frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{9\pi}{4},...$ $\theta=0,\ \frac{\pi}{2},\ 2\pi$	3 Marks: Correct answer. 2 Marks: Finds two angles or makes significant progress towards the solution. 1 Mark: Sets up the sum of two angles or shows some understanding of the problem.
12(e) (i)	$f(x)=\sin^{-1}x+\cos^{-1}x$ $f'(x)=\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-x^2}}=0$	1 Mark: Correct answer.

12(e) (ii)	Since $f'(x)=0$, $f(x)$ is a constant (gradient of tangent is 0) Let $x=0$ then $f(0)=\sin^{-1}0+\cos^{-1}0=0+\frac{\pi}{2}=\frac{\pi}{2}$ Therefore $f(x)=\frac{\pi}{2}$ for $0\le x\le 1$ 	2 Marks: Correct answer. 1 Mark: Recognises that the graph is a horizontal line or shows some understanding of the problem.
13(a) (i)	$f(x)=xe^x-1$ $f(0)=0\times e^0-1=-1<0$ $f(1)=1\times e^1-1=e-1>0$ Therefore the root lies between $x=0$ and $x=1$.	1 Mark: Correct answer.
13(a) (ii)	$f(x)=xe^x-1$ $\quad$ $f'(x)=xe^x+e^x=e^x(x+1)$ $f(0.5)=0.5e^{0.5}-1$ $\quad$ $f'(0.5)=e^{0.5}(0.5+1)=1.5e^{0.5}$ $x_1=x_0-\frac{f(x_0)}{f'(x_0)}$ $=0.5-\left(\frac{0.5e^{0.5}-1}{1.5e^{0.5}}\right)=0.5710204398\ldots\approx 0.57$	2 Marks: Correct answer. 1 Mark: Finds $f(0.5), f'(0.5)$ or shows some understanding of Newton's method.
13(b) (i)	Horizontal Motion $\ddot{x}=0$ $\dot{x}=c_1$ (when $t=0, \dot{x}=v\cos 40^\circ$) $\dot{x}=v\cos 40^\circ$ $x=v\cos 40^\circ t+c_2$ (when $t=0, x=0$) $x=v\cos 40^\circ t$ $\quad$ (1) Vertical Motion $\ddot{y}=-10$ $\dot{y}=-10t+c_1$ (when $t=0, \dot{y}=v\sin 40^\circ$) $\dot{y}=-10t+v\sin 40^\circ$ $y=-5t^2+v\sin 40^\circ t+c_2$ (when $t=0, y=0$) $y=-5t^2+v\sin 40^\circ t$ $\quad$ (2)	3 Marks: Correct answer. 2 Marks: Derives either the horizontal or vertical equations of motion. 1 Mark: States the expressions.

13(b) (ii)	From eqn (1) $t=\frac{x}{v\cos 40^\circ}$ sub into eqn (2) $y=-5\left(\frac{x}{v\cos 40^\circ}\right)^2+v\sin 40^\circ\left(\frac{x}{v\cos 40^\circ}\right)$ $=-\frac{5x^2}{v^2}\sec^2 40^\circ+x\tan 40^\circ$	2 Marks: Correct answer. 1 Mark: Eliminates t or shows some understanding.
13(b) (iii)	To find v for $x=20$ and $y=6$ $6=-\frac{5\times 20^2}{v^2}\sec^2 40^\circ+20\times\tan 40^\circ$ $v^2=\frac{5\times 20^2\times\sec^2 40^\circ}{20\tan 40^\circ-6}$ $v=17.77917137\ldots$ $\approx 17.8\text{ ms}^{-1}$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
13(c) (i)	$(1-x)^n={}^nC_0 1^n+{}^nC_1 1^{n-1}(-x)^1+{}^nC_2 1^{n-2}(-x)^2+\ldots+{}^nC_n 1^1(-x)^n$ $={}^nC_0-{}^nC_1x^1+{}^nC_2x^2-\ldots+(-1)^n\,{}^nC_nx^n$	1 Mark: Correct answer.
13(c) (ii)	Integrate both sides of the identity $\int_0^1(1-x)^n dx=\left[{}^nC_0x-{}^nC_1\frac{x^2}{2}+{}^nC_2\frac{x^3}{3}-\ldots+(-1)^n\,{}^nC_n\frac{x^{n+1}}{n+1}\right]_0^1$ $={}^nC_0-\frac{1}{2}{}^nC_1+\frac{1}{3}{}^nC_2-\ldots+\frac{(-1)^n}{n+1}{}^nC_n$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
13(c) (iii)	$\int_0^1(1-x)^n dx={}^nC_0-\frac{1}{2}{}^nC_1+\frac{1}{3}{}^nC_2-\ldots+\frac{(-1)^n}{n+1}{}^nC_n$ $\left[\frac{(1-x)^{n+1}}{-(n+1)}\right]_0^1=\sum_{r=0}^{n}(-1)^r\frac{1}{r+1}{}^nC_r$ $-\left[0-\frac{1^{n+1}}{n+1}\right]=\sum_{r=0}^{n}(-1)^r\frac{1}{r+1}{}^nC_r$ $\frac{1}{n+1}=\sum_{r=0}^{n}(-1)^r\frac{1}{r+1}{}^nC_r$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
14(a)	$\int\cos^2 2x\,dx=\int\frac{1}{2}(1+\cos 4x)dx$ $=\frac{1}{2}\left[x+\frac{1}{4}\sin 4x\right]+c$ $=\frac{x}{2}+\frac{1}{8}\sin 4x+c$	2 Marks: Correct answer. 1 Mark: Uses double angle formula.

