Sydney Girls High School
2014

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I Pages 3 – 6

10 Marks

- Attempt Questions 1 – 10
- Answer on the Multiple Choice answer sheet provided
- Allow about 15 minutes for this section

Section II Pages 7 – 13

60 Marks

- Attempt Questions 11 – 14
- Answer on the blank paper provided
- Begin a new page for each question
- Allow about 1 hour and 45 minutes for this section

Name: ....

Teacher: ....

THIS IS A TRIAL PAPER ONLY
It does not necessarily reflect the format or the content of the 2014 HSC Examination Paper in this subject.

STUDENT NUMBER/NAME: ....

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

(1) What are the values of p such that $\frac{p+1}{p} \leq 1$?

(A) $p > 0$

(B) $p < 0$

(C) $p \leq 0$

(D) $-1 \leq p \leq 0$

(2) The expression $\tan\left(\frac{\pi}{4} + x\right)$ can also be expressed as:

(A) $\frac{\cos x + \sin x}{\cos x - \sin x}$

(B) $\frac{\cos x - \sin x}{\cos x + \sin x}$

(C) $\frac{\sec^2 x}{1 - \tan^2 x}$

(D) $\frac{\sin x + \cos x}{\sin x - \cos x}$

(3) The acute angle (to the nearest degree) between the lines $x - y = 2$ and $2x + y = 1$ is:

(A) 18°

(B) 27°

(C) 45°

(D) 72°

(4) Two of the roots of the polynomial $4x^3 + 8x^2 + kx - 18 = 0$ are equal in magnitude but opposite in sign. Find the value of k.

(A) $k = -2$

(B) $k = 2$

(C) $k = -9$

(D) $k = 9$

(5) $y = f(x)$ is a linear function with slope $\frac{1}{3}$, find the slope of $y = f^{-1}(x)$.

(A) 3

(B) $\frac{1}{3}$

(C) -3

(D) $-\frac{1}{3}$

(6) In the diagram, AC is a tangent to the circle at the point N, AB is a tangent to the circle at the point M and BC is a tangent to the circle at the point L.

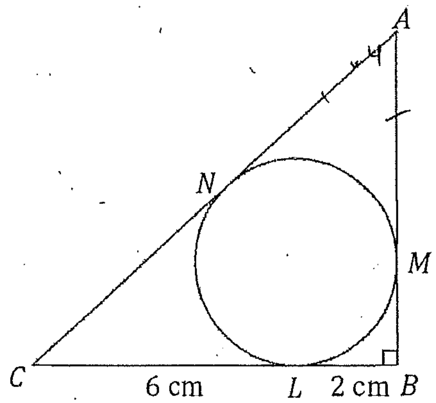
Find the exact length of AM if $CL = 6$ cm and $BL = 2$ cm.

(A) 3 cm

(B) 4 cm

(C) 5 cm

(D) 6 cm



(7) Find $\int \frac{dx}{1+4x^2}$

(A) $\frac{1}{2}\tan^{-1} 2x + C$

(B) $2\tan^{-1} 2x + C$

(C) $2\tan^{-1} \frac{x}{2} + C$

(D) $\frac{1}{2}\tan^{-1} \frac{x}{2} + C$

(8) Evaluate $\lim_{x\to 0} \frac{5x\cos 2x}{\sin x}$.

(A) -10

(B) -5

(C) 5

(D) 10

(9) Using $u = x^2 + 1$, the value that is equal to $\int_0^1 3x(x^2+1)^5 dx$ is:

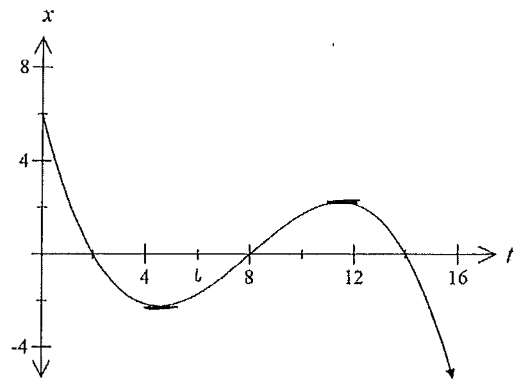
(A) $\frac{1}{4}$

(B) $\frac{16}{3}$

(C) $\frac{63}{4}$

(D) 32

(10) The displacement, x metres, from the origin of a particle moving in a straight line at any time (t seconds) is shown in the graph.



