Sydney Secondary College
Blackwattle Bay Campus

2007
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Board approved calculators may be used Write using black or blue pen
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown in every question
- Write your student number and/or name at the top of every page

Total marks – 120

- Attempt Questions 1 – 10
- All questions are of equal value

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME:

STUDENT NAME / NUMBER.............................

Marks

Question 1 (12 marks)

(a) Evaluate $\dfrac{2.6^2 - 3.9^3}{2\times 2.6\times 3.9}$ correct to 4 significant figures. 2

(b) Factorise: $1+8x^3$ 2

(c) Find the values of x for which $|2x-1| \le 3$ 2

(d) Differentiate $4x - \cos 3x$, with respect to x. 2

(e) Solve: $\dfrac{3x-2}{3} + \dfrac{2-x}{2} = 1$. 2

(f) In a group of 50 students, 12 study Geography, 9 study History and 6 of these students study both Geography and History. What is the probability that a student selected at random from the group, studies neither History nor Geography? 2

Marks

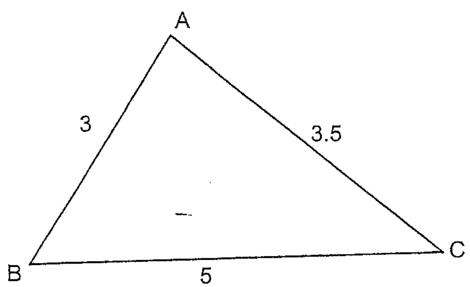
Question 2 (12 marks) *Start a new page*

(a) Differentiate with respect to x.

(i) $\dfrac{x}{\ln x}$. **2**

(ii) $(1+\tan x)^5$. **2**

(b)



In the diagram above AB = 3m, BC = 5m and AC = 3.5m. Find the size of the largest angle, correct to the nearest minute. **3**

(c) (i) Evaluate: $\displaystyle\int_0^2 \frac{6x^2}{1+x^3}dx$. **3**

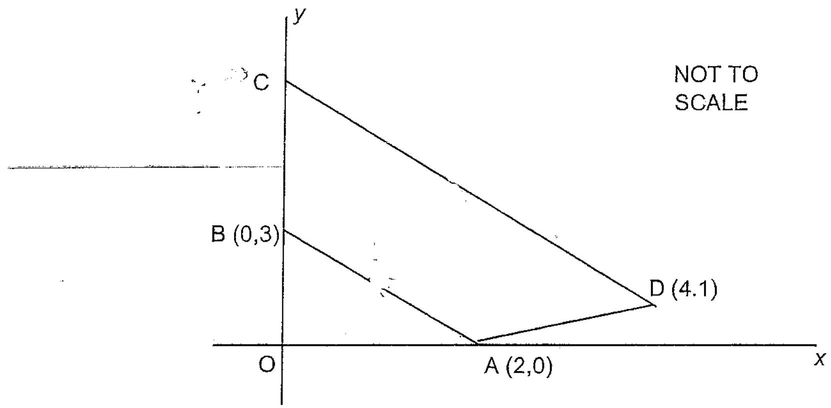
(ii) Find: $\displaystyle\int \left(1+e^{3x}\right)dx$. **2**

STUDENT NAME / NUMBER ....

Marks

Question 3 (12 marks) *Start a new page*

(a)



In the diagram above, the coordinates of A, B and D are (2,0), (0,3) and (4,1) respectively. Point C lies on the y-axis such that AB is parallel to DC. Copy or trace the diagram onto your worksheet.

(i) What type of quadrilateral is ABCD? **1**

(ii) Write down the gradient of AB. **1**

(iii) Show that the equation of DC is $3x+2y-14=0$. **2**

(iv) Find the coordinates of point C. **1**

(v) Using Pythagoras' Theorem or otherwise, show that the length of AB = $\sqrt{13}$ units. **1**

(vi) Find the length of CD. **1**

(vii) Find the perpendicular distance from A to DC. **1**

(viii) Hence or otherwise, find the area of quadrilateral ABDC. **1**

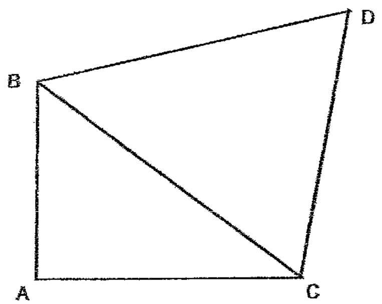
(b) Find the equation of the tangent to the curve $y=e^{2x}+x$ at the point with x-coordinate 0. **3**

STUDENT NAME / NUMBER ....

