Sydney Girls High School

LABOR OMNIA VINCIT

2014 Assessment Task 2

Extension 1 Mathematics

Year 12

Time allowed - 60 minutes + 5 minutes reading time

Topics: Logarithmic & Exponential Functions
Trigonometric Functions and Trigonometric Functions II
Polynomials

Instructions

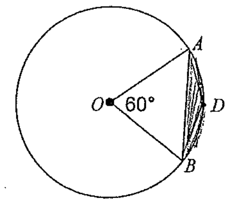
NAME ____________________

TEACHER ____________________

- Attempt all seven questions.
- Questions are not of equal value.
- All necessary working should be shown in every question.
- Marks may be deducted for careless or badly arranged work.
- Write on one side of the paper only.
- A table of standard integrals is attached.

Total Marks=54

Question One (7 Marks)



OA=10 cm

a) Refer to the circle above:

i) Convert 60° to radian measure [1]

ii) Find the exact length of the minor arc AB [1]

iii) Find the exact area of the minor segment ADB [2]

b) Find the value of x if $x = \log_2 8$ [1]

c) Find the volume of the solid formed when the area under the curve $y = \sec x$, bounded by the x-axis, $x = 0$ and $x = \frac{\pi}{4}$ is rotated about the x- axis. [2]

Question Two (7 Marks)

a) Find the derivative of y with respect to x if:

i) $y = e^{\log_e x}$ [1]

ii) $y = \log_e \sqrt{x^2 + 1}$ [2]

b) Find the size of the acute angle between the lines $y = 2x + 3$ and $y = 5 - 3x$ [2]

c) Sketch the graph of $y = 3\cos 2x$ for $0 \le x \le 2\pi$ [2]

Question Three (8 Marks)

a) Solve the equation $x^3 - 3x^2 - 10x + 24 = 0$ [3]

b) If $t = \tan\frac{\theta}{2}$ simplify $\frac{\sin\theta}{\cos\theta + 1}$ in terms of t [3]

c) Find $\int \cos^2 4x\,dx$ [2]

Question Four (8 Marks)

a) Evaluate $\lim_{x \to 0} \frac{\sin(5x)}{2x}$ [1]

b) If $5^{2x} = 51$, find the value of x correct to 2 decimal places [2]

c) Solve the following trigonometric equation:

$$\sin 2x - \cos x = 0 \quad (0 \le x \le 2\pi)$$ [3]

~~d) Find the exact value of p if $\int_1^{4.75} \frac{dx}{4x-3} = \log_e p$~~ [2]

Question Five (8 Marks)

a) If α, β and γ are the roots of the polynomial equation $x^3 + 3x^2 - 1 = 0$ find:

i) $\alpha + \beta + \gamma$ [1]

ii) $\alpha\beta\gamma$ [1]

iii) $\alpha^2 + \beta^2 + \gamma^2$ [2]

b) i) Write $4\sin x + 3\cos x$ in the form $A\sin(x+\alpha)$ [2]

ii) Hence solve $4\sin x + 3\cos x = 4 \left(0^0 \le x \le 360^0\right)$ [2]

Question Six (8 Marks)

a) The equation $\sin(x) - 1 + 2x = 0$ has a root near $x = 0.3$ [2]
Use one application of Newton's Method to find a better approximation.
Give your answer correct to 3 decimal places.

b) Find a polynomial of degree 3 which has a zero at 2, is an odd function and for which $P(3) = 45$ [3]

c) If the roots of the polynomial equation $x^3 - 6x^2 + 3x + k = 0$ are consecutive terms of an arithmetic sequence. Find the value of k. [3]

Question Seven (8 Marks)

a) Given the equation of the curve $y = (1+2x)e^{-2x}$:

i) Find the point where the curve cuts the y- axis [1]

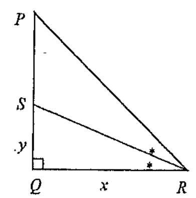
ii) Find the point where the curve cuts the x- axis [1]

iii) Find the coordinates of the stationary point [2]

iv) What happens to y as $x \to +\infty$ [1]

v) Sketch the curve [1]

b) In triangle PQR below, $\angle PQR = 90^\circ$, and SR bisects $\angle PRQ$. [2]
$SQ = y$ and $QR = x$



Find an expression for $\sin \angle PRQ$ in terms of x and y.

