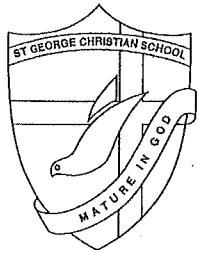


Name : \_\_\_\_\_



**Year 11  
Semester 1 Examination  
2014**

# Mathematics

**General Instructions**

- Reading Time - 5 minutes
- Working Time - 2 hours
- Write using a blue or black pen
- Approved calculators may be used
- All necessary working should be shown
- Marks will be deducted for careless or untidy work
- This exam question paper must be submitted at the end of this examination

**Total marks (65)**

**SECTION I (5 Marks)**

- Answer on the Answer sheet provided
- Allow about 10 minutes for this section

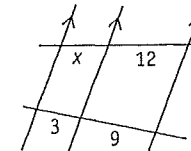
**SECTION II (60 Marks)**

- Begin each question in a new booklet
- Allow about 1 hour 50 minutes for this section

**Section I**

Answer this section on the Multiple Choice answer sheet provided

1. Evaluate  $125^{\frac{1}{3}}$  correct to 3 significant figures  
 A. 2.627      B. 2.63      C. 11.2      D. 25
2. Write  $0.3\bar{6}$  as a fraction in simplest form  
 A.  $\frac{4}{11}$       B.  $\frac{9}{25}$       C.  $\frac{11}{30}$       D.  $\frac{183}{500}$
3. Evaluate  $12 - |4 - 10|$   
 A. -2      B. 6      C. 8      D. 18
4. In the diagram, x equals



- A. 3      B. 4      C. 5      D. 6
5. What is the reciprocal of  $x + \frac{1}{x}$  ?  
 A.  $\frac{1}{x} + x$       B.  $\frac{x+1}{x}$       C.  $\frac{x}{x+1}$       D.  $\frac{x}{x^2+1}$

End of Section I

For Teacher use only:

Section I	Section II	Q6	Q7	Q8	Q9	Q10	Total	%
5		12	12	12	12	12	65	

**Section II**

**QUESTION 6 (12 Marks)**

	Marks
(a) Find $\sqrt{\frac{8.2-6.3}{4.5 \times 3.4}}$ correct to 2 decimal places.	1
(b) Find numbers $a$ and $b$ such that $\frac{6}{5-\sqrt{3}} = a + b\sqrt{3}$	3
(c) Solve the following equations:	
(i) $15x - 5 = 4(x + 3) - x$	2
(ii) $\frac{5a}{6} = \frac{3a-7}{4}$	2
(d) Expand and simplify $6x^3 - 2x^3(5-x)$	2
(e) Simplify: $\frac{x+7}{3} + \frac{2x-1}{2}$	2

**END OF QUESTION 6**

**QUESTION 7 (12 Marks) Start a separate booklet**

	Marks
(a) Factorise fully:	
(i) $xa + 3x - 2a - 6$	2
(ii) $3x^2 - 27$	1
(iii) $2 + x - 10x^2$	2
(b) The mass of 1 atom of oxygen is $2.7 \times 10^{-23}$ grams. What is the mass of $8 \times 10^{29}$ atoms of oxygen? Give your answer in scientific notation.	1
(c) The sides of a right angled triangle are $(x+1)cm$ , $(x+3)cm$ and $(x+5)cm$ . Find the length of each side.	3
(d) Solve $ 8y-9 =5y-6$	3

**END OF QUESTION 7**

QUESTION 8 (12 Marks) Start a separate booklet

Marks

(a) Solve the following equations and graph the solution on the number line.

(i)  $1 + 2x < 5$  2

(ii)  $|4p - 2| \leq 10$  3

(b) Use the quadratic formula to solve  $4x^2 - 2x - 3 = 0$  expressing the answer in surd form. 2

(c) Given the temperature conversion formula  $C = \frac{5}{9}(F - 32)$ , find  $F$  given  $C = 40$ . 2

(d) Solve simultaneously:

$$\begin{aligned} a^2 - b^2 &= 25 \\ a + b &= 3 \end{aligned}$$
3

END OF QUESTION 8

QUESTION 9 (12 Marks) Start a separate booklet

Marks

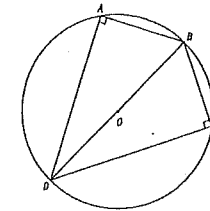
(a) Solve  $4^{2x+1} = 8$  2

(b) The length of a closed rectangular box is three times its width. The volume of the box is to be  $288\text{cm}^3$ . Let the width of the box be  $x$  cm and let the height of the box be  $y$  cm. Show that the surface area ( $S$ ) of the box is given by

$$S = 6x^2 + \frac{768}{x}$$
3

(c) For a regular octagon,  
find (i) the sum of the interior angles 1  
(ii) the size of each interior angle. 1