Marks

Question 4 (12 marks) *Start a new page*

(a)

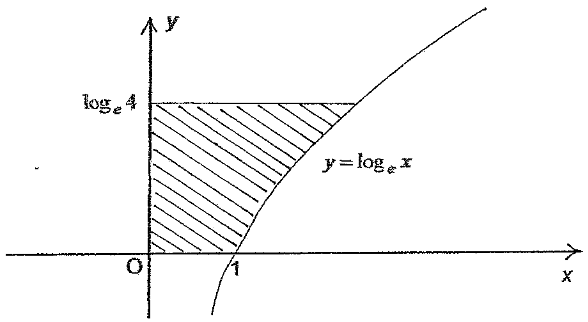


In the diagram above, ABC is an isosceles triangle in which $\angle BAC = 90°$.
BCD is an equilateral triangle.
Copy or trace the diagram onto your worksheet.

(i) Find the size of $\angle ACD$ giving reasons. **2**

(ii) If $BC = 3$ centimetres, find the perimeter of ABDC, giving reasons. **2**

(b) In the diagram below, the shaded region bounded by the curve $y = \log_e x$, the x and y-axes and the line $y = \log_e 4$ is rotated about the y-axis. Find the exact volume of the solid of revolution formed. **3**



(c) The geometric series $1 - x + x^2 - \dots$ has a limiting sum of 4. Find the value of x. **3**

(d) Find the coordinates of the focus of the parabola $(x-2)^2 = 4y$ **2**

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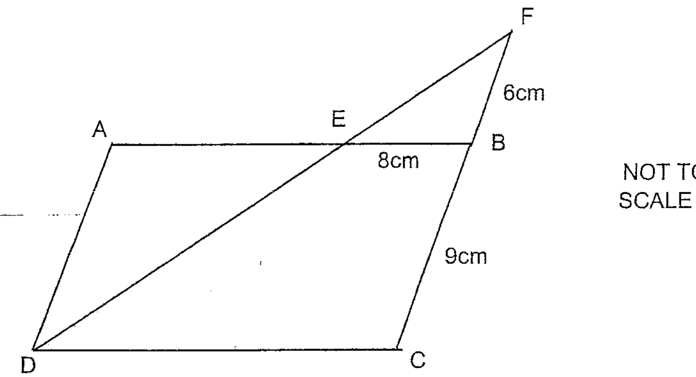
Marks

Question 5 (12 marks) *Start a new page*

(a)



In the diagram above ABCD is a parallelogram. F lies on CB produced such that BF = 6 cm. AB and DF intersect at E. EB = 8 cm and BC = 9 cm.
Copy or trace the diagram onto your worksheet.

(i) Prove that ΔAED is similar to ΔBEF. **3**

(ii) Find the lengths of AE and DC. **2**

(b) Farmer Brown has hired a driller to drill a borehole to enable her to have access to the underground water on her property. The driller quotes a price of \$260 for the first 3 metres drilled, \$280 for the next 2 metres, \$300 for the next 2 metres and so on. The price increases by the same amount for each successive 2 metres of borehole drilled.

(i) Show that the cost of drilling the portion from a depth of 25 metres to 27 metres is \$500. **1**

(ii) Calculate the total cost of drilling to a depth of 27 metres. **1**

(iii) The cost of drilling the borehole to reach water was \$12500. Find the total depth drilled to give access to the water. **2**

(c) Use Simpson's Rule with five function values to find an approximation for the value of $\int_0^1 10^x \, dx$. Give your answer correct to three decimal places. **3**

Marks

Question 6 (12 marks) *Start a new page*

(a) A function $f(x)$ is defined by $f(x)=x\left(x^2-3x-9\right)$.

(i) Find the turning points for the curve $y=f(x)$ and determine their nature. **3**

(ii) Find the coordinates of the point of inflexion. **1**

(iii) Sketch the graph of $y=f(x)$ showing the turning points and point of inflexion. **2**

(iv) Find the values of x for which both $f'(x)<0$ and $f''(x)>0$. **2**

(b) The quadratic equation $2x^2-3x+6=0$ has roots α and β. Find the value of:

(i) $\alpha+\beta$ **1**

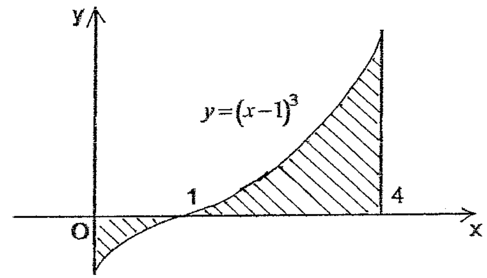
(ii) $\alpha\beta$ **1**

(iii) $\alpha^2+\beta^2$ **2**

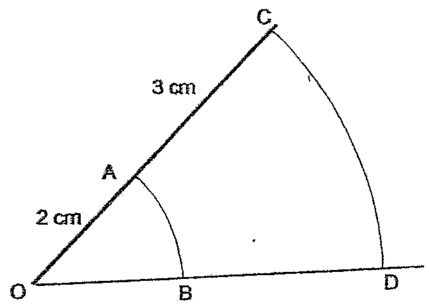
Marks

Question 7 (12 marks) *Start a new page*

(a) The shaded area in the diagram below is the region bounded by the curve $y=(x-1)^3$, the x and y-axes and the line $x=4$. Calculate the shaded area. **4**



(b)



The arcs AB and CD are parts of concentric circles with centre O.
OA = 2 centimetres and AC = 3 centimetres.

