South Sydney High School
TRIGONOMETRY
3 Unit Worksheet

EXERCISE 3: EXAMINATION-TYPE QUESTIONS

1. **(a)** Write an exact expression for:
(i) $\cos^2 60° - \sin^2 60°$ (ii) $\tan 30° \cos 30° \sin 30°$

(b) If θ lies between 0° and 360°, find all values of θ given that:
(i) $\cos\theta = \frac{\sqrt{3}}{2}$ (ii) $2\cos\theta = -\sqrt{3}$

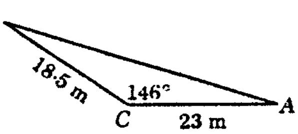
2. **(a)** Simplify to a surd with rational denominator $\frac{2\tan 30°}{1 - \tan^2 30°}$.

(b) Find all values of θ, $0° \le \theta° \le 360°$ when:
(i) $\sec\theta = 2$ (ii) $\sin^2\theta = \frac{1}{2}$

3. **(a)** Express $\sin 30° \cos 45° + \cos 30° \sin 45°$ as a fraction with rational denominator.

(b) Find all values of ϕ, $0° \le \phi° \le 360°$, where:
(i) $\tan\phi = -1$ (ii) $\sin\phi = -\cos\phi$

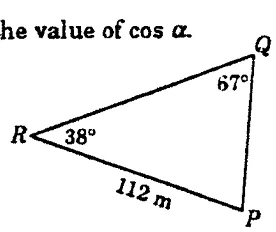
4. Use the Cosine Rule to calculate the length of the side AB (correct to one decimal place).



5. If $\cos x° = \frac{3}{5}$, and $0° < x° < 90°$, calculate the value of $\tan x°$.

6. Given that $\sin\alpha = \frac{5}{13}$, and α is acute, find the value of $\cos\alpha$.

7. Use the Sine Rule to calculate the length of the fence PQ of this triangular paddock. Answer correct to the nearest metre.



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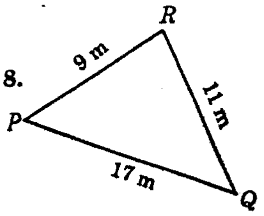
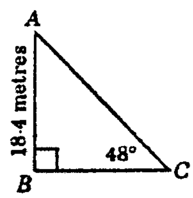


Use the Cosine Rule to calculate the largest angle in the triangle PQR with the sides as shown in the diagram. Answer correct to the nearest minute.

9. A wire connected to the ground at C is fixed to a mast at A such that the wire makes an angle of 48° with the ground. Use trigonometry to calculate the length of wire used (AC) to the nearest tenth of a metre.



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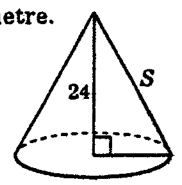
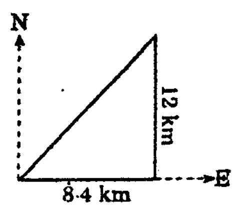
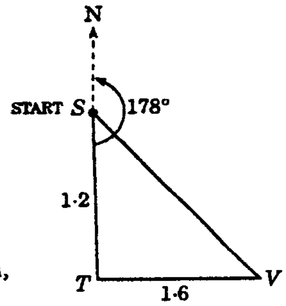


A cone 24 cm high has a semi-vertical angle of 21°. Use trigonometry to calculate the slant height, S, of the cone.

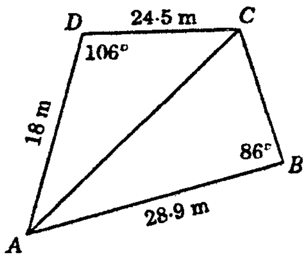
11. Boris leaves Belmont and sails due east for 8·4 km. He then changes direction and sails due north for 12 km. Find his bearing as calculated from Belmont.