When was the particle at rest?

(A) $t = 4.5$ and $t = 11.5$

(B) $t = 0$

(C) $t = 2$, $t = 8$ and $t = 14$

(D) $t = 8$

Section II

60 marks

Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer on the blank paper provided. Begin a new page for each question
In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks)

(a) Evaluate $\int_0^{\frac{\pi}{6}} \sec 2x \tan 2x \, dx$. [2]

(b) $A(-3,7)$ and $B(4,-2)$ are two points. Find the coordinates of the point $P(x,y)$ which divides the interval AB internally in the ratio $3:2$. [2]

(c) The equation $2x^3 - 6x + 1 = 0$ has roots α, β and γ. Evaluate:

i) $\alpha + \beta + \gamma$. [1]

ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. [2]

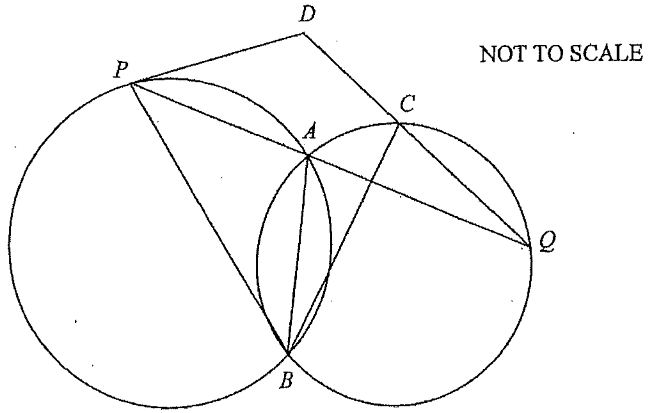
(d) i) Find the domain and range of the function $f(x) = 2\cos^{-1}(1-x)$. [2]

ii) Sketch the graph of the curve $y = 2\cos^{-1}(1-x)$ showing clearly the coordinates of the endpoints. [2]

Question 11 continues on the next page

Question 11 (Continued)

(e)



Two circles intersect at A and B. P is a point on the first circle and Q is a point on the second circle such that PAQ is a straight line. C is a point on the second circle. The line QC produced and the tangent to the first circle at P meet at D.

i) Copy the diagram.

ii) Give a reason why $\angle DPA = \angle PBA$. [1]

iii) Give a reason why $\angle CQA = \angle CBA$. [1]

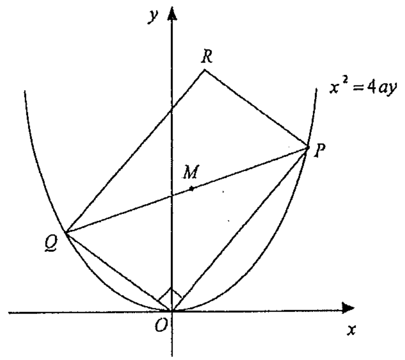
iv) Hence show that $BCDP$ is a cyclic quadrilateral. [2]

End of Question 11

Question 12 (Begin a New Page) (15 Marks)

(a) Use the method of Mathematical Induction to show that $5^n + 12n - 1$ is divisible by 16, for all positive integers $n \ge 1$. [3]

(b)



$P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points which move on the parabola $x^2 = 4ay$ such that $\angle POQ = 90^\circ$, where O is the origin.

$M = \left(a(p+q), \frac{1}{2}a(p^2+q^2)\right)$ is the midpoint of PQ. R is the point such that $OPRQ$ is a rectangle.

i) Show that $pq = -4$. [1]

ii) Show that R has coordinates $\left(2a(p+q), a(p^2+q^2)\right)$. [1]

iii) Find the equation of the locus of R. [2]

Question 12 continues on the next page

Question 12 (Continued)

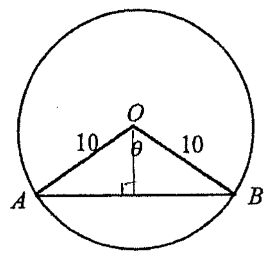
(c)

i) Show that the equation $e^x + x = 0$ has a real root α such that $-1 < \alpha < 0$. [2]

ii) If a is taken as an initial approximation to this real root α, use Newton's method to show that the next approximation a_1 is given by $a_1 = \frac{(a-1)e^a}{e^a + 1}$. [2]

Hence if the initial approximation is taken as $a = -0.5$, find the next approximation for α correct to two decimal places.