(i) Show that the ratio of length of arc AB : length of arc CD = 2 : 5. **1**

(ii) Find the ratio of the area of sector AOB : the area of ABDC. **2**

(c) The probability that the school bus runs late on any particular day is 1 in 8. Find the probability that on three successive days, the bus is:

(i) late on all three days. **1**

(ii) late on exactly two days. **1**

(iii) late on at least two days. **2**

(iv) on time on all three days. **1**

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Marks

Question 8 (12 marks) *Start a new page*

(a) A particle is moving in a straight line and its velocity v metres/second at time t seconds is given by;

$$v = \frac{dx}{dt} = 1 - 2\sin 2t,\ t \geq 0$$

Initially the particle is at the origin.

(i) Express the displacement x, as a function of t. **2**

(ii) Find the position of the particle when $t = \frac{\pi}{6}$. **1**

(iii) Find an expression for the acceleration $a = \frac{d^2x}{dt^2}$. **1**

(iv) Sketch the graph of the acceleration as a function of time, $0 \leq t \leq \pi$. **2**

(v) What is the maximum acceleration of the particle? **1**

(b) For what values of k does the quadratic equation $x^2 - (k+3)x + 4k = 0$

(i) have one root equal to -3? **1**

(iii) have no real roots? **2**

(c) Solve: $2\cos A + \sqrt{3} = 0,\ \ 0 \leq A \leq 2\pi$. **2**

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Marks

Question 9 (12 marks) *Start a new page*

(a) Evaluate: $\sum_{n=2}^{4} 3^{-n}$ (Give your answer in exact form). **1**

(b) Pauline wishes to invest in a superannuation fund. She decides to invest \$2400 in the fund at the beginning of each year. The fund is paying interest at 9% per annum, compounded annually.

(i) Show that the value of the first \$2400 invested when she retires after working for 30 years will be $\$2400(1.09)^{30}$ **1**

(ii) Write down similar expressions for the values of the second and third \$2400 amounts invested, at the end of the thirty year period. **2**

(iii) Calculate the total value of her investment when she reaches retirement. **2**

(c) An industrial plant produces vacuum cleaners. The annual production, P cleaners, at time t years, is given by:

$$P = P_0 e^{kt} \text{ where } P_0 \text{ and } k \text{ are constants.}$$

Initially the production of the plant was 2500 cleaners per annum. Five years later it had increased to 4000 cleaners per annum.

(i) Find the values of P_0 and k. **2**

(ii) What is the predicted production after 10 years? **1**

(iv) How many years will it take for the production to double its original output? **2**

(v) Find the rate of increase in production when the plant has been operating for 5 years. **1**

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Marks

Question 10 (12 marks) ***Start a new page***

(a) Simplify: $\dfrac{\operatorname{cosec} A \sec A}{\tan A}$, expressing your answer in simplest possible form. **3**

(b) The rate of fall in the price $\$D$, of shares in a company is given by:

$$\frac{dD}{dt} = -\frac{16}{t^3},$$

where $t > 0$ is the time in months after the shares were put on the market.

One month after going on the market, the shares were selling for \$24 each

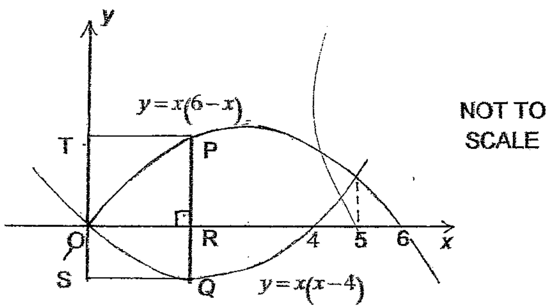
(i) Show that the price of the shares is given by: $D = \dfrac{8}{t^2} + 16$. **2**

(ii) Find the value of the shares after they have been on the market for 2 months. **1**

(iii) Find the rate at which the share price is falling after 3 months. **1**

(iv) Show that the price of the shares will not fall below a certain amount. Give this amount. **1**

(c) In the diagram below, P is a point on the curve $y = x(6-x)$ and Q is a point on the curve $y = x(x-4)$. PQ cuts the x-axis at rightangles at R.
S and T are points on the y-axis such that PQST is a rectangle.