Question 1 - Yr 12 - Ext 1 - Task 2 - 2014

a) i) $\pi = 180°$

$\frac{\pi}{180°} = 1°$ $\therefore 60° \times \frac{\pi}{180} = \frac{\pi}{3}$ radians (1)

ii) $l = r\theta$

$= 10 \times \frac{\pi}{3}$

$= \frac{10\pi}{3}$ cm (1)

iii) Minor segment $ADB = \frac{1}{2}r^2[\theta - \sin\theta]$

$= \frac{1}{2}(10)^2\left[\frac{\pi}{3} - \sin\frac{\pi}{3}\right]$

$= \frac{100}{2}\left[\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right]$

$= \frac{50\pi}{3} - \frac{50\sqrt{3}}{2}$ (2)

$= \frac{50\pi}{3} - 25\sqrt{3}$ units²

b) $x = \log_2 8$

$2^x = 8$

$2^x = 2^3$

$\therefore x = 3$ (1)

c) $V = \pi\int_a^b y^2\,dx$

$= \pi\int_0^{\pi/4} \sec^2 x\,dx$

$= \pi\left[\tan x\right]_0^{\pi/4}$ (2)

$= \pi[\tan\pi/4 - \tan 0]$

$= \pi[1-0] = \pi$ units³

Total

7 marks

2

a) i. $y = e^{\log_e x}$

$y' = \frac{1}{x}e^{\log_e x}$ ✓

$y' = \frac{1}{x}\cdot x = 1$

ii) $y = \log_e\sqrt{x^2+1}$

$y' = \frac{\frac{2x}{2\sqrt{x^2+1}}}{\sqrt{x^2+1}}$

$y' = \frac{x}{x^2+1}$ ✓✓

b) $y = 2x + 3$

$m_1 = 2$

$y = 5 - 3x$

$m_2 = -3$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\tan\theta = \left|\frac{5}{1-6}\right|$ ✓

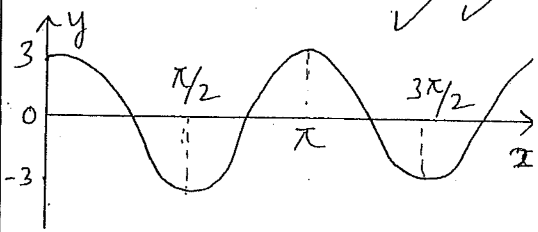
$\theta = 45°$

The angle between $y = 2x+3$ and $y = 5-3x$ is $45°$ ✓

c) $y = 3\cos 2x$ for $0 \le x \le 2$[illegible]

amplitude $= 3$

period $= \frac{2\pi}{2} = \pi$ ✓✓



Q3 3U 2014 Task 2

a)

$$\begin{array}{r} x^2 - x - 12 \\ x-2 \overline{)\, x^3 - 3x^2 - 10x + 24} \\ x^3 - 2x^2 \\ \hline -x^2 - 10x \\ -x^2 + 2x \\ \hline -12x + 24 \\ -12x + 24 \\ \hline \end{array}$$

$(x-2)(x^2-x-12)=0$

$(x-2)(x-4)(x+3)=0$

$x=2,\ 4,\ -3$

b) $\dfrac{\frac{2t}{1+t^2}}{\frac{1-t^2}{1+t^2}+1}$

$= \dfrac{\frac{2t}{1+t^2}}{\frac{1-t^2+1+t^2}{1+t^2}}$

$= \dfrac{2t}{2}$

$= t$

c) $\int \cos^2 4x \, dx$

$= \frac{1}{2}\int \cos 8x + 1 \, dx$

$= \frac{1}{2}\left[\frac{\sin 8x}{8} + x\right]$

$= \frac{\sin 8x}{16} + \frac{x}{2} + c$

Question 4 (8 marks).

a) $\lim_{x\to 0} \frac{\sin(5x)}{2x}$

$= \frac{5}{2} \lim_{x\to 0} \frac{\sin 5x}{5x}$

$= \frac{5}{2} \times 1$ ①

$= \frac{5}{2}$

b) $5^{2x} = 51$

$2x \ln 5 = \ln 51$ ✓

$2x = \frac{\ln 51}{\ln 5}$

$x = \frac{1}{2} \cdot \frac{\ln 51}{\ln 5}$

$x \doteq 1.22$ (2 d.p.) ②

c) $\sin 2x - \cos x = 0$

$2\sin x \cos x - \cos x = 0$ ✓

$\cos x\,(2\sin x - 1) = 0$

$\cos x = 0$ or $\sin x = \frac{1}{2}$ ✓

$x = 90^\circ, 270^\circ$ $\quad x = 30^\circ, 150^\circ$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad x = \frac{\pi}{6}, \frac{5\pi}{6}$ ③

d) $\int_1^{4.75} \frac{dx}{4x-3} = \log_e p$

$\left[\frac{1}{4}\ln(4x-3)\right]_1^{4.75} = \log_e p$ ✓

$\frac{1}{4}\ln 16 - \frac{1}{4}\ln 1 = \log_e p$.