12. **Jillian, during an orienteering exercise, runs 1·2 km on a bearing of 178°, turns due east and runs a further 1·6 km. Calculate her distance from the starting point (*VS*) using the Cosine Rule.**



13. In the figure below, $AD = 18$ m, $AB = 28{\cdot}9$ m, $CD = 24{\cdot}5$ m, $\angle ADC = 106°$, $\angle ABC = 86°$.



(a) In ΔADC use the Cosine Rule to calculate AC correct to one decimal place.

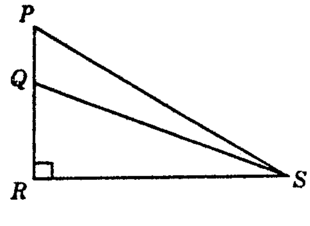
(b) In ΔABC use the Sine Rule to calculate $\angle ACB$ to the nearest degree.

(c) Calculate the area of $ABCD$ to the nearest m^2.

14. From the top of a building (P) the angle of depression of a point S (level with the base of the building) is found to be 32°. A second measurement is taken from a window (Q) 20 m lower down the building. The angle of depression at Q is measured to be 27°.

(a) From ΔPQS use the Sine Rule to calculate the length of QS (correct to one decimal place).

(b) Calculate the distance of S from the foot of the building (nearest metre).

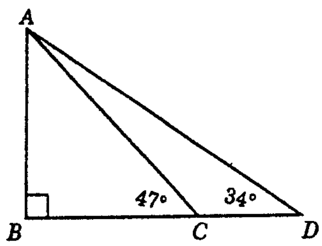


15. Angela measured the angle of elevation of the top of a tower (AB) from two positions C and D 32 metres apart and in a direct line from B, the base of the tower. The angles were 47° and 34°.

(a) Show that $\angle DAC = 13°$.

(b) Use the Sine Rule in ΔACD to calculate AC correct to one decimal place.

(c) Calculate the height of the tower, AB, correct to 3 significant figures.



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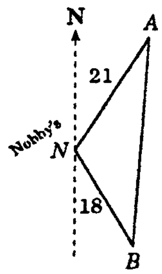
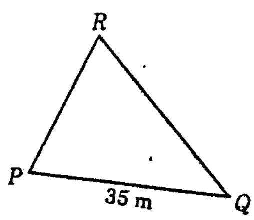


Two trawlers *Arafura* and *Beering* leave Nobby's Headland. *Arafura* steers on a bearing of 049° for 21 km while *Beering* travels along a bearing of 164° for 18 km. Calculate the distance between the two trawlers (AB) correct to 3 significant figures.

17. In a ΔABC, $a = 6{\cdot}4$, $c = 7{\cdot}8$ and $\angle B = 116°$. Calculate b correct to 3 significant figures.

18. Two archers placed at P and Q 35 metres apart fire at a target R. If $\angle RPQ = 71°$ and $\angle RQP = 49°$, calculate the distance between the nearest archer and the target.

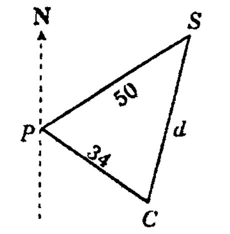


19. A coal ship (S) leaves Port Pathetic (P) and travels 50 km on a bearing of 58°, while a catamaran (C) also leaves P, but on a bearing of 124° and travelling 34 km.

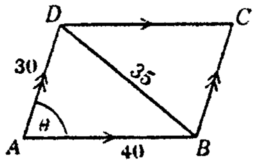
Find:

(a) the distance between the ship and the catamaran;

(b) the bearing of the catamaran as measured from the ship.



20. Find the angles of the parallelogram and the length of the diagonal AC, given that diagonal DB measures 35 cm and a pair of adjacent sides are 30 cm and 40 cm long. Calculate the area of the parallelogram.



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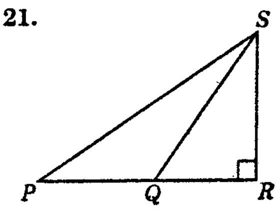
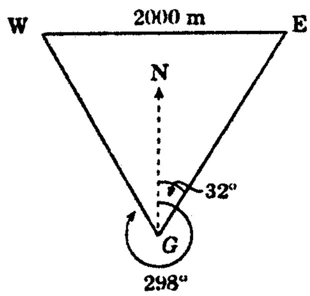


The captain of a yacht at P observes the angle of elevation of the top of a cliff (S) to be 29°. The yacht sails directly towards the cliff to Q where the angle of elevation is calculated as 46°. If the height of the cliff is 140 m calculate the distance PQ.