(i) Show that the length of PQ is given by $10x - 2x^2$. **1**

(ii) Find an expression for the area of PQST as a function of x. **1**

(iii) Find the value of x which gives the maximum area for PQST $(0 \le x \le 5)$ **2**

End of paper

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QU.1 (12 marks)

(a) -2.591666667 (calcul) ①

$= -2.592$ (4 s.f.) ①

(b) $1+8x^3 = 1^3 + (2x)^3$ ①

$= (1+2x)(1-2x+4x^2)$ ①

(c) $|2x-1| \le 3$

"Geometric" method

$-3 \le 2x-1 \le 3$ (if $|x| \le a$ then $-a \le x \le a$)

$-2 \le 2x \le 4$

$-1 \le x \le 2$ ①

① for working

"cases" method

$2x-1 \le 3$ (case 1)

$2x \le 4$

$x \le 2$ (i)

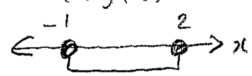
$-(2x-1) \le 3$ (case 2)

$2x-1 \ge -3$

$2x \ge -2$

$x \ge -1$ (ii)

Combine (i) & (ii):



or $-1 \le x \le 2$ ①

(d) $\frac{d}{dx}\left(4x - \cos(3x)\right)$

$= 4 - [-\sin(3x) \times 3]$ ①

$= 4 - [-3\sin(3x)]$

$= 4 + 3\sin(3x)$ ①

(e) $\frac{3x-2}{3} + \frac{2-x}{2} = 1$

$6 \times \frac{(3x-2)}{3} + 6 \times \frac{(2-x)}{2} = 6 \times 1$

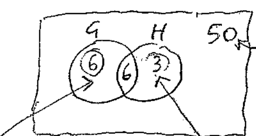
$2(3x-2) + 3(2-x) = 6$ ① for working

$6x - 4 + 6 - 3x = 6$

$3x = 4$

$x = \frac{4}{3}$ ①

(f)



Total

① for working

Geography only $= 12-6$ $= 6$

History only $= 9-6$ $= 3$

Total for G, H and both $= 15$

$P(\text{neither G or H}) = \frac{35}{50}$

$= \frac{7}{10}$ ①

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QU.2 (12 marks)

(a) (i) $\frac{d}{dx}\left(\frac{x}{\ln x}\right) = \frac{1.\ln x - x.\frac{1}{x}}{[\ln x]^2}$ ① quotient rule

$= \frac{\ln x - 1}{\ln^2 x}$ ①

or $= \frac{-1 + \ln x}{\ln^2 x}$

or $= \frac{-1 + \log_e x}{\log_e^2 x}$

either

(ii) $\frac{d}{dx}(1+\tan x)^5 = 5(1+\tan x)^4 (\sec^2 x)$ ② using power and chain rule

$= \frac{5(1+\tan x)^4}{\cos^2 x}$

either

(b) Largest angle is opposite largest side (ie. $B\hat{A}C$)

cosine rule → $5^2 = 3^2 + 3.5^2 - 2 \times 3 \times 3.5 \times \cos B\hat{A}C$ ①

$5^2 - 3^2 - 3.5^2 = -2 \times 3 \times 3.5 \times \cos B\hat{A}C$

$\cos B\hat{A}C = \left[\frac{5^2 - 3^2 - 3.5^2}{-2 \times 3 \times 3.5}\right]$ ① for working

$= -0.178571428$

$\therefore B\hat{A}C = 100°17'$ ①

(c) (i) $\int_0^2 \frac{6x^2}{1+x^3}\,dx = 2\int_0^2 \frac{3x^2}{1+x^3}\,dx = \left[2\log_e(1+x^3)\right]_0^2$ ①

$= 2\log_e 9 - 2\log_e 1$ ①

$= 2\log_e 9$ ①

(ii) $\int (1+e^{3x})\,dx = x + \frac{1}{3}e^{3x} + c$ ②

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QU. 3 (12 Marks)

(a) (i) Trapezium ①

(ii) $M_{AB} = \frac{3-0}{0-2}$

$= -\frac{3}{2}$ ①

(iii) $AB \parallel DC$ (data)

$\therefore M_{DC} = M_{AB}$

$= -\frac{3}{2}$

$D(4,1)$ lies on DC (data)

DC: $(y-y_1) = m(x-x_1)$

Sub $(4,1)$ and 'M': $(y-1) = -\frac{3}{2}(x-4)$ ① for working

$2y - 2 = -3x + 12$

$3x + 2y - 14 = 0$ ①

(iv) 'C' lies on 'y' axis

i.e. where $x = 0$

$\therefore$ $3(0) + 2y - 14 = 0$

$2y = 14$

$y = 7$

$\therefore C(0,7)$ ①

(v) $AB^2 = 3^2 + 2^2$ (Pythagoras)

