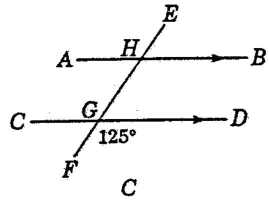
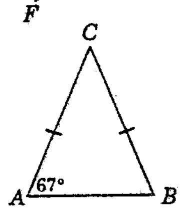
South Sydney High School
GEOMETRY
3 Unit Worksheet

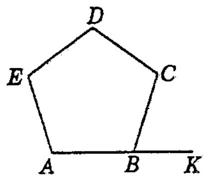
EXERCISE 8: EXAMINATION-TYPE QUESTIONS

1. 

In the figure $AB \parallel CD$ and $D\hat{G}F = 125°$. Find the size of $E\hat{H}B$.

2. 

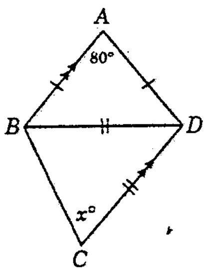
In the diagram $AC = BC$ and $C\hat{A}B = 67°$. Find the size of $B\hat{C}A$.

3. 

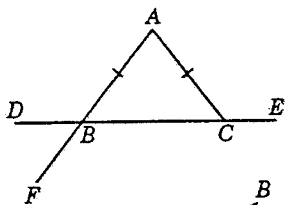
The figure $ABCDE$ is a regular pentagon.

(a) Find the size of $B\hat{C}D$.

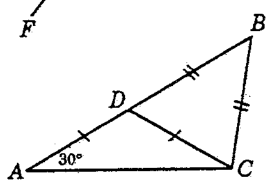
(b) What is the size of $C\hat{B}K$?

4. 

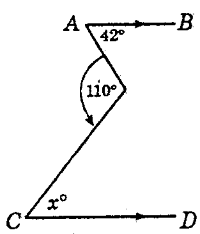
Given $AB = AD$, $BD = CD$, $D\hat{A}B = 80°$ and $AB \parallel DC$, find the value of x.

5. 

In the figure $AB = AC$ and $F\hat{B}D = 52°$. Find the size of $A\hat{C}E$.

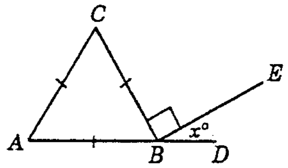
6. 

In the figure A, D and B are collinear. If $BD = BC$, $AD = DC$ and $B\hat{A}C = 30°$, find the size of $C\hat{B}D$.

7. 

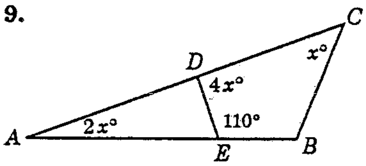
AB and CD are parallel lines.
$B\hat{A}E = 42°$ and $C\hat{E}A = 110°$
Find the value of x. Give reasons.

8.



In the diagram ΔABC is equilateral. A, B and D are collinear. If $C\hat{B}E$ is a right angle, find x, giving reasons.

9.

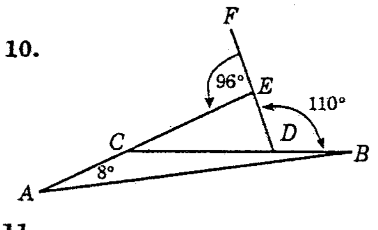


Use the information given in the diagram to find:

(a) the value of x;

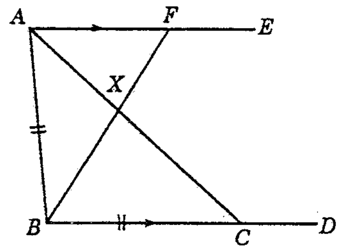
(b) the size of $E\hat{B}C$.

10.



In the diagram, not drawn to scale, $C\hat{A}B = 8°$, $E\hat{D}B = 110°$ and $A\hat{E}F = 96°$. Find the size of $A\hat{B}C$.

11.

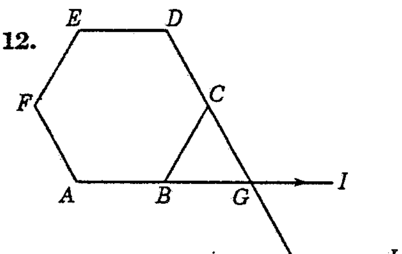


In the diagram ΔABC is isosceles, $AB = BC$, $AE \parallel BD$, $B\hat{C}A = 42°$ and $B\hat{X}A = 100°$.

(a) Draw a neat sketch of the given diagram and mark on it all the given information.

(b) Find: (i) $B\hat{F}A$ (ii) $A\hat{B}F$

12.