(d)



The chord AB of a circle of radius 10 cm subtends an angle θ radians at the centre O of the circle.

i) Show that the perimeter P cm of the minor segment cut off by the chord AB is given by $P = 10\theta + 20\sin\frac{\theta}{2}$. [2]

ii) If θ is increasing at a rate of 0.02 radians per second, find the rate at which P is increasing when $\theta = \frac{2\pi}{3}$. [2]

End of Question 12

Question 13 (Begin a New Page) (15 Marks)

(a) Evaluate $\int_1^{49} \frac{1}{4\left(x+\sqrt{x}\right)}\,dx$ using the substitution $u^2 = x,\ u > 0$. [4]

Give the answer in simplest exact form.

(b) Newton's Law of Cooling states that the rate of change in the temperature, T, of a body is proportional to the difference between the temperature of the body and the surrounding temperature, P.

i) If A and k are constants, show that the equation $T = P + Ae^{kt}$ satisfies Newton's Law of Cooling. [2]

ii) A cup of tea with temperature of $100°\text{C}$ is too hot to drink. Two minutes later, the temperature has dropped to $93°\text{C}$. If the surrounding temperature is $23°\text{C}$, calculate the value of A and k (correct to 3 significant figures). [2]

iii) The tea will be drinkable when the temperature has dropped to $80°\text{C}$. How long in minutes will this take? [1]

(c) A particle's motion is defined by the equation $v^2 = 12 + 4x - x^2$, where x is its displacement from the origin in metres and v its velocity in ms^{-1}.
Initially, the particle is 6 metres to the right of the origin.

i) Show that the particle is moving in Simple Harmonic Motion. [1]

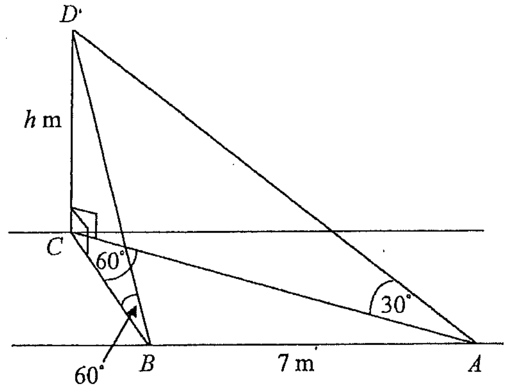
ii) Find the centre, the period and the amplitude of the motion. [3]

iii) The displacement of the particle at any time t is given by the equation $x = a\sin(nt + \theta) + b$. Find the values of θ and b, given $0 \le \theta \le 2\pi$. [2]

End of Question 13

Question 14 (Begin a New Page) (15 Marks)

(a)



A footpath on horizontal ground has two parallel edges. CD is a vertical flagpole of height h metres which stands with its base C on one edge of the footpath. A and B are two points on the other edge of the footpath such that $AB = 7$ m and $\angle ACB = 60°$. From A and B the angles of elevation of the top D of the flagpole are $30°$ and $60°$ respectively.

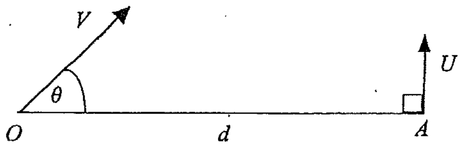
i) Find the exact height of the flagpole. [3]

ii) Find the exact width of the footpath. [2]

Question 14 continues on the next page

Question 14 (Continued)

(b)



O and A are two points d metres apart on horizontal ground. A rocket is projected from O with speed V ms^{-1} at an angle θ above the horizontal where $0<\theta<\frac{\pi}{2}$. At the same instant, another rocket is projected vertically from A with speed U ms^{-1}.

The two rockets move in the same vertical plane under gravity where the acceleration due to gravity is g ms^{-2}.

