South Sydney High School
Applications of Series & Sequences Assignment
2 Unit - HSC Course

Questions :

19. Find the amount, to the nearest cent, to which a sum of \$7500 will accumulate if it is invested at 9% p.a. interest, compounded annually for 4 years.
20. Find the compound interest on \$5000 invested for 3 years, interest compounded:
    (a) annually at 8% p.a., (b) quarterly at 2% per quarter, (c) monthly at $0.\dot{6}\%$ per month.
21. Find the difference between the compound interest and the simple interest on \$11 000 at 7% p.a. for 5 years.
22. A car is valued at \$21 000. Find its value in 9 years if it is depreciated at 5% p.a.
23. Find the time for a sum of money to double if it is invested at 8.5% p.a. compound interest.
24. A man pays \$2600 into a superannuation fund at the beginning of each year. What is his investment worth at the end of 20 years if interest is compounded annually at 9% p.a.?
25. A woman borrows \$20 000 and agrees to repay it over 7 years. Interest is calculated at 0.8% per month and is charged monthly.
    (a) Show that the amount owing after n months is:
    $A_n = 20\,000(1.008)^n - M(1 + 1.008 + \ldots + 1.008^{n-1})$.
    (b) Show that the amount of each monthly instalment is \$328 to the nearest dollar.
26. Find the limiting sum of the series $25 + 10 + 4 + \ldots$.
27. For what values of x does the geometric series $9 + 15x + 25x^2 + \ldots$ have a limiting sum?
28. Find the first term of a geometric series which has a common ratio of $\frac{2}{3}$ and a limiting sum of $\frac{3}{2}$.
29. Express each recurring decimal as an infinite geometric series and hence as a fraction in simplest form: (a) $0.\dot{7}$, (b) $0.\dot{1}4\dot{8}$, (c) $0.3\dot{9}\dot{6}$.

19. $A_n = P\left(1+\frac{r}{100}\right)^n$

$A_4 = \$7500\left(1+\frac{9}{100}\right)^4$

$= \$10\,586.862$ [by calculator]

$= \$10\,586.86$ to the nearest cent.

20. $A_n = P\left(1+\frac{r}{100}\right)^n$

(a) $P = \$5000,\ r = 8,\ n = 3$

$A_3 = \$5000(1.08)^3$

$= \$6298.56$ [by calculator]

C.I. $= A_n - P$

$= \$6298.56 - \$5000 = \$1298.56$

(b) $P = \$5000,\ r = 2,\ n = 12$

$A_{12} = \$5000(1.02)^{12}$

$= \$6341.209$ [by calculator]

$= \$6341.21$ to the nearest cent.

C.I. $= \$6341.21 - \$5000$

$= \$1341.21$ to the nearest cent.

(c) $P = \$5000,\ r = 0.\dot{6},\ n = 36$

$A_{36} = \$5000\left(1.00\dot{6}\right)^{36}$

$= \$6351.185$ [by calculator]

$= \$6351.19$ to the nearest cent.

C.I. $= \$6351.19 - \$5000$

$= \$1351.19$ to the nearest cent.

21. $P = \$11\,000,\ r = 7,\ n = 5$

$A_n = P\left(1+\frac{r}{100}\right)^n$

$A_5 = \$11\,000(1.07)^5$

$= \$15\,428.068$ [by calculator]

$= \$15\,428.07$ to the nearest cent.

C.I. $= \$15\,428.07 - \$11\,000$

$= \$4428.07$ to the nearest cent.

S.I. $= \$11\,000 \times \frac{7}{100} \times 5 = \$3850.00$

Continued

Difference $= \$4428.07 - \$3850.00$

$= \$578.07,$

$\therefore$ the difference between the compound interest and simple interest is \$578.07.

22. $P = \$21\,000,\ r = -5,\ n = 9$

$A_n = P\left(1+\frac{r}{100}\right)^n$

$A_9 = \$21\,000(1-0.05)^9$

$= \$13\,235.237$ [by calculator]

$= \$13\,235$ to the nearest dollar.

$\therefore$ the value of the car after nine years is \$13 235, to the nearest dollar.

23. $r = 8.5$. The sum will have doubled when $A_n = 2P$.

$A_n = P\left(1+\frac{r}{100}\right)^n$

$2P = P(1.085)^n$

$2 = (1.085)^n$

$\log 2 = \log(1.085)^n$

$= n\log 1.085$

$n = \frac{\log 2}{\log 1.085}$

≈ 8.5

$\therefore$ the sum will have doubled in approximately $8\frac{1}{2}$ years.

24. His first investment earns interest for 20 years.

$A_n = P\left(1+\frac{r}{100}\right)^n$

$A_{20} = \$2600(1.09)^{20}$

His second investment earns interest for 19 years.

$A_{19} = \$2600(1.09)^{19}$

His last investment earns interest for 1 year.