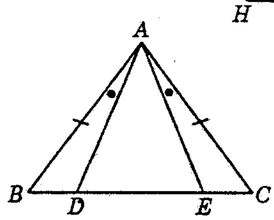


$ABCD$ is a regular hexagon. A, B, G and I lie on the same straight line. D, C, G and H are collinear. $AI \parallel HJ$

(a) Find the size of $F\hat{A}B$, giving reasons.

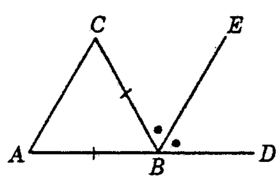
(b) Find the size of $G\hat{H}J$.

13.



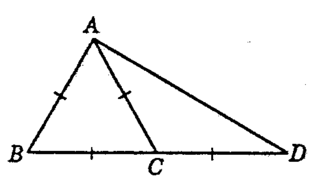
In the figure, ΔABC is an isosceles triangle where $AB = AC$. D and E are points on BC such that $D\hat{A}B = C\hat{A}E$. Prove that ΔADE is isosceles.

14.



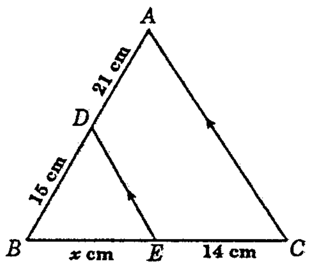
In the figure, not to scale, $AB = BC$ and BE bisects $C\hat{B}D$. Prove that $AC \parallel BE$.

15.



In the figure, ΔACD is an isosceles triangle where $AC = DC$. Triangle ABC is equilateral.
Prove that $AB \perp AD$ (i.e., show that $D\hat{A}B = 90°$).

16.

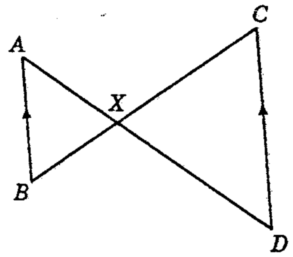


In the figure, $BD = 15$ cm, $EC = 14$ cm, $AD = 21$ cm and $AC \parallel DE$.

(a) Prove ΔABC is similar to ΔDBE.

(b) Hence, find the length of BE.

17.

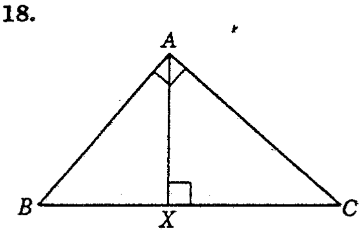


In the figure, $AB \parallel CD$.

(a) Prove that $\Delta ABX \parallel \Delta DCX$.

(b) If $BX = 1{\cdot}5$ cm, $CX = 4{\cdot}5$ cm, and CD is 6 cm, find the length of AB.

18.

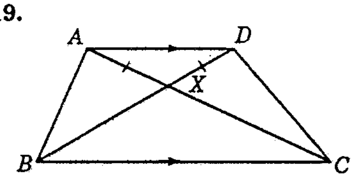


In the figure, $C\hat{A}B = 90°$, $AB = 6$ cm, $BC = 10$ cm and $AX \perp BC$.

(a) Prove that ΔABC is similar to ΔABX.

(b) Hence, or otherwise, find the length of BX.

19.



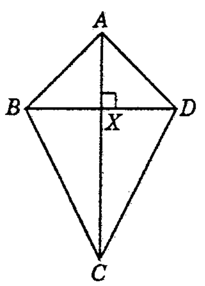
In the figure, $AD \parallel BC$, $AX = DX$ and $A\hat{B}C = B\hat{C}D$.

(a) Show that $A\hat{B}D = A\hat{C}D$.

(b) Prove that $\Delta DAB \equiv \Delta ACD$.

(c) Show that $AB = DC$.

20.

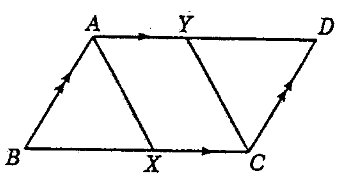


In the diagram, $ABCD$ is a quadrilateral. The diagonal AC bisects BD at right angles.

(a) Prove that $\Delta ABX \equiv \Delta AXD$ and hence show that AC bisects $D\hat{A}B$.

(b) Prove that $DC = BC$.

21.



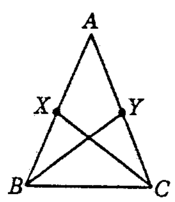
In the figure, $ABCD$ is a parallelogram. $BX = DY$.

(a) Prove that $\Delta ABX \equiv \Delta CDY$.