After time t seconds, the rocket from O has horizontal and vertical displacements x metres and y metres respectively from O, while the rocket from A has vertical displacement Y metres from A. The two rockets collide after T seconds.

i) Derive the expressions for x, y and Y in terms of V, θ, U, t and g. [3]

ii) Show that $d = VT\cos\theta$ and $U = V\sin\theta$. [2]

iii) Show that $V > U$. [1]

iv) Show that the two rockets are the same distance above ground level at all times. [1]

v) Show that $T = \dfrac{d}{\sqrt{V^2-U^2}}$. [2]

vi) If the two rockets collide at the highest points of their flights, show that $d = \dfrac{U\sqrt{V^2-U^2}}{g}$. [1]

End of Exam

Sydney Girls High School
Mathematics Faculty

Multiple Choice Answer Sheet
2014 Trial HSC Mathematics Extension 1

Student Number: SOLUTIONS

Completely fill the response oval representing the most correct answer.

1.	A ○	B ●	C ○	D ○
2.	A ●	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ●
4.	A ○	B ○	C ●	D ○
5.	A ●	B ○	C ○	D ○
6.	A ○	B ●	C ○	D ○
7.	A ●	B ○	C ○	D ○
8.	A ○	B ○	C ●	D ○
9.	A ○	B ○	C ●	D ○
10.	A ●	B ○	C ○	D ○

Multiple choice.

① $\dfrac{p+1}{p} \le 1$

$p^2\left(\dfrac{p+1}{p}\right) \le p^2 \quad (p \ne 0)$

$p\,(p+1) \le p^2$

$p^2 + p \le p^2$

$p \le 0 \quad p \ne 0.$

$\therefore \underline{p < 0}$ (B).

② $\tan\left(\dfrac{\pi}{4} + x\right)$

$= \dfrac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}\tan x}$

$= \dfrac{1 + \frac{\sin x}{\cos x}}{1 - \frac{\sin x}{\cos x}}$

$= \dfrac{\cos x + \sin x}{\cos x} \div \dfrac{\cos x - \sin x}{\cos x}$

$= \dfrac{\cos x + \sin x}{\cancel{\cos x}} \times \dfrac{\cancel{\cos x}}{\cos x - \sin x}.$

$= \dfrac{\cos x + \sin x}{\cos x - \sin x}$ (A)

Multiple choice.

③. $x - y = 2 \Rightarrow y = x - 2 \quad \therefore m_1 = 1$

$2x + y = 1 \Rightarrow y = 1 - 2x \quad \therefore m_2 = -2.$

$\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\dfrac{1-(-2)}{1+(1)(-2)}\right|$

$= \left|\dfrac{3}{-1}\right|$

$\therefore \tan\theta = 3$

$\underline{\theta \doteq 72^\circ}$ (D).

④. $4x^3 + 8x^2 + kx - 18 = 0.$

$\alpha + (-\alpha) + \beta = \dfrac{-8}{4} = -2$

$\therefore \beta = -2.$

$\alpha(-\alpha)\beta = \dfrac{18}{4}$

$-\alpha^2(-2) = \dfrac{9}{2}$

$\alpha^2 = \dfrac{9}{4}$

$\alpha(-\alpha) + \alpha\beta + (-\alpha\beta) = \dfrac{k}{4}$

$-\alpha^2 + \cancel{\alpha\beta} - \cancel{\alpha\beta} = \dfrac{k}{4}$ (C)

$-\alpha^2 = \dfrac{k}{4}$

$-\dfrac{9}{4} = \dfrac{k}{4} \quad \therefore \underline{k = -9}.$

Multiple choice.

(5) $f(x)$: $y = mx + b$

$y = \frac{1}{3}x + b$.

$f^{-1}(x)$: $x = \frac{1}{3}y + b$

$3x = y + 3b$

$y = 3x - 3b$. (A)

$\therefore m = 3$

(6). $CL = CN = 6cm$.

$LB = BM = 2cm$.

$AN = AM = x$ cm

$(x+2)^2 + 8^2 = (x+6)^2$

$\cancel{x^2} + 4x + 4 + 64 = \cancel{x^2} + 12x + 36$.

$4x + 68 = 12x + 36$

$32 = 8x$

$x = 4$

$\therefore AM = 4cm$ (B).

(7). $\int \frac{dx}{1+4x^2}$

$= \int \frac{dx}{4(\frac{1}{4} + x^2)}$

$= \frac{1}{4} \cdot \frac{1}{\frac{1}{2}} \tan^{-1} \frac{x}{\frac{1}{2}}$ (A).

$= \frac{1}{\cancel{4}2} \cdot \cancel{2} \tan^{-1} 2x$.

$= \frac{1}{2} \tan^{-1} 2x + C$.

Multiple choice.