$A_1 = \$2600(1.09)^1$

Total $= \$[2600(1.09) + 2600(1.09)^2 + \ldots$

$\ldots + 2600(1.09)^{19} + 2600(1.09)^{20}]$

This is a geometric series with:

$a = 2600(1.09),\ r = 1.09,\ n = 20$

$S_n = \frac{a\left(r^n - 1\right)}{r-1}$

$= \frac{2600(1.09)\left(1.09^{20}-1\right)}{1.09-1} = 144\,987.78,$

$\therefore$ the value of his investment at the end of 20 years is \$144 987.78.

25. (a) $P = \$20\,000,\ r = 0.8.$

7 years = 84 months

$\therefore\ n = 84.$

If A_n is the amount owing after n months,

$A_1 = 20\,000\left(1+\frac{0.8}{100}\right)^1 - M$

$= 20\,000(1.008) - M$

$A_2 = A_1(1.008) - M$

$= 20\,000(1.008)^2 - M(1.008) - M$

$= 20\,000(1.008)^2 - M(1+1.008)$

$A_3 = A_2(1.008) - M$

$= 20\,000(1.008)^3 - M(1.008 + 1.008^2) - M$

$= 20\,000(1.008)^3 - M(1 + 1.008 + 1.008^2)$

The amount owing after n months will be:

$A_n = 20\,000(1.008)^n - M(1 + 1.008 + \ldots + 1.008^{n-1})$

(b) $A_{84} = 20\,000(1.008)^{84} - M(1 + 1.008 + \ldots + 1.008^{83})$

Now $1 + 1.008 + \ldots + 1.008^{83}$ is a geometric series with: $a = 1$, $r = 1.008$, and $n = 84$.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{84} = \frac{1(1.008^{84} - 1)}{1.008 - 1} = \frac{1.008^{84} - 1}{0.008},$$

$$\therefore A_{84} = 20\,000(1.008)^{84} - M\left(\frac{1.008^{84} - 1}{0.008}\right)$$

If the loan is repaid after 7 years $A_{84} = 0$.

$$0 = 20\,000(1.008)^{84} - M\left(\frac{1.008^{84} - 1}{0.008}\right)$$

$$M = 20\,000(1.008)^{84}\left(\frac{0.008}{1.008^{84} - 1}\right)$$

$= 327.904\,77$ [by calculator]

$= 328$ to the nearest whole number,

$\therefore$ the amount of each monthly instalment, to the nearest dollar, will be \$328.

26. $a = 25$, $r = \frac{10}{25} = \frac{2}{5}$

$$S_\infty = \frac{a}{1 - r}$$

$$= \frac{25}{1 - \frac{2}{5}} = 41\tfrac{2}{3}$$

27. $r = \dfrac{15x}{9} = \dfrac{5x}{3}$. A limiting sum exists when

$-1 < r < 1$

i.e. $-1 < \dfrac{5x}{3} < 1$

$$\frac{-3}{5} < x < \frac{3}{5}$$

28. $r = \dfrac{2}{3}$, $S_\infty = \dfrac{3}{2}$

$$S_\infty = \frac{a}{1 - r}$$

$$\frac{3}{2} = \frac{a}{1 - \frac{2}{3}}$$

$$\frac{3}{2} = \frac{a}{\frac{1}{3}}$$

$3a = 1$

$a = \dfrac{1}{2}$,

$\therefore$ the first term is $\frac{1}{2}$.

29. (a) $0.\dot{7} = 0.777\,777\,77\ldots$

$= 0.7 + 0.07 + 0.007 + \ldots$

An infinite geometric series: $a = 0.7$, $r = 0.1$

$$S_\infty = \frac{a}{1 - r}$$

$$= \frac{0.7}{1 - 0.1} = \frac{0.7}{0.9} = \frac{7}{9}$$

$$0.\dot{7} = \frac{7}{9}$$

(b) $0.\dot{1}4\dot{8} = 0.148\,148\,148\,148\ldots$

$= 0.148 + 0.000\,148 + 0.000\,000\,148 + \ldots$

An infinite geometric series with: $a = 0.148$, $r = 0.001$

$$S_\infty = \frac{a}{1 - r}$$

$$= \frac{0.148}{1 - 0.001} = \frac{0.148}{0.999}$$

$$= \frac{148}{999} = \frac{4}{27}$$

$$0.\dot{1}4\dot{8} = \frac{4}{27}$$

(c) $0.3\dot{9}\dot{6} = 0.396\,969\,696\,96\ldots$

$= 0.3 + 0.096 + 0.000\,96 + \ldots$

This is an infinite geometric series: $a = 0.096$, $r = 0.01$

$$S_\infty = \frac{a}{1 - r}$$

$$= \frac{0.096}{1 - 0.01} = \frac{0.096}{0.99} = \frac{96}{990} = \frac{16}{165},$$

$$\therefore 0.3\dot{9}\dot{6} = 0.3 + \frac{16}{165} = \frac{3}{10} + \frac{16}{165} = \frac{131}{330}.$$