(b) Hence show that $AX = CY$.

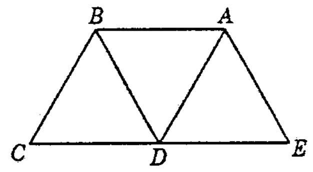
(c) Prove that $AXCY$ is a parallelogram.

22.



In the figure, ABC is an isosceles triangle with base BC (i.e. $AB = AC$). X and Y are the midpoints of sides AB and AC respectively.
Prove that $\Delta XBC \equiv \Delta YBC$.

23.

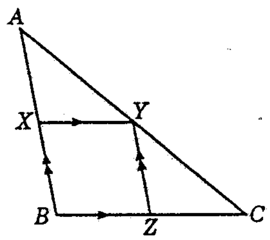


In the figure, $ABCD$ is a parallelogram. The side CD is produced to E so that $DC = ED$.

(a) Prove that $\Delta AED \equiv \Delta BDC$.

(b) Prove that figure $ABDE$ is a parallelogram.

24.

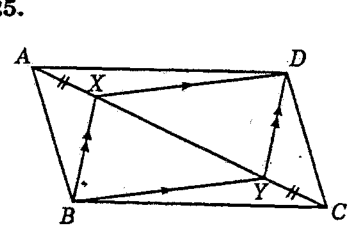


AB and BC are straight lines. $XBZY$ is a parallelogram. $ZC = ZY$ and $AX = BX$. The points A, Y and C lie on the same straight line.

(a) Prove that $\Delta AXY \equiv \Delta CYZ$.

(b) Prove that $XBZY$ is a rhombus.

25.



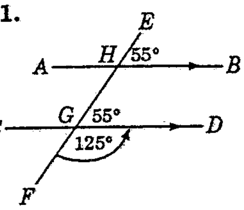
In the figure, AC is the diagonal of the quadrilateral $ABCD$. $BYDX$ is a parallelogram and $AX = CY$.

(a) Prove that $AB = DC$ and $AB \parallel DC$.

(b) Hence, prove that $ABCD$ is a parallelogram.

SOLUTIONS TO EXERCISE 8

1.

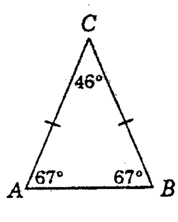


$H\hat{G}D = 55°$ (supplementary to $D\hat{G}F$)

$E\hat{H}B = 55°$ (corresponding to $H\hat{G}D$ and $AB \parallel CD$)

$\therefore\ E\hat{H}B = 55°$

2.

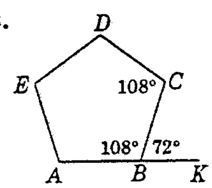


$A\hat{B}C = C\hat{A}B = 67°$ (base angles of isosceles ΔABC)

$B\hat{C}A = 46°$ (angle sum of ΔABC)

$\therefore\ B\hat{C}A = 46°$

3.



(a) Angle is a regular polygon $= \dfrac{(2n-4)\times 90°}{n}$

For pentagon, $n = 5$

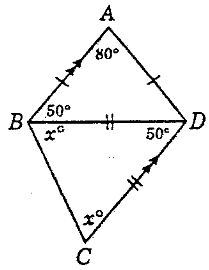
$\therefore\ B\hat{C}D = \dfrac{(2\times 5-4)\times 90°}{5} = 108°$

(b) $A\hat{B}C = 108°$ (angle in a regular pentagon)

$C\hat{B}K = (180-108)°$ (supplementary to $A\hat{B}C$)

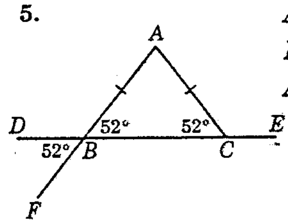
$= 72°$

4.



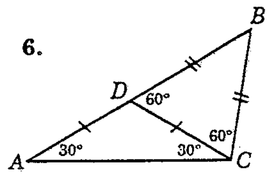
$A\hat{B}D = 50°$ (base angle of isosceles ΔABD)
$C\hat{D}B = 50°$ (alternate to $A\hat{B}D$ and $AB \parallel DC$)
$D\hat{B}C = B\hat{C}D$ (base angles of isosceles ΔBCD)
$x + x + 50 = 180$ (angle sum of ΔBCD)
$2x = 130$
$\therefore \quad x = 65$

5.



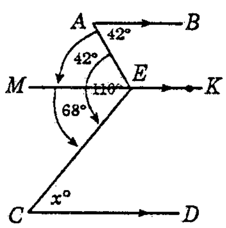
$A\hat{B}C = 52°$ (vertically opposite to $F\hat{B}D$)
$B\hat{C}A = 52°$ (base angle of isosceles ΔABC)
$A\hat{C}E = (180 - 52)°$ (supplementary to $B\hat{C}A$)
$= 128°$

6.