(8). $\lim_{x \to 0} \frac{5x \cos 2x}{\sin x}$.

$= \lim_{x \to 0} \frac{5x(1 - 2\sin^2 x)}{\sin x}$

$= \lim_{x \to 0} \frac{5x}{\sin x} - \frac{10x \sin^2 x}{\sin x}$.

$= \lim_{x \to 0} \frac{5\cancel{x}}{\cancel{x}} - 10 \cancel{x} x^2_0$ ($\sin x \doteq x$, as $x \to 0$)

$= 5$. (C).

(9). $u = x^2 + 1$ $\quad$ $x = 1, u = 2$

$du = 2x\,dx$ $\quad$ $x = 0, u = 1$

$\int_0^1 3x(x^2+1)^5\,dx$

$= \int_1^2 3\cancel{x} \cdot u^5 \cdot \frac{du}{2\cancel{x}}$

$= \frac{3}{2} \int_1^2 u^5\,du$

$= \frac{3}{2 \times 6} \left[u^6\right]_1^2$

$= \frac{1}{4} \times \left[2^6 - 1\right]$

$= \frac{63}{4}$ (C).

(10). (A).

Question 11 - 15 marks - Ext I Mathematics - 2014 - Trials

a) $\int_0^{\pi/6} \sec 2x \tan 2x \, dx = \frac{1}{2} \sec 2x \Big]_0^{\pi/6}$

$= \frac{1}{2} \sec \frac{\pi}{3} - \frac{1}{2} \sec 0$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$ (2 marks)

- Overall this question was done poorly. Many solved it by substitution

b) $m:n$ $\frac{x_2 m + x_1 n}{m+n}, \frac{y_2 m + y_1 n}{m+n}$

$3:2$

$A(-3,7)$ $x_1 y_1$

$B(4,-2)$ $x_2 y_2$

$\frac{4(3)+(-3)(2)}{3+2}, \frac{(-2)(3)+(7)(2)}{3+2}$

$\frac{12-6}{5}, \frac{-6+14}{5}$

$\therefore P\left(\frac{6}{5}, \frac{8}{5}\right)$ or $\left(1\frac{1}{5}, 1\frac{3}{5}\right)$

(2 marks)

- Overall done very well. Most errors were due to carelessness.

c) $2x^3 + 0x^2 - 6x + 1 = 0$

i) $\alpha + \beta + \gamma = -\frac{b}{a}$

$= 0$

ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

$= \frac{c}{a} \div -\frac{d}{a}$

$= -\frac{6}{2} \times \frac{2}{-1}$

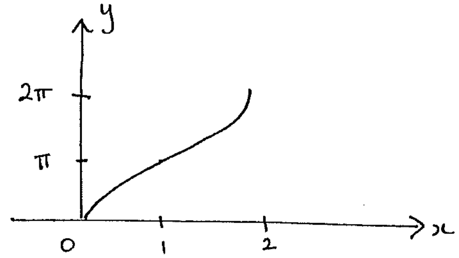
$= 6$

- Overall done very well, except for some who failed to recognise x^2

$2x^3 + \underline{\underline{0x^2}} - 6x + 1 = 0$ was missing!

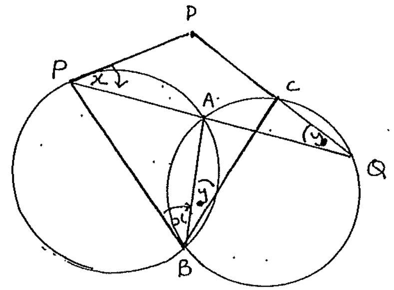
d) i) Domain: $0 \leq x \leq 2$

Range: $0 \leq y \leq 2\pi$



Overall domain & range was found very well. Sketches of the graph varied. Most sketched correctly.

e)



Parts ii) & iii) were completed extremely well.

iv) Most completed question well, some proved it by the exterior ∠ equals interior opposite ∠ of cyclic quad.

ii) $\angle DPA = \angle PBA$ (angle in the alternate segment) $= x°$

iii) $\angle CQA = \angle CBA$ (angles standing on the same arc) $= y°$

iv) $\angle D = 180 - (x+y)$ (angle sum of ΔPDQ)

$\therefore \angle D + \angle PBC = 180 - (x+y) + x + y$

$= 180°$

$\therefore$ opposite ∠'s of quadrilateral BCDP are supplementary, hence, BCDP is a cyclic quadrilateral.