22. Georgio, standing at G, takes the bearing of both ends of an east–west runway (EW) 2000 m long. The bearings are 032° and 298° respectively. Use the Sine Rule to calculate the distance of Georgio to the nearest end of the runway (to the nearest metre).



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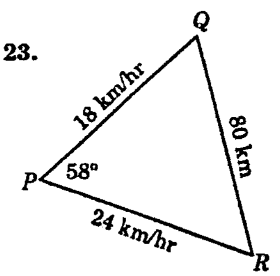
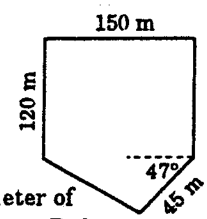


Two straight roads PQ and PR are inclined to each other at 58°. Two bike riders begin simultaneously from P and travel along the roads at 18 and 24 km/hour respectively. How long is it before they are 80 km apart in a direct line? Answer to the nearest minute.

24. Farmer DeSilvio's paddock is rectangular with a triangular section at one end as in the diagram.

(a) Calculate the area of this paddock to the nearest m^2.

(b) Farmer DeSilvio wished to calculate the perimeter of this paddock for fencing purposes. Use the Cosine Rule in the triangle and find the perimeter of this paddock.



25. (a) By adding the fractions, write a simpler expression for

$$\frac{1+\cos\theta}{\sin\theta} - \frac{\sin\theta}{1-\cos\theta}.$$

(b) Simplify the expression $\dfrac{\tan 60° - \tan 30°}{1+\tan 60° \tan 30°}$, writing your answer as a fraction with rational denominator.

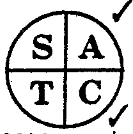
EXERCISE 3: WORKED SOLUTIONS

1. (a) (i) $\left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$

(ii) $\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{1}{4}$

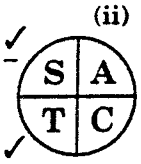
(b) (i) $\cos\theta = \frac{\sqrt{3}}{2}$



$\theta = 30°$ and $(360 - 30)°$

$= 30°$ and $330°$

(ii) $\cos\theta = -\frac{\sqrt{3}}{2}$



Acute angle is 30°
cos negative in 2nd, 3rd quadrants.

$\theta = (180 - 30)°, (180 + 30)°$

$= 150°, 210°$

2. (a) $\frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2}$ $\boxed{\tan 30° = \frac{1}{\sqrt{3}}}$

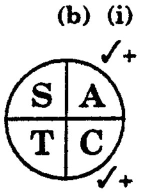
$= \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$

$= \frac{2}{\sqrt{3}} \div \frac{2}{3} = \frac{2}{\sqrt{3}} \times \frac{3}{2}$

$= \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ (rationalise)

$= \frac{3\sqrt{3}}{3} = \sqrt{3}$

(b) (i) $\sec\theta = 2$ (take reciprocal)
or
$\cos\theta = \frac{1}{2}$



$\therefore\ \theta = 60°$ or $(360 - 60)°$
$\theta = 60°$ or $300°$

(ii) $\sin^2\theta = \frac{1}{2}$

$\therefore\ \sin\theta = \pm\frac{1}{\sqrt{2}}$

$\therefore\ \theta = 45°, (180 - 45)°, (180 + 45)°, (360 - 45)°$

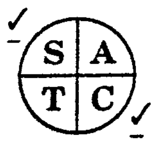
$\theta = 45°, 135°, 225°$ or $315°$.

3. (a) $\frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$

$= \frac{1 + \sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ (rationalise)

$= \frac{\sqrt{2} + \sqrt{6}}{4}$

(b) (i) $\tan\phi = -1$



Acute angle is 45°.
Tan ϕ is negative in 2nd and 4th quadrants.