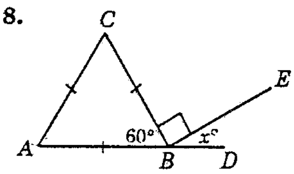
$A\hat{C}D = D\hat{A}C = 30°$ (base angles of isosceles ΔDAC)
$B\hat{D}C = 60°$ (exterior angle of ΔDAC)
$D\hat{C}B = B\hat{D}C = 60°$ (base angles of isosceles ΔBDC)
$C\hat{B}D = 60°$ (angle sum of ΔCBD)

7.



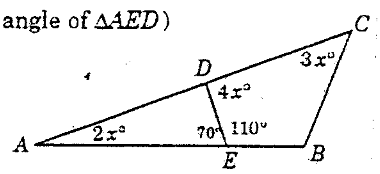
Construct a line MK parallel to AB through E.
$AB \parallel CD$, $MK \parallel AB$ $\therefore$ $MK \parallel CD$
$M\hat{E}A = 42°$ (alternate to $A\hat{B}E$ and $AB \parallel MK$)
$M\hat{E}A + C\hat{E}M = 110°$
$42° + C\hat{E}M = 110°$
$\therefore \quad C\hat{E}M = 68°$
$E\hat{C}D = 68°$ (alternate to $C\hat{E}M$ and $MK \parallel CD$)
$\therefore \quad x = 68°$

8.



$A\hat{B}C = 60°$ (angle in equilateral Δ)
$A\hat{B}C + C\hat{B}E + E\hat{B}D = 180°$ (A, B and D are collinear)
$\therefore \quad 60 + 90 + x = 180$
$x = 30$

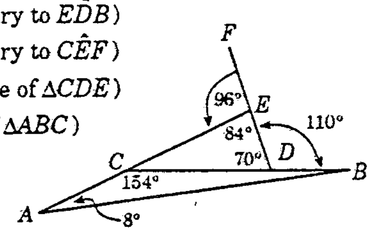
9. (a) $A\hat{E}D = 70°$ (supplementary to $D\hat{E}B$)
$C\hat{D}E = D\hat{A}E + A\hat{E}D$ (exterior angle of ΔAED)
$\therefore \quad 4x = 2x + 70$
$2x = 70$
$x = 35$



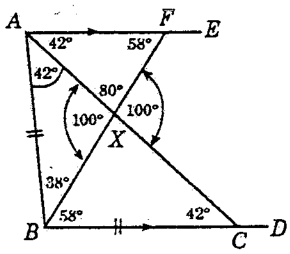
(b) $C\hat{D}E = 4x° = (4 \times 35)° = 140°$
$B\hat{C}D = x° = 35°$
$A\hat{B}C + B\hat{C}D + C\hat{D}E + D\hat{E}B = 360°$
(angle sum of quadrilateral $EBCD$)
$\therefore \quad A\hat{B}C + 35° + 140° + 110° = 360°$
$\therefore \quad A\hat{B}C = 75°$

10.

$C\hat{D}E = 70°$ (supplementary to $E\hat{D}B$)
$D\hat{E}C = 84°$ (supplementary to $C\hat{E}F$)
$D\hat{C}A = 154°$ (exterior angle of ΔCDE)
$A\hat{B}C + 150° + 8° = 180°$ (angle sum of ΔABC)
$\therefore \quad A\hat{B}C = 22°$



11. (a)



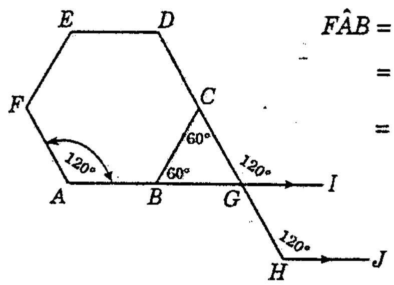
(b) (i)

$E\hat{A}C = 42°$ (alternate to $B\hat{C}A$ and $AE \parallel BC$)
$B\hat{X}A = 100°$ (vertically opposite to $F\hat{X}C$)
$A\hat{X}F = 80°$ (supplementary to $B\hat{X}A$)
$B\hat{F}A + 42° + 80° = 180°$ (angle sum of ΔAXF)
$\therefore \quad B\hat{F}A = 58°$

(ii)