Ext 1 2014 Trial

12a) $n=1$

$= 5+12-1$

$= 16$

Divisible by 16

$5^k + 12k - 1 = 16p$

Prove true for $n=k+1$

$5^{k+1} + 12(k+1) - 1$

$= 5 \cdot 5^k + 12k + 12 - 1$

$= 5(16p - 12k + 1) + 12k + 12 - 1$

$= 80p - 60k + 5 + 12k + 12 - 1$

$= 80p - 48k + 16$

$= 16(5p - 3k + 1)$

$= 16q$

*Most students did this question well, but a few didn't do the last main section well.

b(i) $m_{AQ} \times m_{OP} = -1$

$\frac{aq^2 - 0}{2aq} \times \frac{ap^2 - 0}{2ap} = -1$

$\frac{q \times p}{4} = -1$

$pq = -4$

b(ii)

$a(p+q),\ \frac{1}{2}a(p^2+q^2) = \frac{x+0}{2},\ \frac{y+0}{2}$

$\frac{x}{2} = a(p+q)\ ,\ \frac{y}{2} = \frac{1}{2}a(p^2+q^2)$

$x = 2a(p+q)\ ,\ y = a(p^2+q^2)$

*Many students did more than a page of working for a one mark question

b(iii)

$x = 2a(p+q)$

$y = a(p^2+q^2)$

$p+q = \frac{x}{2a}$

$y = a\left[(p+q)^2 - 2pq\right]$

$= a\left[\left(\frac{x}{2a}\right)^2 + 8\right]$

$y = \frac{x^2}{4a} + 8a$

$x^2 = 4a(y - 8a)$

*This question was done better than part i) and ii)

c) i)

$f(x) = e^x + x$

$f(0) = e^0 + 0$

$= e > 0$

$f(-1) = e^{-1} - 1$

$\doteqdot -0.63 < 0$

since $f(0) > 0$ and $f(-1) < 0$ $\therefore$ there is a root $-1 < \alpha < 0$

*The setting out for this part was very poor.

ii) $x_1 = x - \frac{f(x)}{f'(x)}$

$a_1 = a - \frac{e^a + a}{e^a + 1}$

$= \frac{a(e^a+1) - e^a - a}{e^a + 1}$

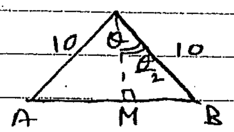
$= \frac{ae^a + \cancel{a} - e^a - \cancel{a}}{e^a + 1}$

$= \frac{e^a(a-1)}{e^a+1}$

$\doteqdot -0.57$

*Please remember for a "show" question you need to show all the steps.

d) i)



$\sin\frac{\theta}{2} = \frac{MB}{10}$

$MB = 10\sin\frac{\theta}{2}$

$AB = 2MB$

$AB = 20\sin\frac{\theta}{2}$

$P = r\theta + 20\sin\frac{\theta}{2}$

$P = 10\theta + 20\sin\frac{\theta}{2}$

*This was the simplest way of doing this question. Some students used a harder method and didn't show the working properly.

ii) $\frac{d\theta}{dt} = 0.02$

$\frac{dP}{dt} = \frac{dP}{d\theta} \cdot \frac{d\theta}{dt}$

$= \left(10 + 20 \times \frac{1}{2} \times \cos\frac{\theta}{2}\right) \times 0.02$

$= \left(10 + 10\cos\frac{\theta}{2}\right) 0.02$

$= 15 \times 0.02$

$= 0.3$ cm/s

* some students didn't get the $\cos\frac{\theta}{2}$ correctly

Question 13:

a) $I = \int_1^{49} \frac{1}{4(x+\sqrt{x})} dx$

$u^2 = x, \ u > 0$

$2u = \frac{dx}{du}$	$x = 1 \Rightarrow u = 1$
$\therefore dx = 2u\,du$	$x = 49 \Rightarrow u = 7$

✓

$\therefore I = \int_1^7 \frac{1}{4(u^2+u)} 2u\,du$

$= \int_1^7 \frac{1}{2(u+1)} du$ ✓ * students who did not factorise at this point, lost unnecessary marks.