$= 9 + 4$ ①

$\therefore AB = \sqrt{13}$

(vi) $CD^2 = (7-1)^2 + (0-4)^2$

$= 36 + 16$

$= 52$ ① either

$CD = \sqrt{52} = 2\sqrt{13}$

(vii) $d = \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$

$= \frac{|3(2)+2(0)-14|}{\sqrt{9+4}}$

$= \frac{|-8|}{\sqrt{13}}$

$= \frac{8}{\sqrt{13}}$ ①

(viii) $A = \frac{1}{2}h(x+y)$

$= \frac{1}{2}\,\frac{8}{\sqrt{13}}\left(2\sqrt{13} + \sqrt{13}\right)$

$= \frac{4}{\sqrt{13}}\left(3\sqrt{13}\right)$

$= 12$ units^2 ①

(b) $y = e^{2x} + x$

$y' = 2e^{2x} + 1$

at $x = 0$: $y = e^0 + 0$

$= 1$

$y' = 2e^0 + 1$

$= 3$

Using $(y-y_1) = m(x-x_1)$

$(y-1) = 3(x-0)$

$y - 1 = 3x$

$y = 3x + 1$

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QU. 4 (12 marks)

(a)

(i) $B\hat{A}C = 90°$ (data)

$A\hat{B}C = A\hat{C}B = 45°$ (base ∠'s of isosceles $\triangle ABC$)

$B\hat{C}D = 60°$ (∠'s in equilateral $\triangle BCD$) ① working

$A\hat{C}D = A\hat{C}B + B\hat{C}D$

$= 45° + 60°$ ①

$= 105°$

(ii) $BC = 3$cm (data)

$BD = 3$cm (sides of equilateral $\triangle BCD$)

$DC = 3$cm (as above)

In $\triangle ABC$: $AB = AC$ (sides of isosceles $\triangle$)

$= x$ (say)

$\therefore$ $x^2 + x^2 = 3^2$ (Pythagoras) ① for working

$2x^2 = 9$

$x^2 = \frac{9}{2}$

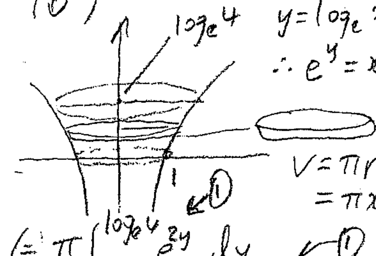
$x = \frac{3}{\sqrt{2}}$

$\therefore$ Perimeter $= \frac{3}{\sqrt{2}} + \frac{3}{\sqrt{2}} + 3 + 3$

$= \frac{6}{\sqrt{2}} + 6$

① $= 3\sqrt{2} + 6$

(b)



$y = \log_e x$

$\therefore e^y = x$

$V = \pi r^2 h$

$= \pi x^2\, dy$ ①

$V = \pi\int_0^{\log_e 4} e^{2y}\, dy$ ①

$= \left[\frac{\pi}{2} e^{2y}\right]_0^{\log_e 4} = (\ldots) - (\ldots) = \frac{15\pi}{\ldots}$ units^3 ①

(c) $S_\infty = \frac{1}{a-r}$ ① $\therefore$ $4 = \frac{1}{1+x}$ ① working

$r = \frac{-x}{1} = -x$

$a = 1$

$4 + 4x = 1$

$4x = -3$

$x = -\frac{3}{4}$ ①

(d) $(x-h)^2 = 4a(y-0)$

$(h,k) = (2,0)$

$\therefore a = 1$ $\therefore$ focus $(2,1)$

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QU.5

(a)

(i) In Δ's AED, BEF:

1. $A\hat{E}D = F\hat{E}B$ (vertically opposite ∠'s) 1
2. $D\hat{A}E = D\hat{C}B$ (opposite ∠'s parallelogram) 1
   $= E\hat{B}F$ (corresponding ∠'s; $AB \parallel DC$ - opposite sides parallelogram)
3. $A\hat{D}E = E\hat{F}B$ (∠ sum of Δ)

∴ $\Delta AED \,|||\, \Delta BEF$ (equiangular) 1

(ii) $\frac{AE}{EB} = \frac{AD}{FB}$ (corresponding sides in similar Δ's)

1 $\frac{AE}{8} = \frac{9}{6}$ (AD = BC opposite sides parallelogram)

∴ AE = 12 cm

also DC = AB (opposite sides parallelogram)

1 = AE + EB
= 12 + 8 (EB = 8 data)
= 20 cm

(b) Cost: 260, 260+20, 260+2×20, … 260+20(n−1)

Depth: 3, 3+2, 3+2×2, … 3+(n−1)2 = 3+2n−2 = 2n+1

(i) 27 = 2n+1 (depth) 260+20(12)
n = 13 = $500 (cost) 1

(ii) $S_{13} = \frac{13}{2}(2\times260+(12)(20)) = \$4940$ 1

(iii) $12500 = \frac{n}{2}(2\times260+(n-1)(20))$ $n^2+25n-1250=0$ 1 for working
n = 25 1 ∴ depth = 2×25+1 = 51 m