$\therefore\ \phi = (180 - 45)°, (360 - 45)°$

$= 135°$ or $315°$.

(ii) $\sin\phi = -\cos\phi$

÷ both sides by cos ϕ.

$\frac{\sin\phi}{\cos\phi} = -\frac{\cos\phi}{\cos\phi}$

$\therefore\ \tan\phi = -1$

from (i),

$\phi = 135°$ or $315°$.

4. $AB^2 = 18{\cdot}5^2 + 23^2 - 2(18{\cdot}5)(23)\cos 146°$

$= 1576{\cdot}761$ (Cosine Rule)

$\therefore\ AB = \sqrt{1576{\cdot}761}$

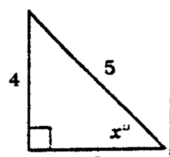
$= 39{\cdot}70845$

$\approx 39{\cdot}7$ (1 dec. pl.)

$\therefore$ AB is 39·7 metres.

5. $\cos x° = \frac{3}{5}$

Using Pythagoras, other side is 4.



$\therefore\ \tan x° = \frac{4}{3}$

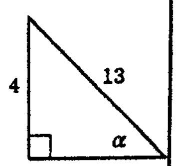
6. $\sin\alpha = \frac{5}{13}$

By Pythagoras,

$x^2 = 13^2 - 5^2$

$= 144$

$x = 12$ Other side is 12.



$\therefore\ \cos\alpha = \frac{12}{13}$ $\boxed{\frac{x}{13}}$

7. Let $PQ = x$ m, then using Sine Rule,

$\frac{x}{\sin 38°} = \frac{112}{\sin 67°}$

$\therefore\ x = \frac{112 \sin 38°}{\sin 67°}$

$= 74{\cdot}908986$

≈ 75 (nearest metre)

Length of fence PQ is 75 m.

8. Largest angle is opposite the longest side, that is, $\angle PRQ$.
Using the Cosine Rule,

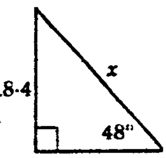
$\cos P\hat{R}Q = \frac{9^2 + 11^2 - 17^2}{2 \times 9 \times 11}$

$= -\frac{87}{198}$

$= -0{\cdot}4393939$

$\therefore\ P\hat{R}Q = 116°4'$

9. $\frac{18{\cdot}4}{x} = \sin 48°$



$\therefore\ \frac{x}{18{\cdot}4} = \frac{1}{\sin 48°}$

(Taking reciprocals of both sides.)

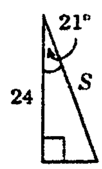
$\therefore\ x = \frac{18{\cdot}4}{\sin 48°}$

$= 24{\cdot}759642$

$\approx 24{\cdot}8$ (1 dec. place)

Length of wire is 24·8 metres.

10. $\frac{24}{s} = \cos 21°$



$\therefore\ \frac{s}{24} = \frac{1}{\cos 21°}$

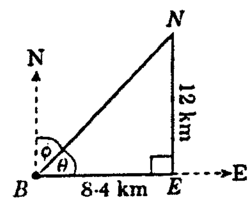
$\therefore\ s = \frac{24}{\cos 21°}$

$= 25{\cdot}70748$

$\approx 25{\cdot}7$ (1 dec. pl.)

Slant height is 25·7 cm.

11. Bearing is $\phi°$.
Find $\theta°$ from ΔBEN.



$\tan\theta = \frac{12}{8{\cdot}4}$

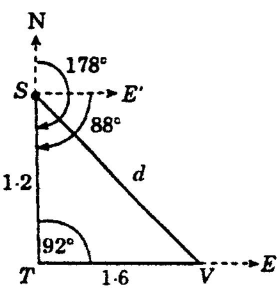
$\therefore\ \theta = 56°53'$

$\therefore\ \phi = 90° - 56°53'$

$= 33°7'$

Bearing is 033°7'.

12.