$= \frac{1}{2}\Big[\ln(u+1)\Big]_1^7$ ✓

$= \frac{1}{2}(\ln 8 - \ln 2)$

$= \frac{1}{2}\ln 4$

$= \ln 2$ (exact answer) ✓ * Carry on error from integration was accepted.

* alternate solutions accepted.

Question 13

b) i) Newton's Law is $\frac{dT}{dt} = k(T-P)$ ✓

If $T = P + Ae^{kt}$

then $\frac{dT}{dt} = k \times Ae^{kt}$

$\therefore \frac{dT}{dt} = k(T-P)$ ✓

ii) When $T = 100$, $P = 23$, $t = 0$:

$100 = 23 + Ae^{0}$

$\therefore A = 77$ ✓

When $t = 2$, $T = 93^{\circ}$

$93 = 23 + 77e^{k \times 2}$

$70 = 77e^{2k}$

$\frac{70}{77} = e^{2k}$

$\therefore k = \frac{1}{2}\ln\frac{70}{77} \doteqdot -0.0477$ (3 sig. figs)

* Rounding-off correct to 3. sig. figs needs ✓ to be REVISED by MOST students.

iii) $80 = 23 + 77e^{-0.0477 \times t}$

$\frac{57}{77} = e^{-0.0477t}$

$\therefore t = \frac{1}{-0.0477} \ln\frac{57}{77}$

$t = 6.31106047$ min

or $t = 7$ min ✓ * Rounding to 6 min is incorrect (without previous work/approximation).

Question 13

c)

$$v^2 = 12 + 4x - x^2$$

i) $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \frac{d}{dx}\left(6 + 2x - \frac{x^2}{2}\right)$

$= 2 - x$

$= -1(x-2) \equiv -n^2(x-b)$ ✓

$\therefore$ particle moves in SHM.

ii) Centre of motion is $x = 2$ (where $a = 0$) ✓

· $n = 1$ so period $T = \frac{2\pi}{n} = 2\pi$ ✓

· Extremes of motion when $v = 0$:

$12 + 4x - x^2 = 0$

$(6-x)(2+x) = 0$

$\therefore x = -2$ and $x = 6$

$\therefore$ amplitude of motion is 4. ✓

iii) $a = 4,\ n = 1,\ b = 2$

$\therefore x = 4\sin(t + \theta) + 2$ ✓

when $t = 0,\ x = 6$:

$6 = 4\sin\theta + 2$

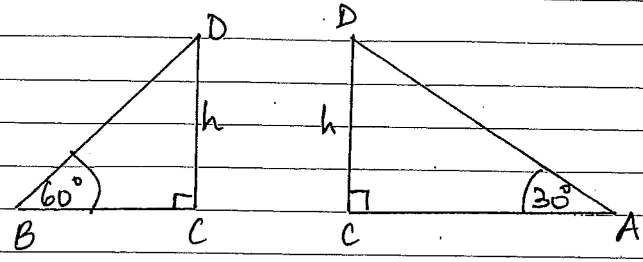
$4 = 4\sin\theta$

$\therefore \theta = \frac{\pi}{2}$ ✓

$\therefore\ x = 4\sin\left(t + \frac{\pi}{2}\right) + 2.$

Question 14:

a)



In ΔBCD: $\frac{h}{BC} = \tan 60°$

$\therefore BC = \frac{h}{\tan 60°} = h\cot 60° = \frac{h}{\sqrt{3}}$ ✓

In ΔACD: $\frac{h}{AC} = \tan 30°$

$\therefore AC = \frac{h}{\tan 30°} = h\cot 30° = \sqrt{3}h$ ✓

Using the cosine rule in ΔABC:

$AB^2 = BC^2 + AC^2 - 2(BC)(AC)\cos 60°$

$7^2 = \frac{h^2}{3} + 3h^2 - 2 \times \frac{h}{\sqrt{3}} \times \sqrt{3}h \times \frac{1}{2}$

$49 = h^2\left(\frac{1}{3} + 3 - 1\right)$

$49 = \frac{7}{3}h^2$

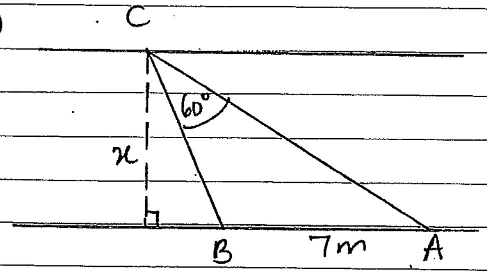
$h^2 = \frac{3 \times 49}{7}$

$h = \sqrt{21}$ m $(h > 0)$ ✓

* Correct answers, not in exact form, were awarded a mark.