(d) The centre of the circle is marked (O).  $AB = BC$  and  $\angle BAD = \angle BCD = 90^\circ$



(i) Prove that  $\triangle ABD \cong \triangle BCD$  3

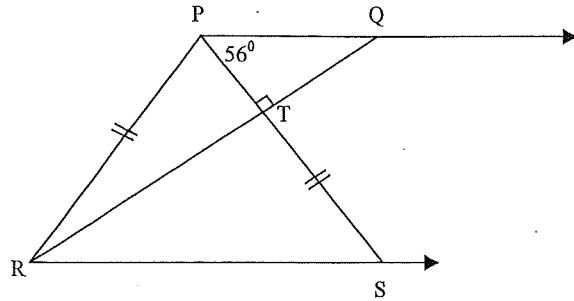
(ii) Hence show that  $AD = CD$  1

(e) What is the supplement of the complement of  $70^\circ$  1

END OF QUESTION 9

QUESTION 10 (12 Marks) Start a separate booklet.

(a)

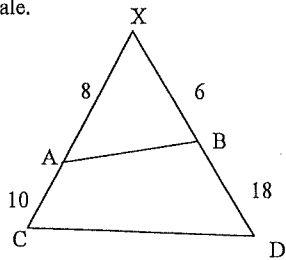


In the diagram above,  $PQ \parallel RS$ ,  $PR = PS$ ,  $\angle QPS = 56^\circ$  and  $PT \perp RQ$

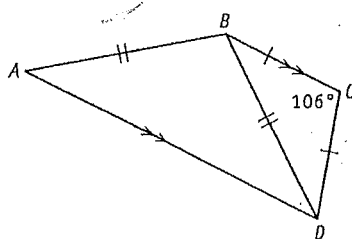
- (i) Find  $\angle PRS$ . Give reasons. 2
- (ii) Show that  $\angle PRQ = 22^\circ$  2

(b) In the figure below, prove  $\triangle XAB \parallel \triangle XDC$ . 3

The figure is not to scale.



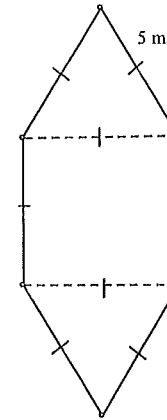
(c) In the diagram  $BC \parallel AD$ ,  $BC = CD$  and  $AB = BD$ . Find the size of  $\angle BAD$ . Give reasons. 3



Marks

(d) Find the area of the shape below

2



END OF EXAMINATION

YEAR 11 (2014) 2U MATHEMATICS 51 EXAM SOLUTIONS

125  $\div 2 \cdot 6265 \dots$   
 $\div 2 \cdot 63$  (55 f.) B

Let  $x = 0.36$

$10x = 3.66666\dots$   
 $9x = 3.6 - 0.3$   
 $= 3.3$

$x = \frac{3.3}{9} \times 10$   
 $= \frac{33}{90} \div 3$   
 $= \frac{11}{30}$

12 - |4 - 10| = 12 - |-6|  
 $= 12 - 6$   
 $= 6$

$\frac{x}{12} \times \frac{3}{9}$   
 $9x = 36$   
 $\div 9$   
 $x = 4$

$\frac{1}{x} + \frac{1}{x} = \frac{x}{x^2 + 1}$   
 $\frac{2}{x} = \frac{x}{x^2 + 1}$

$\sqrt{\frac{8-2-6-3}{4-5 \times 3 \cdot 4}} = 0.352396 \dots$   
 $\div 0.35$  (2 dp)

$\frac{6}{5 - \sqrt{3}} \times \frac{5 + \sqrt{3}}{5 + \sqrt{3}}$   
 $= \frac{6(5 + \sqrt{3})}{5^2 - (\sqrt{3})^2}$   
 $= \frac{30 + 6\sqrt{3}}{25 - 3}$   
 $= \frac{30 + 6\sqrt{3}}{22}$

$= \frac{2(15 + 3\sqrt{3})}{22} \div 2$   
 $= \frac{15 + 3\sqrt{3}}{11}$

$\therefore a = \frac{15}{11}$   
 $b = \frac{3}{11}$

$15x - 5 = 4(x + 3) - x$   
 $= 4x + 12 - x$   
 $= 3x + 12$   
 $-3x + 5 = -3x + 5$

$12x = 17$   
 $\div 12$   
 $x = \frac{17}{12}$   
 $= 1\frac{5}{12}$

$4(5a) = 6(3a - 7)$   
 $20a = 18a - 42$   
 $-18a = -18a$   
 $2a = -42$   
 $\div 2$   
 $a = -21$