$\angle STV = 92°$

As $\angle NSE' = 90°$ ($\angle$ between N and E)

$\therefore\ \angle E'ST = 88°$ $(178 - 90)$

$\therefore\ \angle STE = 92°$ (cointerior $\angle$'s, $SE' \parallel TE$)

Using Cosine Rule,

$d^2 = 1{\cdot}2^2 + 1{\cdot}6^2 - 2(1{\cdot}2)(1{\cdot}6)\cos 92°$

$= 4{\cdot}1340141$

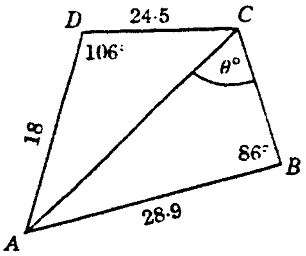
$d = \sqrt{4{\cdot}1340141}$

$= 2{\cdot}0332275$

$\approx 2{\cdot}0$ (1 dec. pl.)

Distance from start is 2·0 km.

13.



(a) In ΔADC,

$AC^2 = 18^2 + 24\cdot5^2 - 2(18)(24\cdot5)\cos 106°$

$= 1167\cdot3621$

$AC = \sqrt{1167\cdot3621}$

$= 34\cdot166\,668\,2$

$\approx 34\cdot2$ (1 dec. pl.)

$\therefore$ AC is 34·2 metres.

(b) In ΔABC, let $\angle ACB = \theta°$

$\frac{28\cdot9}{\sin\theta} = \frac{34\cdot166\,668\,2}{\sin 86°}$

$\therefore \quad \sin\theta = \frac{28\cdot9\sin 86°}{34\cdot166\,668\,2}$

$= 0\cdot843\,793\,1$

$\therefore \quad \theta = 57°33'$

(c) $\angle CAB$

$= 180° - (86° + 57°33')$

$= 36°27'$

Then area of $ABCD$

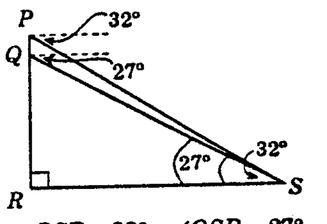
= area ΔACD + area ΔABC

$= \frac{1}{2} \times 18 \times 24\cdot5 \sin 106° + \frac{1}{2} \times 28\cdot9 \times AC \times \sin 36°27'$

$= 505\cdot280\,74$

≈ 505 (nearest whole number)

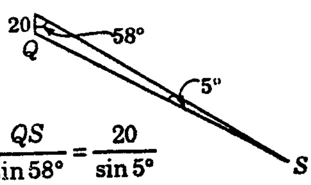
14.



$\angle PSR = 32°$, $\angle QSR = 27°$

(a) $\angle PSQ = 32° - 27° = 5°$

$\angle SPQ = 90° - 32° = 58°$



$\frac{QS}{\sin 58°} = \frac{20}{\sin 5°}$

$\therefore \quad QS = \frac{20\sin 58°}{\sin 5°}$

$= 194\cdot605\,21$

$\approx 194\cdot6$ (1 dec. point)

(b) $\frac{RS}{QS} = \cos 27°$

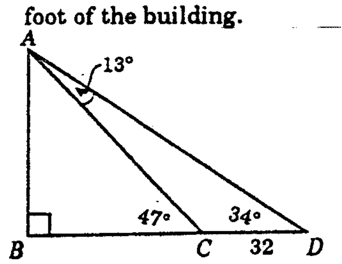
$RS = 194\cdot605\,21 \times \cos 27°$

$= 173\cdot394\,51$

≈ 173 (nearest whole number)

S is 173 metres from the foot of the building.

15.



(a) $\angle DAC = 47° - 34° = 130°$ (external angle of ΔACD)

(b) $\frac{AC}{\sin 34°} = \frac{32}{\sin 13°}$

$\therefore \quad AC = \frac{32\sin 34°}{\sin 13°}$

$= 79\cdot546\,962$

$\approx 79\cdot5$ (1 dec. pl.)

AC is 79·5 metres.