Question 14

a) ii)



Let the width of the footpath be x metres.

$$\text{Area } \Delta ABC = \tfrac{1}{2} ab \sin C$$

$$\tfrac{1}{2} \times 7 \times x = \tfrac{1}{2} \times BC \times AC \times \sin 60^\circ$$

$$7x = \frac{h}{\sqrt{3}} \times \sqrt{3}\, h \times \frac{\sqrt{3}}{2}$$

$$x = \frac{\sqrt{3}}{14} h^2$$

$$x = \frac{21\sqrt{3}}{14}$$

$$x = \frac{3\sqrt{3}}{2} \text{ m} \quad ✓✓$$

* Many students thought that ΔABC is right-angled. This was awarded ZERO marks. The problem was over-simplified.

Question 14:

b) i) Rocket from point O:

horizontal motion:

$$\ddot{x} = 0$$
$$\dot{x} = c$$

when $t=0$, $\dot{x} = V\cos\theta$

$$\therefore \dot{x} = V\cos\theta$$
$$x = V\cos\theta\, t + C_1$$

when $t=0$, $V=0$

$$\therefore x = V\cos\theta\, t \quad ✓$$

Vertical motion:

$$\ddot{y} = -g$$
$$\dot{y} = -gt + c$$

when $t=0$, $\dot{y} = V\sin\theta$

$$\therefore \dot{y} = V\sin\theta - gt$$
$$y = V\sin\theta\, t - \frac{gt^2}{2} + C_1$$

when $t=0$, $V=0$ $\therefore C_1 = 0$

$$\therefore y = V\sin\theta\, t - \tfrac{1}{2} g t^2 \quad ✓$$

* Students who did not DERIVE the equations lost one mark.

Question 14:

bi) Rocket from point A:

$\ddot{Y} = -g$

$\dot{Y} = -gt + c$

when $t=0$, $\dot{Y} = U$

$\therefore \dot{Y} = U - gt$

$Y = Ut - \frac{gt^2}{2} + c_1$

when $t=0$, $Y=0 \therefore c_1 = 0$

$\therefore Y = Ut - \frac{gt^2}{2}$ ✓

ii) When the rockets collide at time T, they must be vertically above A with the same height.

$x = Vt\cos\theta$

when $t=T$, $x=d$:

$\therefore d = VT\cos\theta$ ✓

$y = Vt\sin\theta - \frac{1}{2}gt^2$

when $t=T$, $y=Y$:

$\therefore UT - \cancel{\frac{gt^2}{2}} = VT\sin\theta - \cancel{\frac{gt^2}{2}}$

$UT = VT\sin\theta$

$\therefore U = V\sin\theta$ ✓

Question 14

b) iii) $U = V\sin\theta$

since $0 < \theta < \frac{\pi}{2}$

then $0 < \sin\theta < 1$ ✓

$\therefore V > V\sin\theta$

$\therefore V > U$

iv) $Y = Ut - \frac{1}{2}gt^2$

$= (V\sin\theta)t - \frac{1}{2}gt^2$ ✓

$= y$

Hence the rockets are always at the same height above ground level.

v) $V\cos\theta = \frac{d}{T}$ (from ii)

$V\sin\theta = U$

$\therefore V^2(\cos^2\theta + \sin^2\theta) = \frac{d^2}{T^2} + U^2$ ✓

$V^2 = \frac{d^2}{T^2} + U^2$

$\therefore V^2 - U^2 = \frac{d^2}{T^2}$

$T^2 = \frac{d^2}{V^2 - U^2}$ ✓

$\therefore T = \frac{d}{\sqrt{V^2 - U^2}}$, $(T > 0)$

* alternate solutions accepted.

Question 14:

b) vi) At the highest point of flight of the rocket from A:

$\dot{y} = 0$

$U - gt = 0$

$U = gt$

$\therefore t = \dfrac{U}{g}$

$\therefore$ Rockets collide at highest point if $T = \dfrac{U}{g}$.

Then $d = T\sqrt{V^2 - U^2}$

$\therefore d = \dfrac{U\sqrt{V^2 - U^2}}{g}$, as required. ✓