$6x^3 - 2x^3(5 - x)$   
 $= 6x^3 - 10x^3 + 2x^4$   
 $= -4x^3 + 2x^4$

$\frac{(x+7)x^2}{3 \times 2} + \frac{(2x-1) \times 3}{2 \times 3}$   
 $= \frac{2x+14}{6} + \frac{6x-3}{6}$   
 $= \frac{8x+11}{6}$

$1a + 3x - 2a - 6$   
 $= x(a+3) - 2(a+3)$   
 $= (a+3)(x-2)$

$3x^2 - 27 = 3(x^2 - 9)$   
 $= 3(x^2 - 3^2)$   
 $= 3(x+3)(x-3)$

$2+x-10x^2 = -(10x^2-x-2)$   
 $= -(10x^2-5x+4x-2)$   
 $= -(5x(2x-1) + 2(2x-1))$   
 $= -(2x-1)(5x+2)$

$2 \cdot 7 \times 10^{-2} \times 8 \times 10^{29} = 2 \cdot 16 \times 10^7$

$(x+1)^2 + (x+3)^2 = (x+5)^2$   
 $x^2 + 2x + 1 + x^2 + 6x + 9 = x^2 + 10x + 25$   
 $2x^2 + 8x + 10 = x^2 + 10x + 25$   
 $-x^2 - 2x - 15 = 0$   
 $x^2 - 2x - 15 = 0$   
 $x^2 - 5x + 3x - 15 = 0$   
 $x(x-5) + 3(x-5) = 0$   
 $(x-5)(x+3) = 0$   
 $x-5=0$  or  $x+3=0$   
 $x=5$  or  $x=-3$

$x$  is a length  $\therefore$  length can't be negative  
 $\therefore x = 5$  only  
 sides are  $x+1 = 5+1 = 6$  cm  
 $x+3 = 5+3 = 8$  cm  
 $x+5 = 5+5 = 10$  cm

$18y - 91 = 5y - 6$

$8y - 9 = 5y - 6$   
 $-5y + 9 = -5y + 9$

$3y + 9 = -6$   
 $\div 3$   
 $y = 1$

CHECK:

LHS =  $18(1) - 91$   
 $= 18 - 91$   
 $= -73$

RHS =  $5(1) - 6$   
 $= 5 - 6$   
 $= -1$

$\therefore y = 1$  is not a solution

$-(8y-9) = 5y-6$   
 $-8y+9 = 5y-6$   
 $-8y+9 = 5y-6$   
 $-13y+9 = -6$   
 $-13y = -15$   
 $\div -13$   
 $y = 1\frac{15}{13}$

CHECK  
 LHS =  $18(1\frac{15}{13}) - 91$   
 $= 19\frac{30}{13} - 91$   
 $= 1\frac{15}{13}$

$1 + 2x < 5$   
 $\div 2$   
 $x < 2$



$|4p-2| \leq 10$

$4p-2 \leq 10$   
 $\div 4$   
 $p \leq 3$

CHECK:  $p=0$   
 LHS =  $|4 \times 0 - 2|$   
 $= |-2|$   
 $= 2 \leq 10 \checkmark$

$-(4p-2) \leq 10$   
 $-4p+2 \leq 10$   
 $-4p \leq 8$   
 $\div -4$   
 $p \geq -2$

CHECK:  $p=0$   
 checked above  $\checkmark$   
 $\therefore -2 \leq p \leq 3$

$4x^2 - 2x - 3 = 0$   
 $a=4, b=-2, c=-3$   
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$   
 $= \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 4 \times (-3)}}{2 \times 4}$   
 $= \frac{2 \pm \sqrt{4 + 48}}{4}$   
 $= \frac{2 \pm \sqrt{52}}{4}$   
 $= \frac{2 \pm 2\sqrt{13}}{4}$   
 $= \frac{1 \pm \sqrt{13}}{2}$

$\therefore -2 \leq p \leq 3$



$4x^2 - 2x - 3 = 0$   
 $a=4, b=-2, c=-3$   
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$   
 $= \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 4 \times (-3)}}{2 \times 4}$   
 $= \frac{2 \pm \sqrt{4 + 48}}{4}$   
 $= \frac{2 \pm \sqrt{52}}{4}$   
 $= \frac{2 \pm 2\sqrt{13}}{4}$   
 $= \frac{1 \pm \sqrt{13}}{2}$

$c = \frac{5}{9}(F - 32)$   
 $40 = \frac{5}{9}(F - 32)$   
 $\times 9$   
 $360 = 5(F - 32)$   
 $\div 5$   
 $72 = F - 32$   
 $+32$   
 $104 = F$

$F = 104$   
 $a^2 - b^2 = 25$   
 $a^2 - 9 = 25$   
 $a^2 = 34$   
 $a = \sqrt{34}$