(c)

x	0	0.25	0.50	0.75	1
10^x	1	1.7783	3.1623	5.6234	10

1

$\int_0^1 10^x\,dx = \frac{h}{3}(y_0+4y_1+2y_2+4y_3+y_4)$

1 $= \frac{0.25}{3}(1+4(1.7783)+2(3.1623)+4(5.62…+10)$

QU.6 (12 marks)

(a) $f(x) = x(x^2-3x-9)$

(i) At turning points $f'(x)=0$

$f(x) = x(x^2-3x-9)$
$= x^3-3x^2-9x$

$f'(x) = 3x^2-6x-9$
$= 3(x^2-2x-3)$
$= 3(x-3)(x+1)$ 1 for working

$f'(x)=0$ at $x=3$, $x=-1$

$f(3) = 3(9-9-9) = -27$ $f(-1) = -1(1+3-9) = 5$

Turning points: (3, −27) and (−1, 5)

Nature: $f''(x) = 6x-6$

(3, −27): $f''(3) = 18-6 = 12 > 0$
∴ minimum turning point at (3, −27) 1

(−1, 5): $f''(-1) = 6(-1)-6 = -12 < 0$
∴ maximum turning point at (−1, 5) 1

(ii) Inflexion: $f''(x)=0$; $6x-6=0$ $x=1$

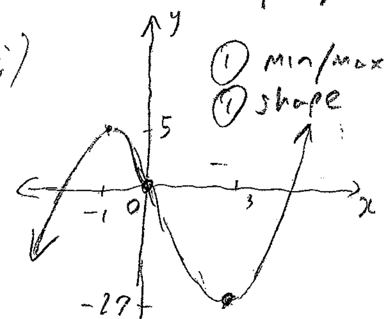
$f(1) = 1(1-3-9) = -11$

Test if (1, −11) is an inflexion point

x	0	1	2
$f''(x)$	−6	0	6

since $f''(x)$ changes sign 1
(1, −11) is an inflexion point

(iii) 1 min/max 1 shape



Note: f(0) = 0

(iv) $f'(x)<0$ and $f''(x)>0$
when $3(x-3)(x+1)<0$; $6x-6>0$
1 working $-1<x<3$; $x>1$
combine: $1<x<3$ 1

(b) $2x^2-3x+6=0$
a = 2 b = −3 c = 6

$\alpha+\beta = \frac{-b}{a} = \frac{3}{2}$ 1

$\alpha\beta = \frac{c}{a} = \frac{6}{2} = 3$ 1

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ 1 either
$= \frac{9}{4} - 6$
$= -15$

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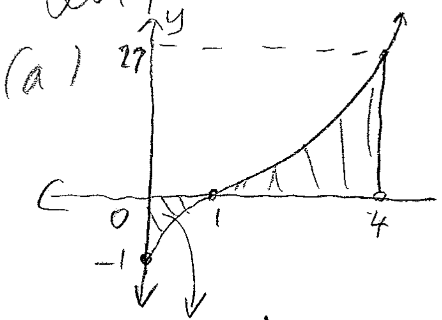
QU.7 (12 marks)

(a)



$y=(x-1)^3$

at $x=0$, $y=-1$

at $x=4$, $y=27$

at $x=1$, $y=0$

$A=\left|\int_0^1 (x-1)^3\,dx\right|+\int_1^4 (x-1)^3\,dx$

$=\left|\left[\frac{(x-1)^4}{4}\right]_0^1\right|+\left[\frac{(x-1)^4}{4}\right]_1^4$

$=\left|\left(0-\frac{1}{4}\right)\right|+\left(\frac{81}{4}-0\right)$

$=\frac{1}{4}+\frac{81}{4}$

$=\frac{41}{2}$ units^2

(b) (i) Arc AB: $l_1=r\theta$
$=2\theta$

Arc CD: $l_2=R\theta$
$=5\theta$

where $\theta=C\hat{O}D$

$\therefore l_1:l_2=2\theta:5\theta$
$=2:5$

(ii) Area AOB $=\frac{\theta}{360}\times\pi(2)^2$
$=\frac{\pi\theta}{90}$

Area COD $=\frac{\theta}{360}\times\pi(5)^2$
$=\frac{25\pi\theta}{360}$

Area ABDC = Area COD − Area AOB

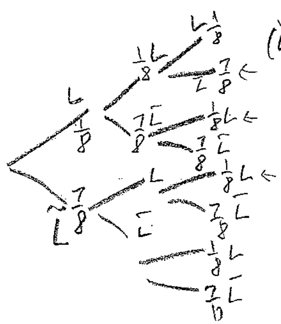
$=\frac{25\pi\theta}{360}-\frac{\pi\theta}{90}=\frac{25\pi\theta-4\pi\theta}{360}=\frac{21\pi\theta}{360}$

$\therefore$ Area AOB : Area ABDC $=\frac{\pi\theta}{90}:\frac{21\pi\theta}{360}=4:21$

(c)