(c) In ΔABC,

$\frac{AB}{AC} = \sin 47°$

$\therefore AB = AC\sin 47°$

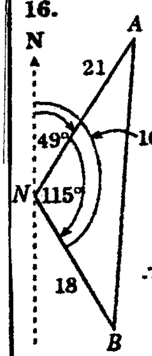
$= 79\cdot546\,962 \sin 47°$

$= 58\cdot176\,965$

$\approx 58\cdot2$ (3 sig. figs.)

Height of tower is 58·2 m.

16.



$\angle ANB = 164° - 49°$

$= 115°$

Using the Cosine Rule,

$AB^2 = 21^2 + 18^2 - 2(21)(18)\cos 115°$

$= 1084\cdot4994$

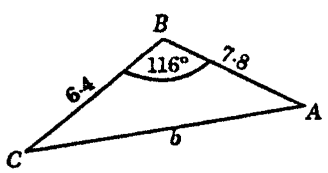
$\therefore AB = \sqrt{1084\cdot4994}$

$= 32\cdot931\,739$

$\approx 32\cdot9$

Distance between trawlers is 32·9 km.

17. $a = 6\cdot4$, $c = 7\cdot8$, $\angle B = 116°$



$b^2 = 6\cdot4^2 + 7\cdot8^2 - 2(6\cdot4)(7\cdot8)\cos 116°$

$= 145\cdot566\,98$

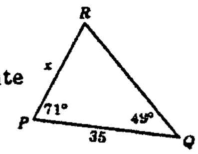
$b = \sqrt{145\cdot566\,98}$

$= 12\cdot065\,114$

$\approx 12\cdot1$ (3 sig. figs.)

$b = 12\cdot1$

18. Shortest side will be opposite the smallest angle.



$\angle PRQ = 180° - (71° + 49°) = 60°$

Using Sine Rule,

$\frac{x}{\sin 49°} = \frac{35}{\sin 60°}$

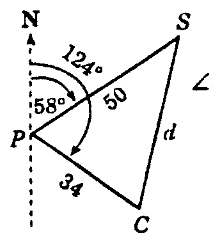
$\therefore \quad x = \frac{35\sin 49°}{\sin 60°}$

$= 30\cdot501\,225$

$\approx 30\cdot5$ (1 dec. place)

Distance of nearest archer to the target is 30·5 metres.

19.



$\angle SPC$

$= 124° - 58°$

$= 66°$

(a) Using the Cosine Rule,

$d^2 = 50^2 + 34^2 - 2(50)(34)\cos 66°$

$= 2273\cdot0954$

$\therefore \; d = \sqrt{2273\cdot0954}$

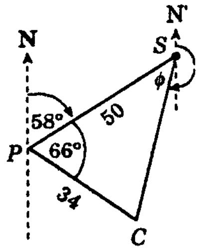
$= 47\cdot676\,99$

$\approx 47\cdot7$ (1 dec. place)

Distance between S and C is 47·7 km.

(b) Bearing is $\angle N'SC$

$\angle N'SP = 122°$ (co-int. $\angle$s, $NP \| N'S$)



$\frac{34}{\sin\phi} = \frac{d}{\sin 66}$

$\sin\phi = \frac{34\sin 66}{47\cdot676\,99}$

$= 0\cdot651\,478\,7$

$\phi = 40°39'$

Then $\angle N'SC$

$= 360° - (122° + 40°39')$

$= 197°21'$

Bearing of C from S is 197°21'.

20. $\cos\theta = \dfrac{30^2 + 40^2 - 35^2}{2 \times 30 \times 40}$

$= \dfrac{1275}{2400}$

$\therefore \theta = 57°55'$

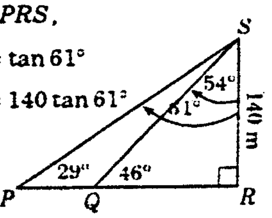
Then $\angle ADC = 122°5'$
(cointerior $\angle$'s, $AB \| DC$)

Angles of the parallelogram are 57°55' and 122°5'.