$F\hat{B}C = 58°$ (alternate to $B\hat{F}A$ and $AE \parallel BC$)
$C\hat{A}B = 42°$ (base angles of isosceles ΔABC)
$F\hat{A}B + A\hat{B}C = 180°$ (cointerior angles and $AE \parallel BC$)
$\therefore \quad 42° + 42° + A\hat{B}F + 58° = 180°$
$\therefore \quad A\hat{B}F = 38°$

12. (a)



$F\hat{A}B$ = angle in a regular hexagon

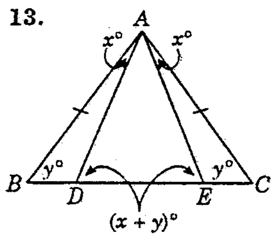
$= \dfrac{(2n-4)\times 90°}{n}$, where $n = 6$

$= \dfrac{(2\times 6-4)\times 90°}{6} = 120°$

(b)

$C\hat{B}G = 60°$ (exterior angle of hexagon)
$G\hat{C}B = 60°$ (exterior angle of hexagon)
$C\hat{G}I = 120°$ (exterior angle of ΔCBG)
$G\hat{H}J = 120°$ (corresponding to $C\hat{G}I$ and $GI \parallel HJ$)

13.

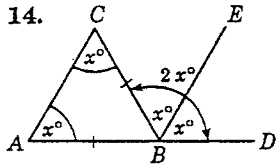


Data
ΔABC is isosceles, where $AB = AC$.
$D\hat{A}B = C\hat{A}E$

Aim
Prove that ΔADE is isosceles.

Proof
Let $C\hat{A}E = x°$ $\therefore$ $D\hat{A}B = x°$ (data)
Let $B\hat{C}A = y°$ $\therefore$ $A\hat{B}C = y°$ (base angles of isosceles ΔABC)
$A\hat{D}E = (x+y)°$ (exterior angle of ΔABD)
$D\hat{E}A = (x+y)°$ (exterior angle of ΔAEC)
$\therefore \quad A\hat{D}E = D\hat{E}A = (x+y)°$
$\therefore$ ΔADE is isosceles (base angles are equal).

14.



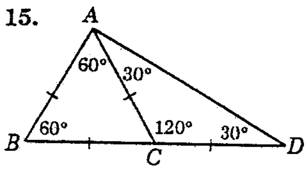
Data
ΔABC is isosceles, where $AB = CB$ and BE bisects $C\hat{B}D$.

Aim
To prove that $AC \parallel BE$.

Proof Let $C\hat{A}B = x°$

$\therefore\ B\hat{C}A = x°$ (base angles of isosceles ΔABC)
$C\hat{B}D = 2x°$ (exterior angles of ΔABC)
$C\hat{B}E = E\hat{B}D = x°$ (BE bisects $C\hat{B}D$)
$C\hat{A}B = E\hat{B}D$ (both $x°$)
$\therefore\ AC \parallel BE$ (a pair of corresponding angles [$C\hat{A}B$ and $E\hat{B}D$] are equal)

15.

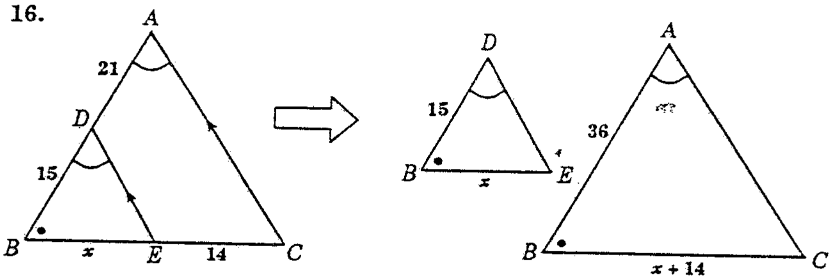


Data
ΔABC is equilateral and ΔACD is isosceles.

Aim
To prove $AB \perp AD$.

Proof $A\hat{B}C = C\hat{A}B = 60°$ (angles in equilateral ΔABC)
$A\hat{C}D = 120°$ (exterior angle of ΔABC)
$D\hat{A}C = C\hat{D}A = 30°$ (base angles of isosceles ΔACD)
$D\hat{A}B = 60° + 30° = 90°$, i.e. $AB \perp AD$

16.



(a) **Data** $AC \parallel DE$, $BE = x$ cm, $EC = 14$ cm, $BD = 15$ cm and $DA = 21$ cm

Aim To prove $\Delta ABC ||| \Delta DBE$.