L: Late

$\tilde{L}$: not late



(i) $P(LLL)=\frac{1}{8}\times\frac{1}{8}\times\frac{1}{8}$
$=\frac{1}{512}$

(ii) $P(LL\tilde{L})+P(L\tilde{L}L)+P(\tilde{L}LL)=\left(\frac{7}{8}\times\frac{1}{8}\times\frac{1}{8}\right)\times 3$
$=\frac{21}{512}$

(iii) $\frac{21}{512}+P(LLL)$
$=\frac{21}{512}+\frac{1}{512}$
$=\frac{22}{512}=\frac{11}{256}$

(iv) $P(\tilde{L}\tilde{L}\tilde{L})$
$=\left(\frac{7}{8}\right)^3=\frac{343}{512}$

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QU.8 (12 marks)

(a) $v=\frac{dx}{dt}=1-2\sin 2t,\ t\geq 0$

$t=0,\ x=0$ (data)

(i) $x=\int 1-2\sin 2t\,dt$
$=t+\cos 2t+c$ 1 working

$t=0\ x=0$

$\therefore 0=0+1+c$
$c=-1$

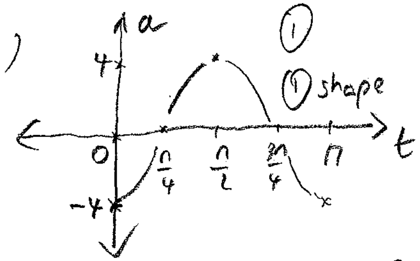
$\therefore x=t-1+\cos 2t$ 1

(ii) $t=\frac{\pi}{6}$

$x=\frac{\pi}{6}-1+\cos\left(\frac{\pi}{3}\right)$
$=\frac{\pi}{6}-1+\frac{1}{2}$
$=\frac{\pi}{6}-\frac{1}{2}$ 1

(iii) $\frac{d^2x}{dt^2}=-4\cos 2t$ 1

(iv)



(v) From the graph $a=4$ 1

(b) $x^2-(k+3)x+4k=0$

$a=1\ b=-(k+3)\ c=4k$

(i) If $x=-3$ (root)

then $(-3)^2-(k+3)(-3)+4k=0$
$9+3k+9+4k=0$
$7k=-18$
$k=-\frac{18}{7}$ 1

(ii) $b^2-4ac<0$ (unreal roots)

$\therefore\left[-(k+3)\right]^2-4(1)(4k)<0$ 1

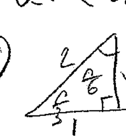
$k^2+6k+9-16k<0$
$k^2-10k+9<0$
$(k-9)(k-1)<0$

$\therefore 1<k<9$ 1

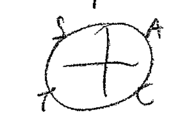
(c) $2\cos A+\sqrt{3}=0\quad 0\leq A\leq 2\pi$

$2\cos A=-\sqrt{3}$ 1

$\cos A=-\frac{\sqrt{3}}{2}$



$\therefore A=\frac{5\pi}{6},\frac{7\pi}{6}$ 1



QU.9

(a) $\sum_{n=2}^{4} 3^{-n} = \sum_{n=2}^{4} \frac{1}{3^n} = \frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}$

$= \frac{3^2+3+1}{3^4}$

$= \frac{13}{81}$ 1

(b)

n	P	r	A
1	2400	0.09	$A_1 = 2400 + 0.09(2400)$ $= 2400(1+0.09)$ $= 2400(1.09)^1$
2	A_1	0.09	$A_2 = A_1 + 0.09A_1$ $= A_1(1+0.09)$ $= A_1(1.09)$ $= 2400(1.09)(1.09)$ $= 2400(1.09)^2$ 1
⋮	⋮	⋮	⋮
30	A_{29}	0.09	$A_{30} = 2400(1.09)^{30}$

(ii) The second '\$2400' will amount to $A_{29} = 2400(1.09)^{29}$ 1

The third '\$2400' " " " $A_{28} = 2400(1.09)^{28}$ 1

⋮

30th year : The last '\$2400' " " " $A_1 = 2400(1.09)^1$

(iii) $\therefore$ Total $= 2400(1.09)^{30} + 2400(1.09)^{29} + 2400(1.09)^{28} + \dots + 2400(1.09)^1$ 1

This is a G.P. with
$a = 2400(1.09)$
$r = 1.09$; $n = 30$

$S = \frac{a(r^n-1)}{r-1} = \frac{2400(1.09)(1.09^{30}-1)}{1.09-1}$

$= \$356\,580.52$ 1

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QU.9 - continued

(c) $P = P_0 e^{kt}$ (data)

$t=0$, $P = 2500$ (data)

$t=5$, $P = 4000$ (data)

(i) at $t=0$ $P = P_0 e^0$
$= P_0$

but $P = 2500$ at $t=0$ (data)

$\therefore P = P_0$
$= 2500$

at $t=5$, $P = 4000$ (data)