$\sqrt{32} = \sqrt{16 \times 2}$   
 $= 4\sqrt{2}$   
 $= 2 \pm 2\sqrt{13}$   
 $= 2 \pm 2\sqrt{13} \div 2$   
 $= 1 \pm \sqrt{13}$

$104 = F$   
 $F = 104$

$a^2 - b^2 = 25$   
 $a^2 - 9 = 25$   
 $a^2 = 34$   
 $a = \sqrt{34}$

$a^2 - b^2 = 25$   
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 $a^2 = 34$   
 $a = \sqrt{34}$

$a^2 - b^2 = 25$   
 $a^2 - 9 = 25$   
 $a^2 = 34$   
 $a = \sqrt{34}$

$$4^{2x+1} = 8$$

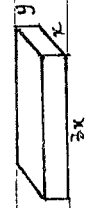
$$(2^2)^{2x+1} = 2^3$$

$$2^{4x+2} = 2^3$$

$$\frac{4x+2}{-2} = \frac{3}{-2}$$

$$\frac{4x}{-4} = \frac{1}{-4}$$

$$x = \frac{1}{4}$$



$$V = lwh$$

$$= 3x \times x \times y$$

$$= 3x^2y$$

$$V = 288 \text{ cm}^3$$

$$288 = 3x^2y$$

$$\frac{288}{3x^2} = y$$

$$\frac{288}{3x^2} \div 3 = \frac{96}{x^2} \div 3$$

$$= \frac{32}{x^2}$$

$$SA = \text{front} + \text{back} + \text{LHS} + \text{RHS} + \text{top} + \text{base}$$

$$= 2 \times 3xy + 2xy + 6x^2$$

$$= 8xy + 6x^2$$

$$= 8x \times \frac{96}{x^2} + 6x^2$$

$$= \frac{768x}{x^2} \div x + 6x^2$$

$$= \frac{768}{x} + 6x^2$$

$$S = (n-2) \times 180^\circ$$

$$= (8-2) \times 180^\circ$$

$$= 1080^\circ$$

$$1080 \div 8 = 135^\circ$$

In  $\triangle ABD \cong \triangle BCD$

BD is common

$$\angle DPB = \angle DCB = 90^\circ \text{ (data)}$$

$$AB = BC \text{ (data)}$$

$\therefore \triangle ABD \cong \triangle BCD \text{ (RHS)}$

AD = CD corresponding sides of congruent  $\triangle$ 's

$$90 - 70 = 20$$

$$180 - 20 = 160^\circ$$

$\angle RTS = 90^\circ$  (vertically opposite to  $\angle PTA$ )

$\angle TSR = 56^\circ$  (alternate to  $\angle QPT$ ,  $PQ \parallel RS$ )

$$\angle TRS = 180^\circ - \angle TSR - \angle RTS \text{ (sum of } \triangle \text{ TRS)}$$

$$= 180^\circ - 56^\circ - 90^\circ$$

$$= 34^\circ$$

$\angle PRS = \angle PSR$  ( $\angle$ 's opposite equal sides in  $\triangle PSR$  are equal)

$$= 56^\circ$$

$$\angle PRQ = \angle PRS - \angle TRS$$

$$= 56^\circ - 34^\circ$$

$$= 22^\circ$$

$\triangle X$  is common

$$\frac{AX}{BX} = \frac{8}{6+18} = \frac{8}{24} = \frac{1}{3}$$

$$\frac{BX}{CX} = \frac{6}{8+18} = \frac{6}{24} = \frac{1}{3}$$

$\therefore$  2 corresponding sides are in the same ratio and the included angle is the same  $\therefore$  the triangles are similar.

$$\angle CBD = \angle CDB \text{ (}\angle\text{'s opposite equal sides in } \triangle CBD \text{ are equal)}$$

$$\angle CBD = \frac{1}{2}(180^\circ - 106^\circ) \text{ (sum of } \angle \text{ BCD)}$$

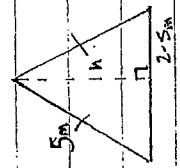
$$= 37^\circ$$

$$\angle BDA = \angle CBD \text{ (Alternate } \angle \text{ s, } BC \parallel AD)$$

$$= 37^\circ$$

$$\angle BAD = \angle BDA \text{ (}\angle\text{'s opposite equal sides in } \triangle ABD \text{ are equal)}$$

$$= 37^\circ$$



$$h^2 = 5^2 - 2.5^2$$

$$= 18.75$$

$$h = \sqrt{18.75} \text{ m}$$

$A = 2 \text{ triangles} + \text{square}$

$$= 2 \times \frac{1}{2}bh + s^2$$

$$= 2 \times \frac{1}{2} \times 5 \times \sqrt{18.75} + 5^2$$

$$= 46.6506 \dots$$

$$\approx 46.7 \text{ m}^2 \text{ (1 d.p.)}$$