$\ln 16^{1/4} = \ln p$

$\therefore p = 16^{1/4} = 2$ ②

5 a) i) $-\frac{3}{1} = -3$ ✓

ii) $-\frac{-1}{1} = 1$ ✓

iii) $(\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$

$= (-3)^2 - 2\times 0$

$= 9$ ✓✓

b) i) $A = \sqrt{4^2+3^2}$

$= 5$

$\tan\alpha = \frac{3}{4}$

$\alpha = 36°52' 11.63''$

$\doteqdot 37°$ ✓✓

ii) $5\sin(x+37°) = 4$

$\sin(x+37°) = \frac{4}{5}$

$x+37° = 53°$ or $127°$

$x = 16°$ or $90°$ ✓✓

Question Six

a) $f(x) = \sin(x) - 1 + 2x$

$f'(x) = \cos x + 2$

$x_1 = 0.3$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ ✓

$= 0.3 - \frac{\sin(0.3) - 1 + 2(0.3)}{\cos(0.3) + 2}$

$= 0.335$ radians ✓ (2)

b) Odd, root at $x=2$ ∴ other roots at $x=-2$, $x=0$ ✓

$P(x) = a(x+2)(x-2)(x)$

when $x=3$, $P(x) = 45$

$45 = a(27-12)$

$15a = 45$

$a = 3$ ✓ (3)

ie $P(x) = 3(x+2)(x-2)(x)$ ✓

c) $P(x) = x^3 - 6x^2 + 3x + k$

Let roots be $a-d$, a, $a+d$

Sum of roots $3a = 6$

$a = 2$ ✓

Now a is a root of $P(x)$ ∴ $P(a) = 0$

ie $8 - 24 + 6 + k = 0$ ✓

$k = 10$ ✓ (3)

Question 7 Solutions

(a)(i) $(0,1)$

(a)(ii) $x-\text{int. when } y=0$

$0=(1+2x)e^{-2x}$

$1+2x=0 \text{ or } e^{-2x}=0$

$\therefore x=-\frac{1}{2}$ only since $e^{-2x}=0$ has no soln.

i.e. $\left(-\frac{1}{2},0\right)$

(a)(iii) $y'=2e^{-2x}+(1+2x)\times -2e^{-2x}$

$=2e^{-2x}(1-1-2x)$

$\therefore y'=-4xe^{-2x}$

Stat. pts when $y'=0$

$-4xe^{-2x}=0$

$-4x=0 \text{ or } e^{-2x}=0$

$\therefore x=0$ only since $e^{-2x}=0$ has no soln.

i.e. stat. pt. at $(0,1)$

$y''=-4x\times -2e^{-2x}+e^{-2x}\times -4$

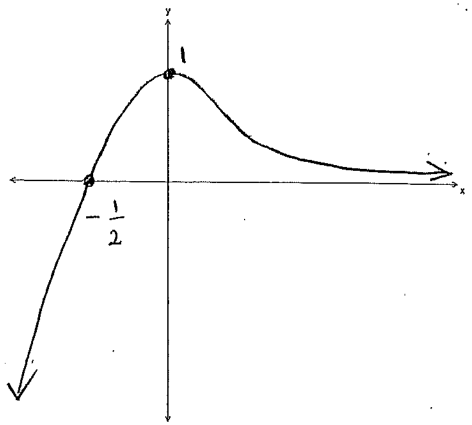
$=8xe^{-2x}-4e^{-2x}$

when $x=0,\ y''=-4<0$

$\therefore$ Max. stat. pt. at $(0,1)$

(a)(iv) As $x\to+\infty,\ y\to 0^{+}$.

(a)(v)



(b) Let $\angle PRS=\angle SRQ=\theta$

$\therefore \sin\angle PRQ=\sin 2\theta$

$=2\sin\theta\cos\theta$

$=2\times\frac{y}{\sqrt{x^2+y^2}}\times\frac{x}{\sqrt{x^2+y^2}}$

i.e. $\sin\angle PRQ=\frac{2xy}{x^2+y^2}$

Sydney Girls High School

Mathematics Faculty

Years 12 HSC Mathematics Extension 1

2014 Task 2

Question	Marker's Comment
1	This question was completed extremely well. The majority of students obtained full marks for this question. It was good to see well set out solutions.
2	The majority of students did very well on this question. Part (b), some students still couldn't remember the angle between two lines formula. You should know this formula well as this kind of question is very common in the HSC. Part (c) some students mixed up between the sinx function and cosx function. Remember to label the x-intercepts clearly.
3	This question was done well. In part a) some students forgot to solve the equation and left the equation in factored form.
4	It was good to see well set-out solutions in this question. In part d) some girls left their answer in exponential form. This could have been simplified further by using a calculator or with deeper knowledge of the conversions between exponentials and logarithms.
5	Both questions were well done by the overwhelming number of students.
6	(a) Over 60% of students incorrectly answered this question in degrees. It is important for students to remember that calculus questions in involving trig functions must be answered in radian measure. (b) Knowledge of the geometrical properties of odd functions led to a quick solution to this question (c) Both this question and (c) above had quick solutions (6-7 lines) or long solutions(15 lines or more)
7	Many students received full marks for this question. (a) (iii) Inability to solve the exponential equation formed by $y'=0$ prevented a number of students from finding the stationary point. (b) A variety of responses given to this question but you should have identified the opportunity to use the double angle formula for sin.