$\therefore 4000 = 2500e^{5k}$

$e^{5k} = \frac{4000}{2500}$
$= \frac{8}{5}$

$\log_e(e^{5k}) = \log_e(\frac{8}{5})$

$5k = \log_e(\frac{8}{5})$

$\therefore k = \frac{1}{5}\log_e(\frac{8}{5})$ (exact)

$= 0.094000725$

$= 0.094$ (3 d.p.)

or $k = \log_e(\frac{8}{5})^{\frac{1}{5}}$

(ii) at $t=10$

$P = 2500e^{10k}$

$= 2500e^{\log_e[\frac{8}{5}]^2}$

$= 2500 \times (\frac{8}{5})^2$

$= 2500 \times \frac{64}{25}$ (100)

$= 6400$

use this result

$k = \frac{1}{5}\log_e(\frac{8}{5})$
$10k = 2\log_e(\frac{8}{5})$
$= \log_e(\frac{8}{5})^2$

Also
if $y = \log_e x$
then $e^y = x$
$\therefore x = e^y = e^{\log_e x}$

(iii) We want $2P_0 = P_0 e^{kt}$
or $2 = e^{kt}$
or $kt = \log_e 2$
$t = \frac{1}{k}\log_e 2$
$= 7.373849237$
$\doteqdot 7.4$ years

9(c)
(iv)
$P = 2500e^{kt}$
$\frac{dP}{dt} = 2500ke^{kt}$
at $t=5$
$\frac{dP}{dt} = 2500ke^{5k}$
$= 2500 \times \log_e(\frac{8}{5})^{\frac{1}{5}} \times e^{5 \cdot \log_e(\frac{8}{5})^{\frac{1}{5}}}$
$= 2500\log_e(\frac{8}{5})^{\frac{1}{5}} \cdot (\frac{8}{5})$
$= 4000\log_e(\frac{8}{5})^{\frac{1}{5}}$
$\doteqdot 376$

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QU.10 (12 marks)

(a) $\dfrac{\operatorname{cosec} A \cdot \sec A}{\tan A} = \dfrac{\frac{1}{\sin A} \cdot \frac{1}{\cos A}}{\frac{\sin A}{\cos A}} \left\{ \times \dfrac{\cos A}{\sin A} \right.$ ①

$= \dfrac{1}{\sin^2 A}$ ①

$= \operatorname{cosec}^2 A$ ①

(b) (i) $\dfrac{dD}{dt} = -\dfrac{16}{t^3}$

$\therefore \displaystyle\int \frac{dD}{dt}\,dt = \int -\frac{16}{t^3}\,dt$ ① for working

$D = -16\displaystyle\int t^{-3}\,dt$

$= -16\left[\dfrac{t^{-2}}{-2}\right] + k$ { k - constant of integration

$D = 8t^{-2} + k$

when $t = 1$ $D = 24$ (Data).

$\therefore 24 = 8(1)^{-2} + k$

$k = 16$

$\therefore D = 8t^{-2} + 16$

or $D = \dfrac{8}{t^2} + 16$ ①

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QU.10 (12 marks) - continued

(b) (ii) at $t = 2$

$D = \dfrac{8}{2^2} + 16$

$= \$18$ ①

(iii) at $t = 3$

$\dfrac{dD}{dt} = -\dfrac{16}{3^3}$

$= -\dfrac{16}{27}$

$\doteqdot -0.59$

('falling' by 59¢ per month)

(iv) $D = 16 + \dfrac{8}{t^2}$ (price of shares)

We note that after 1 month $D = \$24$

after 2 months $D = \$18$

or As t (months) increases D decreases ①

but as 't' increases without bound, $\dfrac{8}{t^2}$ tends toward a value of zero

$\therefore$ D will tend toward a value of \$16 and never fall below this amount

(c)

i) Let $y_1 = x(6-x)$

$y_2 = x(x-4)$

PQ = vertical distance

$= y_1 - y_2$

$= x(6-x) - x(x-4)$

$= 6x - x^2 - x^2 + 4x$

$= 10x - 2x^2$

①

(ii) Area $= OR \times PQ$

$= x(10x - 2x^2)$

$= 10x^2 - 2x^3$

①

(iii) $A = 10x^2 - 2x^3$

$\dfrac{dA}{dx} = 20x - 6x^2$

For max $\dfrac{dA}{dx} = 0$

① for working

$\therefore 20x - 6x^2 = 0$

$2x(10 - 3x) = 0$

$\therefore x = 0$ or $x = \dfrac{10}{3}$

We take $x = \dfrac{10}{3}$ and test if maximum

$\dfrac{d^2A}{dx^2} = 20 - 12x$

at $x = \dfrac{10}{3}$ $\dfrac{d^2A}{dx^2} = -20 < 0$

① $\therefore x = \dfrac{10}{3}$ gives max area