Proof In Δ's BED and BCA
$D\hat{B}E = A\hat{B}C$ (common to both triangles)
$E\hat{D}B = C\hat{A}B$ (corresponding angles and $AC \parallel DE$)
$\therefore\ \Delta BED ||| \Delta BCA$ (equiangular)

(b) As $\Delta BED ||| \Delta BCA$,

$\frac{BE}{BC} = \frac{BD}{BA}$ (corresponding sides of similar triangles are in the same ratio)

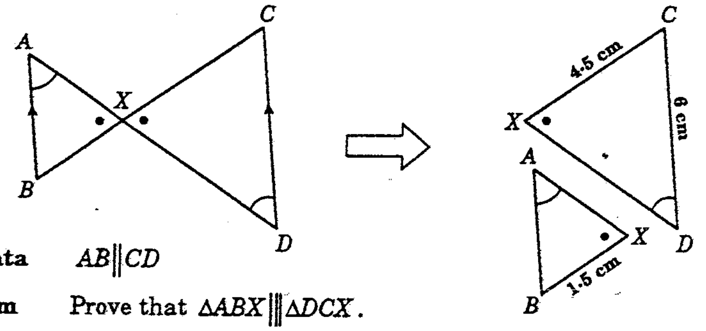
$\therefore\ \frac{x}{x+14} = \frac{15}{36}$

$\therefore\ \frac{x}{x+14} = \frac{5}{12}$.

$12x = 5(x+14)$
$12x = 5x + 70$
$7x = 70$
$\therefore\ x = 10$

$\therefore$ the length of BE is 10 cm.

17.



(a) Data $AB \parallel CD$

Aim Prove that $\triangle ABX\,|||\,\triangle DCX$.

Proof In Δ's ABX and DCX

$B\hat{X}A = C\hat{X}D$ (vertically opposite angles)

$X\hat{A}B = X\hat{D}C$ (alternate angles and $AB \parallel CD$)

$\therefore\ \triangle ABX\,|||\,\triangle DCX$ (equiangular)

(b) As $\triangle ABX\,|||\,\triangle DCX$,

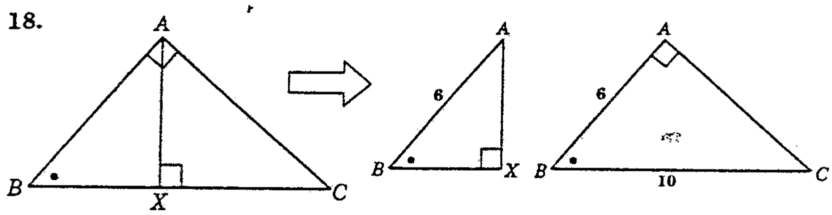
$\dfrac{AB}{CD} = \dfrac{BX}{CX}$ (corresponding sides of similar triangles are in the same ratio)

$\therefore$ since $CD = 6$ cm, $BX = 1{\cdot}5$ cm and $CX = 4{\cdot}5$ cm,

then $\dfrac{AB}{6} = \dfrac{1{\cdot}5}{4{\cdot}5} \Rightarrow AB = \dfrac{1{\cdot}5}{4{\cdot}5} \times 6 = 2$

$\therefore$ the length of AB is 2 cm.

18.



(a) Data $AX \perp BC$, (i.e. $A\hat{X}C = 90°$) and $C\hat{A}B = 90°$

Aim To prove that $\triangle ABC\,|||\,\triangle ABX$

Proof In Δ's ABC and ABX

$C\hat{A}B = B\hat{X}A = 90°$ (data)

$A\hat{B}C = A\hat{B}X$ (common to both triangles)

$\therefore\ \triangle ABC\,|||\,\triangle ABX$ (triangles ABC and ABX are equiangular)

(b) As $\triangle ABC\,|||\,\triangle ABX$,

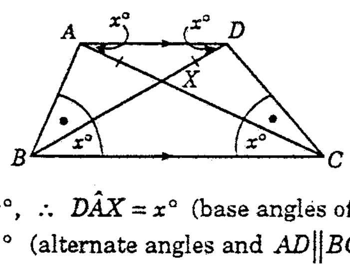
$\dfrac{BX}{AB} = \dfrac{AB}{BC}$ (corresponding sides in similar triangles are in the same ratio)

$\therefore$ since $AB = 6$ cm, $BC = 10$ cm,

then $\dfrac{BX}{6} = \dfrac{6}{10} \Rightarrow BX = \dfrac{6}{10} \times 6 = 3{\cdot}6$

$\therefore$ the length of BX is $3{\cdot}6$ cm.