14(b) (i)	Simple harmonic motion occurs when $\ddot{x}=-n^2x$ Now $x=3\cos 2t+4\sin 2t$ $\dot{x}=-3\times 2\sin 2t+4\times 2\cos 2t$ $\ddot{x}=-3\times 2^2\cos 2t-4\times 2^2\sin 2t$ $=-2^2(3\cos 2t+4\sin 2t)$ $\ddot{x}=-2^2x$	2 Marks: Correct answer. 1 Mark: Recognises the condition for SHM.
14(b) (ii)	Maximum speed at $\ddot{x}=0$ or $x=0$ (centre of motion) $x=3\cos 2t+4\sin 2t=0$ $4\sin 2t=-3\cos 2t$ $\tan 2t=-\frac{3}{4}$ $2t=\tan^{-1}(-0.75)+n\pi$, where n is an integer $2t=-0.6435011088\ldots+0,\pi,2\pi$ Smallest positive value of t for maximum speed $t=\frac{1}{2}(-0.6435011088\ldots+\pi)=1.249045772\ldots$ $\dot{x}=-3\times 2\sin(2\times 1.24\ldots)+4\times 2\cos(2\times 1.24\ldots)=-10$ Maximum speed is 10 Alternatively show $\dot{x}=-10\sin(2t+\tan^{-1}0.75)$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
14(c) (i)	$T=Ae^{-kt}-12$ or $Ae^{-kt}=T+12$ $\frac{dT}{dt}=-kAe^{-kt}$ $=-k(T+12)$	1 Mark: Correct answer.
14(c) (ii)	Initially $t=0$ and $T=24$, $T=Ae^{-kt}-12$ $24=Ae^{-k\times 0}-12$ $A=36$	1 Mark: Correct answer.
14(c) (iii)	Also $t=15$ and $T=9$ $9=36e^{-k\times 15}-12$ $e^{-15k}=\frac{21}{36}=\frac{7}{12}$ $-15k=\log_e\frac{7}{12}$ $k=-\frac{1}{15}\log_e\frac{7}{12}$ $=0.03593310005\ldots$ We need to find t when $T=0$	3 Marks: Correct answer. 2 Marks: Determines the value of e^{-kt} or makes significant progress. 1 Mark: Finds the value of k or shows some understanding.

	$0=36e^{-kt}-12$ $e^{-kt}=\frac{12}{36}=\frac{1}{3}$ $-kt=\log_e\frac{1}{3}$ $t=-\frac{1}{k}\log_e\frac{1}{3}$ $=30.5738243\ldots\approx 31$ minutes It will take about 31 minutes for the water to cool to 0°C	
14(d) (i)	Facing front: Number of ways $=5\times 4\times 7!$ Facing back: Number of ways $=4\times 3\times 7!$ Total number of ways $=(5\times 4+4\times 3)\times 7!$ $=161\ 280$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
14(d) (ii)	Alex facing front and Bella facing back Number of ways $=5\times 4\times 7!$ Bella facing front and Alex facing back Number of ways $=5\times 4\times 7!$ Total number of ways $=(5\times 4\times 7!)\times 2$ $=201\ 600$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.