21. From ΔPRS,

$\dfrac{PR}{140} = \tan 61°$

$\therefore PR = 140 \tan 61°$



From ΔQRS, $\dfrac{PR}{140} = \tan 54°$

$\therefore QR = 140 \tan 54°$

Now PQ

$= PR - QR$

$= 140 \tan 61° - 140 \tan 54°$

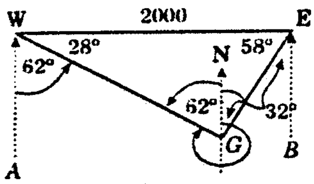
$= 140(\tan 61° - \tan 54°)$

$= 59 \cdot 873217$

$\approx 59 \cdot 9$

PQ is 59·9 metres.

22.



$\angle WGN = 360° - 298°$
$= 62°$

Then $\angle AWG = 62°$ (alt. $\angle$'s, $AW \| GN$)

$\angle BEG = 32°$ (alt. $\angle$'s, $EB \| GN$)

$\therefore \angle EWG = 28°$ $(90 - 62)$

$\angle WEG = 58°$ $(90 - 32)$

Shortest distance will be GE (opposite smallest angle).

$\dfrac{GE}{\sin 28°} = \dfrac{2000}{\sin 94°}$ $[62 + 32]$

$\therefore GE = \dfrac{2000 \sin 28°}{\sin 94°}$

$= 941 \cdot 23593$

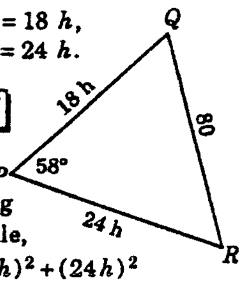
≈ 941 (nearest whole number)

Shortest distance is 941 metres.

23. Let time taken be h hours.

Then $PQ = 18\,h$,
$PR = 24\,h$.

$D = S \times T$



Then using Cosine Rule,

$80^2 = (18h)^2 + (24h)^2 - 2(18h)(24h)\cos 58°$

$= h^2[18^2 + 24^2 - 2(18)(24)\cos 58°]$

$\therefore h^2 = \dfrac{80^2}{18^2 + 24^2 - 2(18)(24)\cos 58°}$

$= 14 \cdot 474734$

$\therefore h = 3 \cdot 8045675$

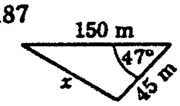
$= 3$ hours 48 minutes

Time taken is 3 hours 48 minutes.

24. (a) Area of Δ

$= \dfrac{1}{2} \times 45 \times 150 \times \sin 47°$

$= 2468 \cdot 3187$



Area of paddock

$= 150 \times 120 + 2468 \cdot 3187$

$= 20\,468 \cdot 319$

$\approx 20\,468$

Area is $20\,468\text{ m}^2$.

(b) $x^2 = 150^2 + 45^2 - 2(150)(45)\cos 47°$

$= 15\,318 \cdot 022$

$\therefore x = 123 \cdot 766$

Perimeter $= (120 \times 2) + 150 + 45 + 123 \cdot 766$

$= 558 \cdot 766$

≈ 559 (nearest metre)

Perimeter is 559 metres.

25. (a)

$\dfrac{(1 + \cos\theta)(1 - \cos\theta) - (\sin\theta)(\sin\theta)}{\sin\theta(1 - \cos\theta)}$

$= \dfrac{1 - \cos^2\theta - \sin^2\theta}{\sin\theta(1 - \cos\theta)}$

$= \dfrac{\sin^2\theta - \sin^2\theta}{\sin\theta(1 - \cos\theta)}$

$= 0$ (using $\sin^2\theta = 1 - \cos^2\theta$)

(b) $\dfrac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}}$

$= \dfrac{\sqrt{3} - \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{3}}{1 + 1} = \dfrac{\sqrt{3} - \frac{\sqrt{3}}{3}}{2}$

$= \dfrac{6\sqrt{3} - 2\sqrt{3}}{3}$ × each term by 6

$= \dfrac{4\sqrt{3}}{3}$