19. (a)



Let $X\hat{D}A = x°$, $\therefore\ D\hat{A}X = x°$ (base angles of isosceles $\triangle AXD$)

$D\hat{B}C = x°$ (alternate angles and $AD \parallel BC$)

Also $B\hat{C}A = x°$ (alternate angles and $AD \parallel BC$)

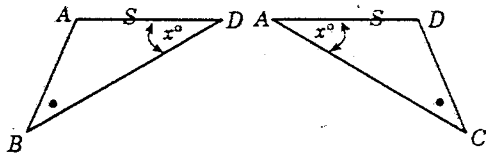
It was given that $A\hat{B}C = B\hat{C}D$

$\therefore \quad A\hat{B}D + D\hat{B}C = B\hat{C}A + A\hat{C}D$

$\therefore \quad A\hat{B}D + x° = x° + A\hat{C}D$

$\therefore \quad A\hat{B}D = A\hat{C}D$

(b)



Data In the figure $AD \| BC$,
$AX = DX$ and $A\hat{B}C = B\hat{C}D$.

Aim To prove $\Delta DAB \equiv \Delta ACD$.

Proof

In Δ's DAB and ACD

$B\hat{D}A = D\hat{A}C$ (A) (base angles of isosceles ΔDAX)

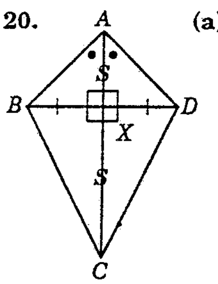
$A\hat{B}D = A\hat{C}D$ (A) (proved in part a)

$AD = AD$ (S) (common to both triangles)

$\therefore \Delta DAB \equiv \Delta ACD$ (AAS)

(c) $AB = DC$ (a pair of corresponding sides in congruent triangles DAB and ACD).

20.



(a) **Data** In the figure, $ABCD$ is a quadrilateral where the diagonal AC bisects BD at right angles.

Aim To prove $\Delta ABX \equiv \Delta AXD$.

Proof

In Δ's ABX and AXD,

$BX = DX$ (S) (AC bisects BD)

$B\hat{X}A = D\hat{X}A$ (A) (AC bisects BD at right angles)

$AX = AX$ (S) (common to both triangles)

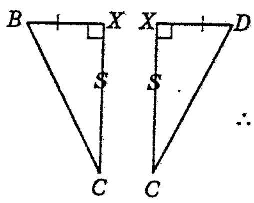
$\therefore \Delta ABX \equiv \Delta AXD$ (SAS)

$B\hat{A}X = D\hat{A}X$ (corresponding angles in congruent triangles ABX and AXD)

$\therefore$ AC bisects $D\hat{A}B$, since $D\hat{A}B = B\hat{A}X + D\hat{A}X$ and $B\hat{A}X = D\hat{A}X$

(b) To prove that $DC = BC$, prove that $\Delta BCX \equiv \Delta DXC$ and then deduce the required result.

In Δ's BCX and DXC,



$BX = DX$ (S) (AC bisects BD)

$C\hat{X}B = D\hat{X}C$ (A) (AC bisects BD at right angles)

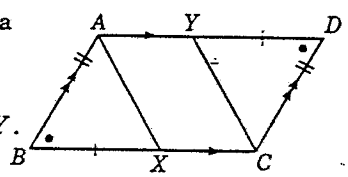
$CX = CX$ (S) (common to both triangles)

$\therefore \Delta BCX \equiv \Delta DXC$ (SAS)

$DC = BC$ (corresponding sides in congruent triangles BCX and DXC)

21. (a) **Data** In the figure, $ABCD$ is a parallelogram and $BX = DY$.

Aim To prove $\Delta ABX \equiv \Delta CDY$.



Proof

In Δ's ABX and CDY,

$AB = DC$ (S) (opposite sides of a parallelogram)

$A\hat{B}X = C\hat{D}Y$ (A) (opposite angles of a parallelogram)

$BX = DY$ (S) (given)

$\therefore \Delta ABX \equiv \Delta CDY$ (SAS)

(b) $AX = CY$ (corresponding sides in congruent Δ's ABX and CDY).

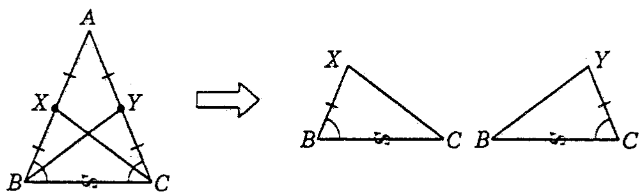
(c) $BC = AD$ (opposite sides of a parallelogram)

$\therefore BX + CX = DY + AY$, but $BX = DY$

$\therefore CX = AY$

Figure $AXCY$ is a parallelogram because both pairs of opposite sides are equal, i.e. $AX = CY$ and $CX = AY$.

22.



(a) Data ABC is an isosceles triangle, $AB = AC$. X and Y are the midpoints of sides AB and AC respectively.

Aim To prove $\Delta XBC \equiv \Delta YBC$.

Proof In Δ's XBC and YBC,

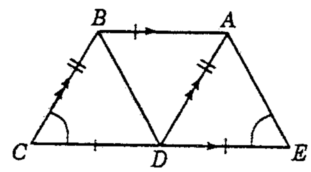
$BX = CY$ ($AB = AC$ and X and Y are the mid-points of AB and AC respectively)

$X\hat{B}C = B\hat{C}Y$ (given in data)

$BC = BC$ (common to both triangles)

$\therefore \Delta XBC \equiv \Delta YBC$ (SAS)

23.



(a) Data In the figure, $ABCD$ is a parallelogram. CD is produced to E so that $DC = ED$.

Aim To prove $\Delta AED \equiv \Delta BDC$.

Proof

In Δ's AED and BDC,

$AD = BC$ (S) (opposite sides of parallelogram)

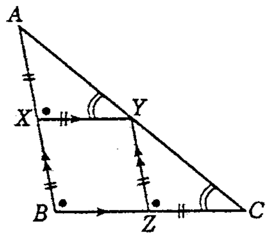
$A\hat{D}E = B\hat{C}D$ (A) (corresponding angles, $CB \parallel AD$)

$DE = DC$ (S) (given in data)

$\therefore \Delta AED \equiv \Delta BDC$ (SAS)

(b) $AB = CD$ (opposite sides of a parallelogram), but $CD = DE$ $\therefore AB = DE$. Figure $ABDE$ is a parallelogram, since a pair of opposite sides ($AB = DE$ and $AB \parallel DE$) is both parallel and equal.

24.



(a) **Data** In the figure, $XBZY$ is a parallelogram, $ZC = ZY$, $AX = BX$ and points A, Y and C are collinear.

Aim To prove that $\Delta AXY \equiv \Delta CYZ$.

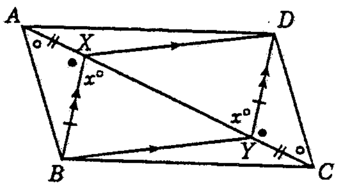
Proof

$X\hat{B}Z = Y\hat{Z}C$ (corresponding angles and $XB \parallel YZ$)
$X\hat{B}Z = A\hat{X}Y$ (corresponding angles and $XY \parallel BZ$)
$\therefore$ $Y\hat{Z}C = A\hat{X}Y$ (both equal to $\angle XBZ$)
$X\hat{Y}A = Y\hat{C}Z$ (corresponding angles and $XY \parallel BZ$)
$XB = YZ$ (opposite sides of a parallelogram)
but $AX = XB$, $\therefore$ $AX = YZ$

In Δ's AXY and CYZ,
$A\hat{X}Y = Y\hat{Z}C$ (S) (proved above)
$X\hat{Y}A = Z\hat{C}Y$ (A) (proved above)
$AX = YZ$ (S) (proved above)
$\therefore$ $\Delta AXY \equiv \Delta CYZ$ (AAS)

(b) $XY = ZC$ (corresponding sides in congruent Δ's AXY and CYZ)
But $ZY = ZC$ $\therefore XY = ZY$.
$XBZY$ is a parallelogram with a pair of adjacent sides equal,
i.e. $XY = ZY$ (and therefore all sides are equal),
so $XBZY$ is a rhombus.

25.



(a) **Data** In the figure, AC is the diagonal of $ABCD$. $BYDX$ is a parallelogram and $AX = CY$.

Aim To prove that $AB = DC$ and $AB \parallel DC$.

Proof

$Y\hat{X}B = X\hat{Y}D = x°$ (alternate angles and $BX \parallel YD$)
$B\hat{X}A = (180 - x)°$ (supplementary to $Y\hat{X}B$)
$D\hat{Y}C = (180 - x)°$ (supplementary to $X\hat{Y}D$)
$\therefore$ $B\hat{X}A = D\hat{Y}C$

In Δ's ABX and CDY,
$AX = CY$ (S) (given in data)
$B\hat{X}A = D\hat{Y}C$ (A) (proved above)
$BX = DY$ (S) (opposite sides of parallelogram $BYDX$)
$\therefore$ $\Delta ABX \equiv \Delta CDY$ (SAS)
$\therefore$ $AB = DC$ (corresponding sides in congruent Δ's)
also $X\hat{A}B = Y\hat{C}D$ (corresponding angles in congruent Δ's)
$\therefore$ $AB \parallel DC$ (a pair of alternate angles $X\hat{A}B$ and $Y\hat{C}D$ are equal)

(b) $ABCD$ is a parallelogram since a pair of opposite sides are equal and parallel, i.e. $AB = DC$ and $AB \parallel DC$.